# ALNOOR UNIVERSITY COLLEGE 

# Dept. of Building Engineering and 

Projects Management

Reinforced Concrete (1)

FIRST SEMESTER 2023-2024

# CHATER ONE PROPERTIES OF CONCRETE AND STEEL 

LECTURER<br>Dr. SA'AD A. AL-TA'AN

## CHAPTER ONE

## INTRODUCTION: PROPERTIES OF CONCRETE AND STEEL

### 1.1 Concrete, Reinforced Concrete and Prestressed Concrete

The materials used in most of the structures are, wood, steel, reinforced concrete, precast concrete, and prestressed concrete. Other lightweight materials like aluminum and polymers are used lately. The material that is most widely used in civil engineering structures is reinforced concrete due to its properties as compared with other materials like relatively low cost, moldability, durability, and rigidity.
Reinforced concrete is a composite material composed of concrete that has large compressive strength but small tensile strength, and embedded reinforcing bars in concrete which provide the required tensile strength of the member. The reason that make concrete and steel working effectively are:
i- Bond or the interaction between the reinforcing bars and the surrounding concrete, that prevent slipping or sliding of the two materials relative to each other.
ii- The concrete mixtures after its hardening, has low permeability that prevents rusting of the reinforcing bars.
iii- The coefficient of thermal expansion of concrete is $\left(10-13 \times 10^{-6} / \mathrm{C}^{\circ}\right)$ and $\left(12 \times 10^{-6} / \mathrm{C}^{\circ}\right)$ for steel make the difference between the stresses created due to temperature variation small and can be neglected.
For these reasons, reinforced concrete is suitable for use in ordinary buildings, bridges, tanks, retaining walls, tunnels, conduits, and other structures.

### 1.2 CONCRETE AND REINFORCED CONCRETE

Ordinary concrete is usually made from certain proportions of cement, fine aggregates, coarse aggregates, water and sometimes mineral or chemical admixtures that provide the fresh or hardened concrete with certain properties. After hardening in the forms around the reinforcing bars, it is called reinforced concrete. It is possible to get concrete with different strengths by changing the proportions of the four constituent materials, using certain types of cement, or using certain curing process. The factors that make concrete widely used as a construction material are:
i- Its moldability when it is fresh,
ii- Relative resistance to fire,
iii-Resistance to environmental conditions, and
iv- Availability and cheapness of the constituent materials except cement.

The compression strength of concrete is relatively high as in natural rocks which make it suitable for use in members subjected to compression forces like columns and arches. On the other hand, concrete is a brittle material and has low tensile strength ( $8-20 \%$ of its compression strength) like natural rocks. This will limit its use in structural members subjected completely to tension like ties, or partially as in slabs and beams. To compensate this, steel reinforcement that has high tensile strength compared to concrete can be used to in places subjected to tensile stresses. The steel reinforcement is usually bars with circular section and lugs or protrusions on the external surface to increase the bond between the bars and concrete.

### 1.2.1 CEMENT

The bonding material used to manufacture concrete is called hydraulic cement because it requires water to react with and harden. The materials used to manufacture cement are, limestone and clay. There are many types of cement as shown in Table (1) below.

### 1.2.2 AGGREGATES

The aggregates occupy about $70-80 \%$ of the total volume of concrete, and for this reason it has a great effect on the properties of fresh and hardened concrete. Therefore, the aggregates must have a good strength, durability, and resistance to the environmental conditions. Its external surface should be free from silt, loam, and organic matters that may weaken the bond with the cement paste. It should be ascertained that the aggregates are not reactive which react with the cement and cause expansion of concrete and consequently disintegration.

Aggregates is classified as fine (sand with size $\leq 5.0 \mathrm{~mm}$ ) and coarse with size > 5.0 mm like gravel or any other crushed stones. Maximum aggregate size is limited by ACI Code (26.4.2) to:
i- One fifth of the least distance of the lateral forms (columns, walls, and beams web),
ii- One third the slab depth, and
iii- Three quarter the clear spacing between bars.
Concrete made with gravel or crushed stones result in a density of $23 \mathrm{kN} / \mathrm{m}^{3}$, and when reinforced with steel gives a density of $\approx 24 \mathrm{kN} / \mathrm{m}^{3}$.

There is also lightweight concrete which is made from natural or manufactured lightweight aggregates that result in densities ranging from 8 to $19 \mathrm{kN} / \mathrm{m}^{3}$.

Table (1) Types of Cements

| Designation | Cement type | Uses |
| :--- | :--- | :--- |
| Type I | Normal | Pavements, floors, reinforced concrete buildings, bridges, <br> tanks, reservoirs, pipe, masonry units, and precast concrete <br> products. |
| Type IA | Normal, air- <br> entraining | As Type I and produce concrete with improved resistance to <br> freezing and thawing |
| Type II | Moderate <br> sulfate <br> resistance | Protection against moderate sulfate attack is necessary |
| Type IIA | Moderate <br> sulfate <br> resistance, air <br> entraining | As Type II, improved resistance to freezing and thawing |
| Type <br> (MH) | Moderate heat <br> of hydration and <br> moderate sulfate <br> resistance | Large piers, large foundations, and thick retaining walls |
| Type <br> (MH)A | Moderate heat <br> of hydration, <br> moderate sulfate <br> resistance, air <br> entraining | As Type II (MH) and produce concrete with improved <br> resistance to freezing and thawing |
| Type III | High early <br> strength | Structure must be put into service quickly, In cold weather, <br> reduction in the length of the curing period |
| Type IIIA | High early <br> strength, air- <br> entraining | As Type III, improved resistance to freezing and thawing |
| Type IV | Low heat of <br> hydration | Massive concrete structures, such as large gravity dams |
| Type V | High sulfate <br> resistance | Concrete exposed to severe sulfate environments - <br> principally where soils or ground waters have a high sulfate <br> content |

### 1.2.3 WATER

Water is one the important constituent of concrete, without it the hydration and hardening processes will not occur. Mixing and curing water should be clean, free from acids, alkalis, salts, organic matter, or any other material that may affect concrete or steel. In general, the drinking water is suitable for use in mixing and curing.

Water cement ratio is one the most important factor that affects the concrete strength, Fig. (1.1). The amounts of mixing water is more than that necessary for the hydration process by at least $10 \%$ so that the fresh concrete mix has an acceptable workability and take the shape of the formwork. Curing of concrete
is necessary to substitute or prevent the mixing water from evaporation so that the hydration process will continue.


Figure (1.1) Effect of water cement ratio on the flexural and compression strength of concrete

### 1.2.4 ADMIXTURES

Admixtures are either:
a. Chemical or
b. Mineral materials.

It is added to the four concrete constituents before or after mixing to change or improve some of the concrete properties during the fresh or hardened state. The effects of admixtures can be summarized as follow:
i- Improve the workability without increasing water, or decrease water and keep the same workability,
ii- Retard the initial setting process, and therefore decreasing the heat resulting from the hydration process. This is necessary also when casting in hot weather.
iii- Accelerating the strength development when casting in cold weather, and calcium chloride is usually used.
iv- Increase the compression strength, like the silica fume.
v - Increase the resistance of concrete to freezing and thawing, where the concrete member may be subjected to frequent freezing and thawing. This is achieved by either using air entrained cement or some chemicals that create air voids inside the concrete mass.

### 1.3 WORKABILITY

To get a concrete with good quality, adequate amount of water should be used to make the fresh concrete consistent, easy to finish, moldable, and surround the reinforcing bars. If excessive amount of water is added, the concrete becomes porous, weak, and exhibit high shrinkage. There are many factors that affect the workability of concrete:
i- Amount of water,
ii- Aggregate to cement ratio,
iii-Coarse aggregate to fine aggregate ratio, and
iv-Maximum aggregate size.
There are many methods for measuring the workability of concrete indirectly:
i- Slump test,
ii- Compacting factor test,
iii-Kelly ball test, and
iv- V-B test.

### 1.4 QUALITY CONTROL FOR CONCRETE

As mentioned before, concrete is composed of four constituents and its properties in the fresh and hardened state depend on the quality and quantity of these constituents in addition to the curing regime after casting.
Before any concrete member (slabs, beams, columns, and foundation) a concrete compression strength must be specified. This strength is that of a concrete cylinder ( 150 mm in diameter and 300 mm in height) at the age of 28 days after casting. A trial mix design is usually conducted in the laboratory to get the specified compression strength by carefully selecting the proportions of the four concrete constituents. In the laboratory, the used materials are very small and their quality, gradation, weights, and the curing regime (temperature and humidity) can be controlled not like the he large quantities in the site. Therefore, it is expected that the quality and strength of the concrete in the site is less than those in the laboratory. For these reasons, there should be a quality control measures for the quality and quantities of the used materials, mixing method, transportation method, casting, compaction, and curing regime so that the specified compression strength can be obtained.
The ACI Code (26.12.1.1), recommends that a strength test (represent the average of (two concrete cylinders $150 \times 300 \mathrm{~mm}$ ) or (three cylinders $100 \times 200$ mm ) must be conducted for the following cases:
i- For a certain type of concrete once a day for each $155 \mathrm{~m}^{3}$ of concrete or once for each $450 \mathrm{~m}^{2}$ of slabs and walls.
ii- In any project, if the number of tests is less than five, tests should be conducted on five random concrete batches or five tests if the number of batches is less than five.
iii- If the volume of concrete is less $40 \mathrm{~m}^{3}$, there is no need to conduct any test, if the supervision authority proves that a satisfactory strength can be obtained according to ACI Code (2.12.26).

The ACI Code (3.12.26) considered that the concrete quality is acceptable in the following cases:
i- If the average of three consecutive tests equal to or greater than $f_{c}^{\prime}$,
ii- Any compression strength test (average of two cylinders) is not less than the required compression test by no more than 3.5 MPa if the compression strength is not greater than 35 MPa , or $0.1 f_{c}^{\prime}$ if the compression strength is greater than 35 MPa .

### 1.5 MECHANICAL PROPERTIES OF CONCRETE

Since concrete is composed of four constituents, its properties depend on the proportions and properties of these four constituents mainly the water/cement ratio. There are many methods for designing concrete mixtures of certain slump and strength and can be referred to in the concrete technology textbooks.

### 1.5.1 Compression Strength

In the last decades, it was possible to make concrete with compression strength up to 100 MPa or even more. However, such concrete has many limitations on its use. In ordinary reinforced concrete buildings, the compression strength may range from 20 to 40 MPa . In prestressed and precast concrete, higher strength is required and may range from 30 to 60 MPa . In multistory buildings, It is preferable to use concrete of high strength in the columns.

Compression strength usually measured by means of cylinders or cubes, and the cylinder will be used in this course as per the ACI Code.

The compression strength of a cylinder is that for a ( $150 \times 300 \mathrm{~mm}$ ) 28 days after casting under a certain rate of loading. The ACI Code (19.2.1.1) limited the strength of concrete according to the type of building as shown in the Table below.

Table (2) Concrete strength for various Buildings and Types of Concrete

| Type of building | Concrete | $f_{c}^{\prime}$ |  |
| :--- | :---: | :---: | :---: |
|  |  | Minimum strength | Maximum strength |
| General | Normal and <br> lightweight | 17 | No limit |
| Moment resisting | Normal weight | 17 | No limit |


| frames and some <br> structural walls | Lightweight | 17 | $35^{(1)}$ |
| :--- | :---: | :---: | :---: |

${ }^{(1)}$ Higher compression strength may be used, if it is demonstrated that the lightweight members have strength and rigidity equal to or more than those constructed using normal weight concrete of the same strength.
The behavior of concrete and the stress-strain relationship depends on the compression strength, age, rate of loading, cement and aggregates properties, and the type and size of the tested specimens. Figure (1.2), shows some stressstrain relationships of concrete with various strength tested concentrically at the age of 28 days.

It can be noticed from the Figure that the peak strength is attained at a strain of 0.002 to 0.0025 for normal concrete, and the maximum strain range from 0.003 to 0.008 . The ACI Code (22.2.2.1) limit the maximum strain by 0.003 .

Poisson's ratio, range from 0.11 for high strength concrete to 0.21 for low strength concrete.

### 1.5.2. Modulus of Elasticity

The modulus of elasticity of concrete depends on the compression strength, cement and aggregates properties, rate of loading, size and type of the tested specimen. Figure (1.3) shows a typical stress-strain curve for concrete. The figure shows the initial modulus, secant modulus, and tangent modulus. The secant modulus represents the slope of the straight line from the origin to about $50 \%$ of the compression strength and usually used as a modulus of elasticity for concrete according to the ACI Code. Paragraph (19.2.2.1) of the ACI Code, define the modulus of elasticity for concrete with density ranging from 1450 to $2550 \mathrm{~kg} / \mathrm{m}^{3}$ as follow:
$E_{c}=0.043 w_{c}^{1.5} \sqrt{f_{c}^{\prime}}$
Where $E_{c}$ and $f_{c}^{\prime}$ are in MPa , and $w_{c}$ is the concrete density in $\mathrm{kg} / \mathrm{m}^{3}$. For normal weight concrete whose density about $2300 \mathrm{~kg} / \mathrm{m}^{3}$, Equation (1.1) becomes:

$$
\begin{equation*}
E_{c}=4750 \sqrt{f_{c}^{\prime}} \tag{1.2}
\end{equation*}
$$



Figure (1.2) Typical stress-strain curves for normal concrete tested concentrically


Figure (1.3) Definition of the Elastic

It was found that the above equations give modulus of elasticity values more than the true values by around $29 \%$ when the compression strength values ranges from 40 to 80 MPa for normal and lightweight concrete. The following equation gives a fair estimation of elastic modulus for normal concrete with compression strength between 20 to 80 MPa and lightweight concrete with compression strength between 20 to 60 MPa :

$$
\begin{equation*}
E_{c}=\left(3320 \sqrt{f_{c}^{\prime}}+6895\right)\left(w_{c} / 2300\right)^{1.5} \tag{1.3}
\end{equation*}
$$

### 1.5.3 Tensile Strength

Tensile stresses are created in reinforced concrete members due to the shear forces, bending moments, and torsional moments. The tensile strength of concrete is very small ( $<20 \%$ ) of its compression strength. This low tensile strength causes cracks initiation even in the working conditions and causes redistribution of the stresses and internal forces.

There are three methods to measure the tensile strength of concrete:
i- Direct tensile strength,
ii- $\quad$ Splitting tensile strength (Brazilian test), and
iii- Flexural strength.
The following Table show the range of the three tensile strength tests.
Table (3) Range of Tensile Strength of Concrete for different tests

| Type of strength | Normal Concrete | Lightweight Concrete |
| :--- | :---: | :---: |
| Direct tensile strength | $0.25-0.4 \sqrt{ } f_{c}^{\prime}$ | $0.17-0.25 \sqrt{ } f_{c}^{\prime}$ |
| Splitting strength | $0.50-0.67 \sqrt{c} f_{c}^{\prime}$ | $0.33-0.50$ |
| Flexural strength | $0.67-1.0 \sqrt{ } f_{c}^{\prime}$ | $0.50-0.67 \sqrt{ } f_{c}^{\prime}$ |

### 1.5.4 Creep of Concrete

Creep is the increase in strain under sustained stress; these strains are inelastic or plastic strains and increase with time at a reducing rate. The mechanism of creep or this plastic flow may be due to one or more of the following:
i- Crystal flow of the aggregates and the hardened cement paste,
ii- Plastic flow of the hardened cement paste surrounding the aggregates,
iii- Decrease of the voids or pores in the concrete mass, and
iv- Water flow from the cement paste due to stresses and evaporation.
The factors that increase the creep of concrete, are; (a) increase of the water/cement ratio, (b) increase in temperature and decrease of humidity, (c) loading the member in an early age, (d) increase of the load duration, (e) increase in stresses, (f) decrease in the volume/surface ratio of the member, and (g) type of cement and aggregates.

Creep does not decrease the capacity of strength of the reinforced concrete members during the service conditions, but causes redistribution of stresses in concrete and steel. The followings are the effects of creep on reinforced concrete members:
i- Increase in the long-term deflection of flexural members (slabs and beams),
ii- In reinforced concrete columns, it causes increase of the stresses on steel and decrease of the stresses on concrete, and
iii- In prestressed members, it causes reduction of the prestressing forces with time.


Figure 1.4: Deformation in a loaded concrete cylinder: (a) specimen unloaded, (b) elastic deformation, (c) elastic plus creep deformation, and (d) permanent deformation after release of load.

### 1.5.5 SHRINKAGE OF CONCRETE

As mentioned before, to get a workable fresh concrete the amount of mixing water should be more than that necessary for the hydration process. After the concrete cast, the excess water (not hydrated with cement) starts evaporating, resulting in shrinkage of concrete. This shrinkage could be attributed to the capillary action of water remaining in the pores.
The concrete shrinkage increases with, (a) water / cement ratio, (b) cement content. (c) temperature and decrease with humidity, (d) surface / volume ratio, and (e) aggregates porosity. Figure (1.4) shows some shrinkage time relationships.
Shrinkage creates mostly compressive stresses in steel and tensile stresses in concrete leading to cracking. Shrinkage causes increase in long time deflection in flexural members, which decrease with the presence of steel, it causes also reduction in the prestressing forces in prestressed concrete members.


Figure (1.5) Variation of shrinkage strains with time for normal and lightweight concrete

### 1.5.6. TEMPERATURE CHANGE

Concrete like other materials, expands with temperature and contract with the temperature decrease. This temperature change creates internal stresses (tension and compression) depending upon the temperature gradient between the top and bottom surface. The treatment of temperature change is similar to that of shrinkage. The designer should take these stresses into consideration, such as providing expansion or contraction joints to decrease the internal stresses.

### 1.6 REINFORCING STEEL

The axial forces (tension and compression), shear forces, bending moments, and torsional moments create tensile stresses in concrete members; since the tensile strength of concrete is small compared with compression it will limit the strength of concrete members. When steel with higher tensile strength (100 times than the tensile strength of concrete) is added to concrete and high ductility, the reinforced concrete member becomes stiffer and more ductile than the unreinforced member and capable of resisting high tensile stresses. In reinforced concrete members, concrete in the compression zone carries the compression stresses and steel in the tension zone carries the tensile stresses. Steel reinforcement, are used also in compression zones of columns and beams to carry compression stresses and in beams (in the form of stirrups) to carry diagonal tensile stresses created by shear and torsion that lead to shear failure.
The reinforcing steel are present in the form:
i- Bars that are used in all reinforced concrete members,
ii- Wire fabrics that are used in flat, curved, and folded surfaces,
iii- Wires that are used in prestressing members, and
iv- Strands that are used in prestressing members.
The ordinary reinforcing bars are usually deformed with lugs or protrusions to increase the bond and decrease the slippage with the surrounding concrete.
Bars diameter usually range from 6 to 57 mm , Figure (1.6).


Figure (1.6) Typical reinforcing bars


Figure (1.7) Typical wire fabrics


Figure (1.8) Typical seven wire strands

### 1.6.1 STRESS-STRAIN CURVES

The factors that determine the properties of the reinforcing bars are:
i- $\quad$ Yield strength $f_{y}$,
ii- Ultimate strength $f_{u}$, and
iii- Elastic modulus $E_{s}$.
The elastic modulus is usually constant for all the reinforcing bars, Figure (1.7) and equal to $200 \mathrm{GPa}(200000 \mathrm{MPa}$ ), as in ACI Code (20.2.2.2). The prestresssing steel has elastic modulus slightly less than that for the ordinary reinforcing steel.

The yield strength of the reinforcing bars is limited to 520 MPa for flexural members (slabs and beams) and 400 MPa for bars used shear and torsion.


Figure (1.9) Typical stress-strain curves of reinforcing steel, (a) complete curves, (b) first part enlarged 10 times


# ALNOOR UNIVERSITY COLLEGE 

# Dept. of Building Engineering and 

Projects Management Third Year

## Reinforced Concrete (I)

FIRST SEMESTER 2023-2024

Chapter Two<br>Design Methods and Requirements

LECTURER

Dr. SA'AD A. AL-TA'AN

## CHAPTER TWO <br> DESIGN METHODS AND REQUIREMENTS

### 2.1 INTRODUCTION

Reinforced concrete is a composite, nonhomogeneous material and the properties and behavior of the two materials (concrete and steel) differs from each other completely. Concrete is a nonhomogeneous material strong in compression with nonlinear behavior, weak in tension, cracked during the service conditions, shrink due to evaporation of water, and creep when subjected to sustained load. While the reinforcing steel, is a homogeneous material with a high compression and tensile strength and high ductility also. Therefore, it is not possible to use the methods and formulas used in mechanics of materials (for homogeneous materials) to find the strains, stresses, and deformation in the elastic or plastic stages, since reinforced concrete is not homogeneous, elastic, or completely plastic even when the external loads are small. For these reasons, most of the analysis and design methods used used for reinforced concrete members are empirical, i.e.; methods and formulas based on experimental reslts or results demonstrated with experience and time. These formulas, methods, and specifications are usually gathered in a code revised from time to time when theoretical or experimental results are available. This code contains also minimum and maximum limits for some factors used in the design related to the behavior and safety of the reinforced concrete members or the the structure as a whole.

To design a cross-section, reinforced concrete member, or structure the nature and magnitude of the acting loads and its influence on the structure should be determined. These effects, may be a shear force, bending moments, torsional moments, and axial compressive or tensile forces. The methods of structural analysis are usually used to determine these internal forces.

After this stage, the structural design is begin which include choosing a suitable cross-sections and finding the necessary area of steel.

### 2.2 LOADS

The loads acting on structures in general, can be classified into three categories, dead, live, and environmental loads.

### 2.2.1 DEAD LOADS

Sometimes called static or constant loads represent the weight of the structural member or any other load acting on the structure permanently. It is so called, since its magnitude and direction of action is constant. It represents also, the finishing loads like, plastering, tiles, cement mortar under the tiles, and any other permanent electrical or mechanical fixtures. The magnitude of these loads can be determined after a preliminary design when all the cross-sections and
dimensions of the members are assumed. After analyzing the structure and determining the internal forces, these assumed dimensions can be fixed or revised depending on the structural design.

### 2.2.2 LIVE LOADS

As the name implies, these are the loads acting on the structure and represent the occupants of the structure, in roads and bridges it represent the traffic loads. It is called live since its magnitude, duration, and direction of action is variable. Its magnitude and place should be chosen to create maximum effects on the structure; these effects could be shearing force, bending moment, torsional moment, and axial forces individually or in combination.
Values of these loads are usually recommended in codes of practice.

### 2.2.3. ENVIRONMENTAL LOADS

It represents the environmental effects such as, wind pressure, snow and rain loads, earth pressure, hydrostatic pressure, forces created by temperature change, force created by shrinkage and creep of concrete, forces created by differential settlement, and seismic forces.

### 2.3. ELASTIC AND STRENGTH DESIGN METHODS

There are two methods for designing reinforced concrete members. The elastic or working stress design method (working or service conditions) was used since 1900 to the late 1950's. After that the strength design method (ultimate conditions) was developed.

In the elastic design method, the member is designed for the working or service conditions under the effect of working or service loads (true value of the loads). In this stage, the stresses in concrete and steel should not exceed certain allowable limits (about 40-45\% of their strength) so that the deformations at this stage are small and the structure is serviceable. The methods used for the analysis and design are those used in mechanics of materials for elastic homogeneous materials.

Using this method does not give reliable results for the following reasons:
i- $\quad$ The magnitude and distribution of the stresses created due to shrinkage cannot be known accurately,
ii- With time creep occur and result in redistribution of stresses in concrete and steel,
iii- Since the stress-strain relationship of concrete is not linear, it is not possible to determine the factor of safety between the ultimate (failure) and working conditions, and
iv- Since the limitation is on the concrete and steel stresses, it is not possible to determine the difference between the applied loads.

The second method is the strength design method. In this method, the working (service) loads are increased by a factors to get the ultimate or factored loads.

The dimensions of the cross-sections are chosen so that its design strength equal to or greater than the required strength obtained from the structural analysis. When calculating the design strength, the strength of the materials are assumed equal and $f_{y}$ for steel in tension and compression, and to $f_{c}^{\prime}$ for concrete in compression and the nonlinear relationship between stress and strain in concrete should be taken into account.
Since 1971, the majority of the ACI Code was devoted to the strength design method, and the working stress design method can be used as alternative method.
Even when the strength design method is used, the conditions of the members comprising the structure must be checked against the limiting serviceability criteria, like deflection, cracking, and vibration. These limitations take into account the aesthetic, functional aspects of the members, and comfort to the occupants.

### 2.4. SAFETY PROVISIONS

All the structural members or structures must be designed to resist loads more than that expected in the normal conditions. The reason is that there are many factors that let the designer takes into account the increase in the design loads that may occur. These factors can be summarized as follow:
i- The true value of the loads may differ from that used in the design,
ii- The true loads distribution may differ from that used in the design,
iii- The magnitude and distribution of the loads acting on the structure during construction, like construction materials, and forms cannot be estimated accurately,
iv- Changing the function of part or the whole structure may differ from the originally assumed loads,
v- The unavoidable assumptions and simplifications used for the analysis of the structure may give effects of the actual loads differ from the actual effects, and
vi- The true behavior of the structure may differ from that assumed during analysis and design.

Based on these factors, all the codes of practice recommended what is called load safety factor which is usually greater than 1.0 . These factors represent the ratio of the ultimate (factored) load to the working (service) load, and this factor differ with the type of load (dead, live, wind, etc.). The value of these load factors depends on statistical analysis and to some extent on experience.

### 2.4.1 ACI CODE LOAD FACTORS OF SAFETY

The ACI Code load factors are used for working (service) loads (dead, live, and environmental). After analyzing a structure under the action of these factored (ultimate) loads, the internal forces or ( $\mathrm{U}=$ required strength) can be found which may include, $V_{u}$ shear force, $P_{u}$ axial force, $M_{u}$ bending moment, and $T_{u}$ torsional moment. The subscript $(\mathrm{u})$ is used to denote that the action is ultimate or (factored).

Table (2.1) shows the load factors, it can be noted that the loads that can be estimated accurately like dead loads has load factors less than those with more variation like live load.

Using these factors means that there is a probability of $(1 / 1000)$ that the true ultimate loads can exceed the calculated ultimate loads.

Table (2.1) Summation of the factored loads to determine the required strength (U) according to the ACI Code

| Load state | Factored load |
| :--- | :--- |
| Dead load | $\mathrm{U}=1.4 \mathrm{D}$ |
| Basic state | $\mathrm{U}=1.2 \mathrm{D}+1.6 \mathrm{~L}$ |
| Dead load and fluid pressure | $\mathrm{U}=1.4 \mathrm{D}+1.4 \mathrm{~F}$ |
| Dear load, fluid pressure, temperature change, <br> creep, shrinkage, and differential settlement | $\mathrm{U}=1.2(\mathrm{D}+\mathrm{F}+\mathrm{T})+1.6(\mathrm{~L}+\mathrm{H})+$ <br> $0.5\left(\mathrm{~L}_{\mathrm{r}}\right.$ or S or R$)$ |
| Dead load, live load, snow load, or rain and <br> wind load | $\mathrm{U}=1.2 \mathrm{D}+1.6\left(\mathrm{~L}_{\mathrm{r}}\right.$ or S or R$)$ <br> $+(1.0 \mathrm{~L}$ or 0.8 W$)$ |
| Dead load, wind load, live load, snow or rain | $\mathrm{U}=1.2 \mathrm{D}+1.6 \mathrm{~W}+1.0 \mathrm{~L}+$ <br> $0.5\left(\mathrm{~L}_{\mathrm{r}}\right.$ or S or R$)$ |
| Dead load, live load, and seismic load or snow | $\mathrm{U}=1.2 \mathrm{D}+1.0 \mathrm{E}+1.0 \mathrm{~L}+$ <br> 0.2 S |
| Dead load, wind load, and earth pressure | $\mathrm{U}=0.9 \mathrm{D}+1.6 \mathrm{~W}+1.6 \mathrm{H}$ |
| Dead load, seismic load, and earth pressure | $\mathrm{U}=0.9 \mathrm{D}+1.0 \mathrm{E}+1.6 \mathrm{H}$ |

$\mathrm{D}=$ Dead loads, $\mathrm{E}=$ Load effects of seismic forces, $\mathrm{F}=$ Loads due to weight and pressures of fluids, $\mathrm{H}=$ Loads due to weight and pressure of soil, $\mathrm{L}=$ Live loads, $\mathrm{L}_{\mathrm{r}}=$ Roof live loads, $\mathrm{R}=$ Rain loads, $\mathrm{S}=$ snow loads, $\mathrm{T}=$ cumulative effect of temperature, creep, shrinkage, differential settlement, and shrinkage.

## Example 6:

The frame in Example (1) is subjected to a uniformly distributed working dead and live loads equal to $15 \mathrm{kN} / \mathrm{m}$ and $20 \mathrm{kN} / \mathrm{m}$ respectively and a concentrated working wind load of 25 kN at point (f). Find the ultimate load and moment that should be carried by one of the side columns.

## Solution:

According to the ACI Code, the frame must be analyzed for four loads combinations (vertical and horizontal loads):
(a) $1.4 D L$
(b) $1.2 D L+1.6 L L$
© $1.2 D L+1.6 W L+1.0 L L$
(d) $0.9 \mathrm{DL}+1.6 \mathrm{WL}$

Figure (a) below shows the factored loads, reactions and the moments created from the vertical DL only.

(a) 1.4 DL

Figure (b) below shows the factored loads, reactions and the moments created from the vertical DL and LL.

(b) $1.2 \mathrm{DL}+1.6 \mathrm{LL}$

Figure (c) below shows the factored wind load, reactions and the moments created from the wind load.


Figure © below shows the load combination (1.2DL+1.6WL+1.0LL) and reactions and the moments created:


Figure (d) below shows the load combination (0.9DL+1.6WL) and reactions and the moments created:
$\mathbf{W}_{\mathrm{u}}=13.5 \mathrm{kN} / \mathrm{m}$


$$
\mathrm{d}=0.9 \mathrm{DL}+1.6 \mathrm{WL}
$$

Therefore, the column must be designed three times:
1- For a factored axial load and moment of 142.7 kN and 35.5 kN .m respectively, and
2- For a factored axial load and moment of 118.174 kN and $76.3 \mathrm{kN} . \mathrm{m}$ respectively.
3- For a factored axial load and moment of 48.25 kN and $46.34 \mathrm{kN} . \mathrm{m}$ respectively.

### 2.4.2. STRENGTH REDUCTION FACTORS

The dimensions of the structural members are chosen so that their design strength $\left(\varphi S_{n}\right)$ equal to or greater than the required strength (U). The design strength is a reduced value of the nominal (ideal or theoretical strength) $\left(\mathrm{S}_{\mathrm{n}}\right)$.
The nominal strength is calculated according to the assumptions and requirements recommended by the ACI Code and subscripted by ( n ).
The factors that make the design strength less than the nominal strength are as follow:
i- The true strength of the materials especially concrete may be less than that adopted in the design if the quality control measure is poor.
ii- The dimensions of the constructed structural members may differ from that fixed by the designer or that fixed on the structural drawings, and this depend on the site supervision.
iii- The reinforcing bars may not be placed in the right positions.
iv- Accuracy of the design calculations and assumptions, and empirical equations used may not give the true value of the strength.

Therefore, if any structural member is subjected to shear force, bending moment, axial forces, or torsional moment, the design strength must equal to or greater than the required strength as follow:

$$
\begin{aligned}
\varphi V_{n} & \geq V_{u} \\
\varphi M_{n} & \geq M_{u} \\
\varphi P_{n} & \geq P_{u} \\
\varphi T_{n} & \geq T_{u}
\end{aligned}
$$

The left hand side represent the design strength and the right hand side the required strength which is found from the structural analysis under the action of the factored loads, $\phi$ is the strength reduction factor which is usually less than 1.0. There are other factors in addition to those mentioned above that effect the value of $\phi$, these are:
i- Type of expected failure, whether in concrete or steel,
ii- Importance of the member (column, beam, slab, etc.), and
iii- Type of the building (school, warehouse, residential building, etc.).
Table (2.2) shows the strength reduction factors as recommended by the ACI Code.
Using these factors give a probability of $(1 / 100)$ that the strength of a member is less the design strength. Therefore failure, whether due to increase in loads or strength reduction may occur with a probability of;

$$
\frac{1}{100} \times \frac{1}{1000}=\frac{1}{100000}
$$

Table (2.2) Strength reduction factors according to ACI Code

| Type of strength | $\phi$ عامل تخفيض المقاومة |
| :--- | :---: |
| Members where tension is controlled | 0.9 |
| Members where compression is controlled, <br> members with spiral reinforcement, <br> Other members | 0.75 |
| Shear or shear and torsion | 0.65 |
| Bearing on concrete | 0.75 |
| Bending, compression, shear, and loading on <br> plain concrete | 0.65 |

### 2.5. DUCTILITY IN REINFORCED CONCRETE BUILDINGS

Ductility in reinforced concrete buildings is one of the safety measures that should be provided. Ductility means, maintaining strength while sizeable deformation occurs, (deflection, curvature, rotation, cracks, etc.) the failure occurs slowly and gradually like rubber and steel. On the contrary, brittle materials fail suddenly with small deformation before failure, like rock and concrete, i.e.; failure occurs without warning.

Ductility is considered important for many reasons:
i- In statically indeterminate structures, ductility allows stressed parts to retain its strength in spite of the large deformation, and let parts with
smaller stresses take additional loads. This means that failure of one section or member does not lead to failure of the whole structure.
ii- Structures located in seismic areas, or may be subjected to explosion, the structure should have a capacity to absorb energy by providing the members with reasonable ductility.
iii- Beams and slabs with large ductility, will give warnings before the member reach its ultimate or failure stage. These warnings could be wide cracks, large deflection, and curvatures and rotation which lead to evacuation of the building or reducing the applied loads.


## ALNOOR UNIVERSITY COLLEGE

## Dept. of Building Engineering and Projects Management

Reinforced Concrete (I)
FIRST SEMESTER 2023-2024

# Chapter Three WORKING STRESS DESIGN METHOD 

(ELASTIC DESIGN)<br>LECTURER Dr. SA'AD A. AL-TA'AN

# CHAPTER THREE Analysis by the Working Stress Design Method (Elastic design method) 

### 3.1 Introduction

In chapter two, it was mentioned that there are two methods for analysis and design of reinforced concrete structures; the working stress design and strength design method. Whichever method is used, the structural member or the structure should be:
i. Durable, i.e.; sustain the environmental conditions without reduction in strength or needs maintenance frequently,
ii. Have adequate strength when subjected to overloads (ultimate loads), and
iii. Serviceable, the deflection and cracks are within the limits set by the codes of practice.

In this chapter, a brief description of the working stress design method will be presented.

### 3.2. Fundamental Assumptions

The working stress design method (straight line or elastic design method) for flexural members (slabs and beams) depends on four basic assumptions:
i. Plane section before bending remains plane after bending, which means that the strains are proportional to the distance from the neutral axis. This assumption will be used also in the strength design method.
ii. Stresses and strains are elastic and proportional to each other in concrete and steel. For concrete this assumption is valid up to about $50 \%$ of $f_{c}^{\prime}$ and for steel up to the yielding point.
iii. Tensile strength of concrete is neglected if cracks initiate at the tension face. This assumption will be used also in the strength design method.
iv. There is a perfect bond between the reinforcing bars and the surrounding concrete. There is no slip between concrete and steel and the strains in both are equal. This assumption will be used also in the strength design method.

### 3.3. Stresses in Reinforced Concrete Members

### 3.3.1 Axial Compression in the Elastic State

Consider the reinforced concrete column in Fig. (3.1) from the fourth assumption, the strain at any point in concrete $\epsilon_{\mathrm{c}}=f_{c} / E_{c}$ equal to the strain in steel $\epsilon_{\mathrm{s}}=f_{s} / E_{s}$,


Figure (3.1) Reinforced Concrete Column subjected to Axial Load

$$
\begin{equation*}
\frac{f_{c}}{E_{c}}=\frac{f_{s}}{E_{s}} \tag{3.1}
\end{equation*}
$$

$f_{c}$ and $f_{s}$ are the stresses in concrete and steel respectively, Eq.(3.1) can be rewritten as follow:

$$
\begin{equation*}
f_{s}=n . f_{c} \tag{3.2}
\end{equation*}
$$

n is called the modular ratio $=E_{s} / E_{c}$ (usually rounded to the nearest integer).
Consider the column of Fig. (3.1) subjected to a load (P), this load will be shared by concrete $\left(\mathrm{P}_{\mathrm{c}}\right)$ and steel $\left(\mathrm{P}_{\mathrm{s}}\right)$. The load carried by concrete equal to:

$$
\begin{equation*}
P_{c}=f_{c} A_{c}=f_{c}\left(A_{g}-A_{s t}\right) \tag{3.3}
\end{equation*}
$$

Where $f_{c}$ is the concrete stress, and $A_{c}$ is the net concrete area and equal to:

$$
\begin{equation*}
A_{c}=A_{g}-A_{s t} \tag{3.4}
\end{equation*}
$$

$A_{g}=$ gross concrete area $(\mathrm{b} \times \mathrm{h})$, and $A_{s t}=$ total area of steel. The load carried by the steel reinforcement equal to:

$$
\begin{equation*}
P_{s}=f_{s} A_{s t} \tag{3.5}
\end{equation*}
$$

Adding equations (3.3) and (3.5) to get the total load carried by the column:

$$
\begin{equation*}
P=f_{c} A_{c}+f_{s} A_{s t} \tag{3.6}
\end{equation*}
$$

Substituting $\left(f_{s}\right)$ from Equation (3.2) into Equation (3.6), the following Equation is obtained:

$$
\begin{equation*}
P=f_{c} A_{c}+n f_{c} A_{s t}=f_{c}\left(A_{c}+n A_{s t}\right)=f_{c}\left(A_{g}-A_{s t}+n A_{s t}\right)=f_{c}\left[A_{g}+(n-1) A_{s t}\right]=f_{c} A_{t} \tag{3.7}
\end{equation*}
$$

Where $A_{t}$ is the transformed area and equal to:

$$
\begin{equation*}
A_{t}=A_{c}+n A_{s t}=A_{g}+(n-1) A_{s t} \tag{3.8}
\end{equation*}
$$

The transformed area $A_{t}$ can be interpreted as the area of a fictitious concrete cross section, Fig. (3.2).


Actual section
(a)


Transformed section $A_{t}=A_{c}+n A_{s t}$
(b)


Transformed section $A_{t}=A_{g}+(n-1) A_{s t}$
(c)

Figure (3.2) Gross and transformed are, (a) reinforced concrete section, (b) transformed section, (c) transformed section

## Example 3.1

A reinforced concrete column $350 \times 450 \mathrm{~mm}$ reinforced with four bars ( $\$=25$ mm ) and subjected to a compressive axial load of $\mathrm{P}=1.5 \mathrm{MN} . f_{c}^{\prime}=30 \mathrm{MPa}$ and $f_{y}=420 \mathrm{MPa}$. Calculate the stresses and strains in concrete and steel.

## Solution

$$
\begin{aligned}
& E_{c}=4750 \sqrt{30}=26017 \mathrm{MPa} \\
& =n=\frac{200000}{26017}=7.69 \approx 8 \\
& A_{s t}=4 \times 491=1964 \mathrm{~mm}^{2} \\
& A_{c}=350 \times 450-1964=155536 \mathrm{~mm}^{2}
\end{aligned}
$$



$$
A_{t}=155536+8 \times 1964=171248 \mathrm{~mm}^{2}=0.171248 \mathrm{~m}^{2}
$$

Using Equation (3.7) to calculate the concrete stress:
$f_{c}=P / A_{t}=1.5 / 0.171248=8.76 M P a$
$\varepsilon_{c}=f_{c} / E_{c}=8.7625907=0.000338 \ll 0.002$
$P_{c}=A_{c} f_{c}=0.155536 \times 8.76=1.362 M N \quad(90.8 \%)$
$f_{s}=n f_{c}=8 \times 8.76=70.1 \mathrm{MPa}$
$\varepsilon_{s}=f_{s} / E_{S}=70.1 / 200000=0.000351<\varepsilon_{y}=f_{y} / E_{S}=400 / 200000=0.002$
$P_{S}=A_{s t} f_{s}=0.001964 \times 70.1=0.138 M N \quad(9.2 \%)$

## Example 3.2

For Example 3.1 calculate the load and the steel stress that can be carried by the column if the concrete stress $=13.5 \mathrm{MPa}$.

## Solution

$P=f_{c} A_{t}=13.5 \times 0.171248=2.311 \mathrm{MN}$
$f_{s}=n f_{c}=8 \times 13.5=108 M P a$

### 3.3.2 Axial Compression in the Ultimate State

Strength is the maximum load that the structure or member will carry. When the load on the column increases; the stresses and strains in concrete and steel will increase also. When the concrete reach a strain of 0.003 (stress $=f_{c}^{\prime}$ ), crushing of concrete will occur and the steel will reach a stress of $\left(f_{s}=\mathrm{E}_{s} \times \varepsilon_{s}=200000 \times 0.003=600 \mathrm{MPa}\right)$ exceed the yield strength $f_{y}=400 \mathrm{MPa}$. Therefore; the maximum load that will be carried by the column equal to:

$$
\begin{equation*}
P_{n}=0.85 f_{c}^{\prime} A_{c}+A_{s t} f_{y} \tag{3.9}
\end{equation*}
$$

## Example 3.3

Calculate the maximum load that the column of Example (3.1) will carry?.

## Solution

$$
\begin{aligned}
& P_{n}=0.85 f_{c}^{\prime} A_{c}+A_{s t} f_{y}=0.85 \times 30 \times 0.155536+0.001964 \times 400=4.752 \mathrm{MN} \\
& P_{c}=0.85 f_{c}^{\prime} A_{c}=0.85 \times 30 \times 0.155536=3.966 \mathrm{MN}(83.5 \%) \\
& P_{s}=A_{s t} f_{y}=0.001964 \times 400=0.786 \mathrm{MN}(16.5 \%)
\end{aligned}
$$

### 3.3.3 Axial Tension in the Elastic Uncracked State

When the tension force is small; the stresses and strains are also small (less than the tensile strength of concrete) and both materials will behave elastically.

The tensile force will be carried by concrete and steel:

$$
\begin{equation*}
P=f_{c t} A_{c}+A_{s t} f_{s}=f_{c t} A_{c}+A_{s t}\left(n f_{c t}\right)=f_{c t}\left(A_{c}+n A_{s t}\right) \tag{3.10}
\end{equation*}
$$

Where $f_{c t}=$ tensile stress in concrete .

## Example 3.4

Calculate the stresses in concrete and steel for the column of Example (3.1), if it is subjected to a tensile load of 500 kN .

## Solution

$$
\begin{aligned}
& f_{c t}=P / A_{t}=0.5 / 0.171248=2.92 M P a<f_{r}=0.62 \sqrt{30}=3.4 M P a \text { (uncracked member). } \\
& P_{c}=A_{c} f_{c t}=0.155536 \times 2.92=0.454 M N(0.908) \\
& f_{s}=n f_{c t}=8 \times 2.92=23.4 M P a \\
& P_{s}=A_{s t} f_{s}=0.001964 \times 23.4=0.046 M N \quad(0.092)
\end{aligned}
$$

## Example 3.5

What is the maximum tensile load that the member of Example (3.1) will carry before the concrete crack?

## Solution

$$
\begin{aligned}
& f_{c t}=f_{r}=0.62 \sqrt{30}=3.4 \mathrm{MPa} \\
& P_{c}=A_{c} f_{c t}=0.155536 \times 3.4=0.5288 \mathrm{MN} \\
& f_{s}=n f_{c t}=8 \times 3.4=27.2 \mathrm{MPa} \\
& P_{s}=A_{s t} f_{s}=0.001964 \times 27.2=0.0534 \mathrm{MN} \\
& P_{n t}=P_{c}+P_{s}=0.5288+0.0534=0.5822 \mathrm{MN} \\
& P_{c}=90.8 \% \\
& P_{s}=9.2 \%
\end{aligned}
$$

### 3.3.3 Axial Tension in the Ultimate State

When the tension force increases; the stresses and strains are also increases and when the tensile stress in concrete exceeds the tensile strength, the concrete will crack and the steel will carry the entire tensile load. The maximum tensile load that the member will carry equal to:

$$
\begin{equation*}
P_{n t}=A_{s t} f_{y} \tag{3.11}
\end{equation*}
$$

## Example 3.6

What is the maximum tensile load that the member of Example (3.1) will carry?

## Solution

$P_{n t}=A_{s t} f_{y}=0.001964 \times 400=0.7856 \mathrm{MN}$

### 3.4. Analysis of Rectangular Uncracked Reinforced Concrete Sections

Figure (3.3) shows a rectangular single reinforced section at the onset of cracking stage which is before the working or service conditions. The cracking stage starts when the tensile stress at the tension face equal to the modulus of rupture $f_{r}=0.62 \checkmark f_{c}^{\prime}$. If the reinforcing steel is neglected, the neutral axis for rectangular section is located at the mid depth of the section (h/2), and the stresses can be calculated using the bending of elastic homogeneous sections,

$$
\begin{equation*}
f=\frac{M \times y}{I} \tag{3.13}
\end{equation*}
$$

By substituting $f=f_{r}$ and $M=M_{c r}$ (cracking moment) can be found as follow:

$$
\begin{equation*}
M_{c r}=\frac{f_{r} \times I_{g}}{y_{t}} \tag{3.1.}
\end{equation*}
$$

$I_{g}=$ moment of inertia of the gross section (ignoring the steel) $=b . h^{3} / 12$ and $y_{t}=$ distance from the N.A. to the tension face, which is in this case $=h / 2$.
Substituting $I_{g}$ and $y_{t}$ in Eq.(3.14), the cracking moment for rectangular section becomes:

$$
\begin{equation*}
M_{c r}=\frac{f_{r} \times b . h^{3}}{12(h / 2)}=f_{r} \frac{b \cdot h^{2}}{6} \tag{3.15}
\end{equation*}
$$



Figure (3.3) Strain and stress distribution in uncracked section, (a) cross-section dimension, (b) strain distribution, (c) stress distribution.

If the reinforcing steel is to be taken into account in calculating the moment of inertia, the steel has to be transformed into equivalent area of concrete, Figure (3.4).

For steel in the compression zone the equivalent area of concrete $=(2 n-1) A_{s}{ }^{\prime}$, and for steel in the tension zone $=(n-1) A_{s}$

(a)

(b)

Figure (3.4) Uncracked reinforced concrete section, (a) reinforced concrete section, (b) transformed section

To find the neutral axis, use principle of the moment of area for the whole section about the compression face:

$$
\begin{equation*}
c=\frac{b . h^{2} / 2+(2 n-1) A_{s}^{\prime} d^{\prime}+(n-1) A_{s} \cdot d}{b . h+(2 n-1) A_{s}^{\prime}+(n-1) A_{s}} \tag{3.16}
\end{equation*}
$$

The moment of inertia of the uncracked transformed section $I_{u t}$ equal to:

$$
\begin{equation*}
I_{u t}=b \cdot c^{3} / 3+b(h-c)^{3} / 3+(2 n-1) A_{s}^{\prime}\left(c-d^{\prime}\right)^{2}+(n-1) A_{s}(d-c)^{2} \tag{3.17}
\end{equation*}
$$

## EXAMPLE (3.7)

A rectangular single reinforced section with $b=300 \mathrm{~mm}, \mathrm{~h}=450 \mathrm{~mm}, \mathrm{~d}=390$ $\mathrm{mm}, \mathrm{A}_{\mathrm{s}}=4 \# 20=1256 \mathrm{~mm}^{2} . f_{c}^{\prime}=20 \mathrm{MPa}$, and $f_{y}=276 \mathrm{MPa}$. Calculate the cracking moment, the steel, and concrete stresses.

## SOLUTION

$$
\begin{aligned}
& E_{c}=4750 \sqrt{f_{c}^{\prime}}=21243 M P a \\
& n=200000 / 21243=9.4 \approx 9
\end{aligned}
$$

Using Eq.(3.16) to calculate the neutral axis depth:
$\mathrm{c}=\frac{300(450)^{2} / 2+(9-1) 1256 \times 390}{300 \times 450+(9-1) 1256}=236 \mathrm{~mm}$
The modulus of rupture of concrete equal to:
$f_{r}=0.62 \sqrt{f_{c}^{\prime}}=2.8 \mathrm{MPa}$
The compression stress in concrete $f_{c}$ equal to:
$f_{c}=\frac{c . f_{r}}{(h-c)}=\frac{236 \times 2.8}{450-236}=3.1 \mathrm{MPa}$
To calculate the stress in the tension steel, calculate the concrete stress at that level ( $f_{c s}$ ) and then multiply it by (n)
$f_{s}=n . f_{c S}=n \frac{(d-c)}{c} f_{c}=9 \frac{(390-236)}{236} 3.1=18.2 \mathrm{MPa}$
Calculate the moment of inertia of the uncracked transformed section $I_{u t}$ use Equation (3.8):
$I_{u t}=300(236)^{3} / 3+300(450-236)^{3} / 3+9 \times 1256(390-236)^{2}=2.53 \times 10^{9} \mathrm{~mm}^{4}$
$=2.53 \times 10^{-3} \mathrm{~m}^{4}$
$M_{c r}=\frac{f_{r} \times I_{u t}}{(h-c)}=\frac{2.8 \times 0.00253}{(0.450-0.236)}=0.0332 \mathrm{MN} . \mathrm{m}=33.2 \mathrm{kN} . \mathrm{m}$
If the tension steel is neglected, $c=h / 2=0.45 / 2=0.225 \mathrm{~m}$, and $I_{g}=b . h^{3} / 12=$ $0.3 \times 0.45^{3} / 12=0.3 \times 0.45^{3} / 12=0.002278 \mathrm{~m}^{4}\left(10 \%\right.$ less than $\left.I_{u t}\right)$, the cracking moment equal to:
$M_{c r}=\frac{f_{r} \times I_{g}}{(h / 2)}=\frac{2.8 \times 0.002278}{(0.450 / 2)}=0.0284 M N . m=28.4 \mathrm{kN} . \mathrm{m}(15 \%$ less than the exact value).

### 3.4 Flexural Stresses in Beams in the Elastic Cracked Stage

Figure (3.5) show a beam in a cracked state, the tensile force carried by the tension steel equal to:

$$
\begin{equation*}
T=A_{s} \cdot f_{s}=A_{s}\left(n \cdot f_{c s}\right)=f_{c s}\left(n \cdot A_{s}\right) \tag{3.18}
\end{equation*}
$$

The subscript (s) is added to $f_{c}$ to refer to the steel location. The other form of the formula relates $(n)$ to $\left(A_{s}\right)$ instead of the stress, this means that the force in the tension steel equal to the concrete stress at the steel level $\left(f_{c s}\right)$ multiplied by a fictitious area of concrete $\left(n A_{s}\right)$. This area is called the transformed area, i.e.; area of steel $\left(A_{s}\right)$ transformed to equivalent area of concrete $\left(n A_{s}\right)$.
When the transformation process is done, the cross-section become homogeneous, i.e.; composed of one material which is concrete and the formula
for the bending of elastic homogeneous sections can be used for the analysis of the cross-section.

### 3.5. Allowable Service Load Stresses

During the working (service) conditions, the stresses should not exceed the following limits:
i. In concrete, the compression stress $f_{c} \leq 0.45 f_{c}^{\prime}$
ii. In steel, the tensile stress $f_{s} \leq 140 M P a$ for $f_{y} \leq 345 \mathrm{MPa}$, and
iii. $f_{s} \leq 165 \mathrm{MPa}$ for $f_{y} \geq 400 \mathrm{MPa}$, and
iv. $f_{s}=0.5 f_{y} \leq 200 \mathrm{MPa}$ for steel in flexural members with diameter $\leq 10$ mm in one-way slabs with span $\leq 3.6 \mathrm{~m}$.

For steel in compression zones, the steel stress assumed equal to ( 2 n ) times the concrete compression stress adjacent to the steel, but $\leq$ allowable $f_{s}$.

### 3.6 Analysis of Rectangular Cracked Reinforced Concrete Sections

When the moment acting on the section exceeds the cracking moment $\mathrm{M}_{\mathrm{cr}}$, and the tensile stress at the tension face exceeds the tensile strength of concrete and the section becomes cracked section. In this case the concrete in tension zone will no longer carry any stress and the whole tensile stresses will be resisted by the tension steel. The stress distribution in this case is as shown in Figure (3.4) below. The neutral axis depth is determined by equating the moment of the area under compression to that under tension:

$$
\begin{equation*}
b c^{2} / 2=n A_{s}(d-c) \tag{3.19}
\end{equation*}
$$



Figure (3.4) Stress Distribution in Cracked Reinforced Concrete Section

Solving the above equation for c , the moment of inertia of the section and the stresses in steel and concrete can be found.

## Example (3.8)

The section of Example (3.7) is subjected to a bending moment of $60 \mathrm{kN} . \mathrm{m}$, determine the concrete and steel stresses.

## Solution

Since the moment is greater than the cracking moment found in Example (3.7), the section is cracked. Find the neutral axis depth:
$b c^{2} / 2=n A_{s}(d-c)$
$300 c^{2} / 2=9 \times 1256(390-c)$
$c=138 \mathrm{~mm}$
$I_{c t}=b c^{3} / 3+n A_{s}(d-c)^{2}=0.3(0.138)^{3} / 3+9 \times 0.001256(0.390-0.138)^{2}$
$=9.8 \times 10^{-4} \mathrm{~m}^{4}$
$f_{c}=\frac{M \times c}{I_{c t}}=\frac{0.06 \times 0.138}{0.00098}=8.4 \mathrm{MPa}<\mathrm{All} . f_{c}=9 \mathrm{MPa}$
$f_{s}=n \frac{M \times(d-c)}{I_{c t}}=9 \frac{0.06 \times(0.39-0.138)}{0.00098}=138.9 \mathrm{MPa}<$ All. $f_{S}=140 \mathrm{MPa}$.

## Problems

P.3.1. A circular reinforced concrete column 400 mm diameter reinforced with six bars ( $\phi=25 \mathrm{~mm}$, $\mathrm{A}_{\mathrm{st}}=6 \times 491=2946 \mathrm{~mm}^{2}$ ). The column is subjected to a compressive axial load of $\mathrm{P}=1.5 \mathrm{MN} . f_{c}^{\prime}=35 \mathrm{MPa}$ and $f_{y}=400 \mathrm{MPa}$. Calculate the stresses and strains in concrete and steel.
P.3.2. For P.3.1., calculate the load and the steel stress that
 can be carried by the column if the concrete stress $=0.45 f_{c}^{\prime}=15.75 \mathrm{MPa}$.
P.3.3. For P.3.1., calculate the maximum load that the column will carry?
P.3.4. A square reinforced concrete column with a side length of 400 mm reinforced with eight bars ( $\phi=20 \mathrm{~mm}, \mathrm{~A}_{\mathrm{st}}=8 \times 314=2512 \mathrm{~mm}^{2}$ ).
Calculate the stresses in concrete and steel, if it is subjected to a tensile load of 500 kN . $f_{c}^{\prime}=30 \mathrm{MPa}$ and $f_{y}=276 \mathrm{MPa}$.

P.3.5. What is the maximum tensile load that the column of P.3.4. will carry before the concrete crack?.
P.3.6. What is the ultimate tensile load that the column of P.3.4. will carry?.
P.3.7. A rectangular single reinforced section with $\mathrm{b}=300 \mathrm{~mm}, \mathrm{~h}=500 \mathrm{~mm}, \mathrm{~d}=440 \mathrm{~mm}, \mathrm{~A}_{\mathrm{s}}=4 \# 20$ $=1256 \mathrm{~mm}^{2} . f_{c}^{\prime}=25 \mathrm{MPa}$, and $f_{y}=276 \mathrm{MPa}$. Calculate the cracking moment, steel, and concrete stresses two times, by neglecting the tension steel ( $\mathrm{I}_{\mathrm{g}}$ ), and taking the tension steel into account ( $\mathrm{I}_{\mathrm{ut}}$ ).
P.3.8. The section of $\mathbf{P} .3 .7$. is subjected to a bending moment of $65 \mathrm{kN} . \mathrm{m}$, determine the concrete and steel
 stresses.

ALNOOR UNIVERSITY COLLEGE

## Dept. of Building Engineering and Projects Management

## REINFORCED CONCRETE (I)

C.E. 3234

FIRST SEMESTER 2023-2024

## CHATER FOUR

# FLEXURAL ANALYSIS OF BEAMS USING THE STRENGTH DESIGN METHOD 

LECTURER<br>Dr. SA'AD A. AL-TA'AN

## Flexural Analysis of Beams using the Strength Design Method

### 4.1 Fundamental Assumptions

i. Plane section before bending remains plane after bending,
ii. Stress- strain relationship for concrete and steel are known,
iii. Tensile strength of concrete is neglected,
iv. There is a perfect bond between the reinforcing bars and the surrounding concrete.


Figure (4.1) Variation of strains and stresses with increasing loads, Beam elevation, (b) cross-section, (c) uncracked stage, (d) working stage, (e) ultimate stage

### 4.2 Equivalent Rectangular Stress Block

The actual stress distribution in compression is replaced by an equivalent rectangular stress block, with average stress of $0.85 f_{c}^{\prime}$ and depth $\left(a=\beta_{1} \times c\right)$ :
For $f_{c}^{\prime} \leq 28 \mathrm{MPa} \beta_{1}=0.85$, for each increase of 6.89 MPa in $f_{c}^{\prime}, \beta_{1}$ is decreased by 0.05 :

$$
\begin{equation*}
\beta_{1}=0.85-0.00725\left(f_{c}^{\prime}-28\right) \geq 0.65 \tag{4.1}
\end{equation*}
$$

Table (4.1) Variation of $\beta 1$ with $f_{c}^{\prime}$

| $f_{\mathcal{c}}^{\prime}$ | $\leq 28$ | 30 | 35 | 40 | 50 | 55 | $\geq 55.6$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\beta 1$ | 0.85 | 0.836 | 0.799 | 0.763 | 0.690 | 0.654 | 0.650 |

### 4.3 The Balanced Rectangular Beam

The balanced strain condition is defined as that at which the compression strain in concrete $=0.003$ and that in the tension steel $\varepsilon_{y}=f_{y} / E_{s}$, Figure (4.2). From the similar strain triangles, $\mathrm{c}_{\mathrm{b}}$ :
$\frac{c_{b}}{d}=\frac{0.003}{0.003+f_{y} / E_{s}}$
Substitute the value of $\mathrm{E}_{\mathrm{s}}=200000 \mathrm{MPa}$ :
$c_{b}=\frac{600}{600+f_{y}} \times d$
The depth of the stress blocks $a_{b}$ :
$a_{b}=\beta_{1} \times c_{b}=\frac{600 \beta_{1}}{600+f_{y}} d$


Figure (4.2) Rectangular beam in a balanced strain condition, (a) strain distribution, (b) actual stress distribution, (c) equivalent stresses and resultants

Using the second equation of equilibrium $\sum F x=0$, when there is no external horizontal force, $N_{c b}=N_{t b}$

$$
\begin{equation*}
0.85 f_{c}^{\prime} \cdot \beta_{1} \cdot c_{b} \cdot b=A_{s b} \cdot f_{y} \tag{4.5}
\end{equation*}
$$

substituting $A_{s b}=\rho_{b} b . d$ in the above equation, where $\rho_{b}$ is the balanced reinforcement ratio, equation (4.5) becomes:

$$
\begin{align*}
& 0.85 f_{c}^{\prime} \cdot \beta_{1} \cdot \frac{600 d}{600+f_{y}} \cdot b=\rho_{b} b \cdot d \cdot f_{y}  \tag{4.6}\\
& \rho_{b}=0.85 \beta_{1} \frac{f_{c}^{\prime}}{f_{y}} \frac{600}{600+f_{y}} \tag{4.7}
\end{align*}
$$

The resultant of the compression stresses $N_{c b}$ is located at the center of the stress block, at a distance $\left(\mathrm{a}_{\mathrm{b}} / 2\right)$ from the compression face, the lever arm z (distance between $N_{c b}$ and $N_{t b}$ ) equal to:
$z_{b}=d-a_{b} / 2$
If the depth of the beam is constant (tension face parallel to the compression face) $N_{c b}$ and $N_{t b}$ are parallel, equal, and opposite and creating a couple $M_{n b}$ (internal resisting moment or nominal flexural strength):

$$
\begin{equation*}
M_{n b}=N_{c b} \times z_{b}=N_{t b} \times z_{b} \tag{4.9}
\end{equation*}
$$

If $f_{c}^{\prime}=20 M P a$ and $f_{y}=276 M P a$, the following values of the variables in Eqs.
(4.3) to (4.9) can be obtained:
$c_{b}=\frac{600}{600+276} \times d=0.685 d$
$a_{b}=\beta_{1} \times c_{b}=0.85 \times 0.685 d=0.582 d$
$\rho_{b}=0.85 \beta_{1} \frac{f_{c}^{\prime}}{f_{y}} \frac{600}{600+f_{y}}=0.85 \times 0.85 \frac{20}{276} \frac{600}{600+276}=0.0359$
$z_{b}=d-a_{b} / 2=d-0.582 d / 2=0.709 d$
$N_{c b}=0.85 f_{c}^{\prime} . a . b=0.85 \times 20 \times 0.582 d . b=9.9 b . d$
$N_{t b}=\rho_{b} . b . d . f_{y}=0.0359(b . d) 276=9.9 b . d$
$M_{n b}=N_{c b} \times z_{b}=N_{t b} \times z_{b}=9.9 b . d(0.709 d)=7.02 b . d^{2}$
For other shapes like T, L, I, double reinforced beams, and other special shapes, the same procedure can be followed to calculate the nominal balanced flexural strength by assuming that the compression stress is constant across the depth and equal to $0.85 f_{c}{ }^{\prime}$ whatever the shape of the compression zone.

### 4.4 Modes of Failure according to ACI Code

The ACI Code considers the structural reinforced concrete member reach its nominal flexural strength when the compression strain attains a value of 0.003 .

The strain $\left(\epsilon_{\mathrm{t}}\right)$ in the first layer of tension steel (adjacent to the tension face) can be found from the strain triangles, Figure (4.4). When $\left(\epsilon_{\mathfrak{t}}\right) \leq 0.002$, the crosssection is in the compression controlled zone ( $\varphi=0.65$ ), and the failure is sudden without visual warnings like large deflection and cracks. When $\left(\epsilon_{\mathrm{t}}\right) \geq 0.005$, the cross-section is in the tension controlled zone ( $\varphi=0.90$ ), and failure is preceded by large deflection and cracks. When the strain $\left(0.002<\epsilon_{\mathrm{t}}<0.005\right)$ the crosssection is in the transition zone and the strength reduction factor $\varphi$ equal to:
$\phi=0.65+\left(\varepsilon_{t}-0.002\right) 250 / 3$ tied members
$\phi=0.75+\left(\varepsilon_{t}-0.002\right) 50 \quad$ spiral members


Figure (4.3) Modes of Failure according to ACI Code


Figure (4.4) Variation of $\varphi$ with the strain $\epsilon_{t}$

### 4.5 Maximum Reinforcement Ratio

The balanced beam fails by yielding of the tension steel ( $\left.\epsilon_{s}=\epsilon_{y}=f_{y} / E_{s}\right)$ and crushing of concrete $\left(\epsilon_{\mathrm{c}}=\epsilon_{\mathrm{cu}}=0.003\right)$, this failure is not preceded by visual warning like large deflection and wide cracks, while tension failure is preceded by visual warning, like large deflection and wide cracks or ductile failure. Ductility means, maintenance of strength while sizable deformation occurs. To ensure that this failure will occur, the ACI Code limits ( $\epsilon_{t} \leq 0.004$ ) in beams and columns subjected to small axial load $\left(\leq 0.1 \mathrm{f}_{\mathrm{c}}^{\prime} \mathrm{A}_{\mathrm{g}}\right)$. the following example shows the method of calculation:
If $f_{c}^{\prime}=20 M P a$ and $f_{y}=276 M P a$, the neutral axis depth ( $\mathrm{c}_{\text {max }}$.) equal to:

$$
\begin{gather*}
c_{\max }=\frac{0.003}{0.007} d=\frac{3}{7} d  \tag{4.11a}\\
a_{\max }=\frac{3}{7} \beta_{1} d=(3 / 7) 0.85 d=0.364 d \tag{4.11b}
\end{gather*}
$$

The subscript (max.) is used to refer to the maximum conditions.

$$
\begin{align*}
& N_{c \max }=0.85 f_{c}^{\prime} a_{\max } \cdot b=0.85 f_{c}^{\prime}\left(\frac{3}{7} \beta_{1} \cdot d\right) b=6.193 b \cdot d \\
& N_{t \max .}=\rho_{\max .} f_{y} b \cdot d=0.85 f_{c}^{\prime}\left(\frac{3}{7} \beta_{1} \cdot d\right) b \\
& \rho_{\max .}=\frac{3}{7} \beta_{1}\left(\frac{0.85 f_{c}^{\prime}}{f_{y}}\right)=0.0224  \tag{4.12}\\
& z_{\text {max }}=d-a_{\max .} / 2=d-\frac{3}{7} \beta_{1} d / 2=0.818 d
\end{align*}
$$

The nominal maximum flexural strength, (Max. $\mathrm{M}_{\mathrm{n}}$ ) equal to:
$\operatorname{Max} . M_{n}=N_{c \text { max } .} z=5.06 b d^{2}=k_{n} b d^{2}$
Where $\mathrm{k}_{\mathrm{n}}=5.06 \mathrm{MPa}$
The design maximum flexural strength equal to:

$$
\phi=0.65+\left(\varepsilon_{t}-0.002\right) 250 / 3=0.65+(0.004-0.002) 250 / 3=0.817
$$

$\phi M_{n}=0.817(5.06) b d^{2}=4.14 b d^{2}=k_{m} b d^{2}$
Where $k_{m}=4.14 M P a$. Table (4.2) shows the variables, $k_{n}, k_{m}, a_{\max }$, and $\rho_{\max }$. for different values of concrete and steel strengths when $\epsilon_{t}=0.004$ which is the maximum strain recommended by the ACI Code. Table (4.3) shows the variables, $k_{n}, k_{m}, a_{\text {max }}$, and $\rho_{\text {max. }}$ for different values of concrete and steel strengths when $\epsilon_{\mathrm{t}}=0.005$ which is the minimum strain recommended by the ACI Code for the tension controlled zone.

Table (4.2) Maximum constants for $\epsilon_{t}=0.004(\phi=0.817)$

| $f_{y}(M P a)$ |  |  |  |  | 276 | 345 | 400 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f_{c}^{\prime}$ | $k_{n}$ | $k_{m}$ | $a / d$ | $100 \rho$ | $100 \rho$ | $100 \rho$ | $100 \rho$ |
| 20 | 5.06 | 4.14 | 0.36 | 2.24 | 1.80 | 1.55 | 1.19 |
| 25 | 6.33 | 5.17 | 0.36 | 2.80 | 2.24 | 1.94 | 1.49 |
| 30 | 7.50 | 6.12 | 0.36 | 3.31 | 2.65 | 2.28 | 1.76 |
| 35 | 8.45 | 6.90 | 0.34 | 3.70 | 2.95 | 2.55 | 1.96 |
| 40 | 9.30 | 7.60 | 0.33 | 4.03 | 3.22 | 2.78 | 2.14 |

Table (4.3) Maximum constants for $\epsilon_{\mathrm{t}}=0.005(\phi=0.9)$

| $f_{y}(M P a)$ |  |  |  |  | 276 | 345 | 400 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f_{c}^{\prime}$ | $k_{n}$ | $k_{m}$ | $a / d$ | $100 \rho$ | $100 \rho$ | $100 \rho$ | $100 \rho$ |
| 20 | 4.56 | 4.10 | 0.32 | 1.96 | 1.57 | 1.35 | 1.04 |
| 25 | 5.69 | 5.12 | 0.32 | 2.45 | 1.96 | 1.69 | 1.30 |
| 30 | 6.74 | 6.06 | 0.31 | 2.89 | 2.32 | 2.00 | 1.54 |
| 35 | 7.58 | 6.82 | 0.30 | 3.23 | 2.58 | 2.23 | 1.71 |
| 40 | 8.34 | 7.50 | 0.29 | 3.52 | 2.82 | 2.43 | 1.87 |

### 4.6. Minimum Reinforcement Ratio according to the ACI Code

When the area of steel is small due to small value of the external bending moment or the cross-section is larger than necessary, it is possible that the concrete resistance to the tensile stresses is more than that of the tension steel. In other words, the cracking moment $M_{c r}>M_{n}$. This mean that the structural member will lose its strength once cracking occurs. To prevent such failure, the ACI Code put a minimum limit of the reinforcement ratio unless the provided reinforcement area exceeds the required by $33 \%$. The minimum reinforcement ratio is derived by assuming that the resultant of the tensile stresses at the cracking stage is carried out by the tension steel $\left(\rho_{\text {min }} b . d\right)$ :
$\rho_{\text {min. }}=\frac{\sqrt{f_{c}^{\prime}}}{4 f_{y}} \geq \frac{1.4}{f_{y}}$
When the flange of a T beam under tension, the minimum reinforcement area equal to whatever is greater from the equations below:

$$
\begin{align*}
& A_{s \text { min. }}=\frac{\sqrt{f_{c}^{\prime}}}{4 f_{y}}\left(2 b_{w}\right) d \geq \frac{1.4}{f_{y}}\left(2 b_{w}\right) d  \tag{4.13b}\\
& A_{s \text { min. }}=\frac{\sqrt{f_{c}^{\prime}}}{4 f_{y}}(b) d \geq \frac{1.4}{f_{y}}(b) d \tag{4.13c}
\end{align*}
$$

For slabs and foundation of constant thickness, the minimum reinforcement area is that of shrinkage and temperature, according to ACI (24.4.3.2):
(a) When $f_{y}<400 \mathrm{MPa}$ and for deformed bars

$$
\rho_{\mathrm{g}, \text { min. }}=0.002
$$

(b) When When $f_{y} \geq 400 \mathrm{MPa}$ deformed bars and welded wire fabrics (smooth or deformed)

$$
\rho_{\mathrm{g}, \min .}=0.0018\left(400 / \mathrm{f}_{\mathrm{y}}\right) \geq 0.0014
$$

The minimum reinforcement area is calculated with respect to the gross area:

$$
\begin{equation*}
A_{s, \text { min }}=\rho_{g, \text { min } .} b . h \tag{4.14}
\end{equation*}
$$

The maximum spacing for shrinkage and temperature reinforcement should not exceed 5 times the slab thickness or 450 mm .

### 4.7 Flexural Analysis of Reinforced Concrete Sections

When analyzing any reinforced section subjected to bending moment, the following steps must be followed:
i. Compare $\rho_{\text {actual }}$ with $\rho_{\min }$. if $\rho_{\text {actual }} \geq \rho_{\min }$, proceed to the next step, if $\rho_{\text {actual }}<\rho_{\text {min }}$, multiply the provided $A_{s}$ by (3/4) and proceed to the next step, (in slabs, the provided area of steel should not be less that required for temperature and shrinkage).
ii. Compare $\rho_{\text {actual }}$ with $\rho_{\text {max. }}$, if $\rho_{\text {actual }} \leq \rho_{\max }$, proceed to the next step, if not ( $\rho_{\text {actual }}>\rho_{\max }$ ) assume $\mathrm{A}_{\mathrm{s}}=\rho_{\max }$ (b.d) and proceed to the next steps.

### 4.8 Analysis of Single Reinforced Rectangular Sections

When any reinforced concrete section reach its flexural strength, the strain and stress distribution are as shown in Figure (4.5), the stress resultant in tension equal to:

$$
\begin{equation*}
T=A_{s} \cdot f_{y} \tag{4.15}
\end{equation*}
$$

The stress resultant in compression equal to:

$$
\begin{equation*}
C=0.85 f_{c}^{\prime} a . b \tag{4.16}
\end{equation*}
$$

By equating the tensile and compressive forces ( C and $\mathrm{T}, 0.85 f_{c}^{\prime} a \cdot b=A_{s} f_{y}$ ), the value of (a) can be found:

$$
\begin{equation*}
a=A_{s} f_{y} /\left(0.85 f_{c}^{\prime} b\right) \tag{4.17}
\end{equation*}
$$

the lever arm (between C and T ) z equal to:

$$
\begin{equation*}
z=d-a / 2 \tag{4.18}
\end{equation*}
$$

The internal moment (nominal flexural strength) $M_{n}$ of the section equal to: $M_{n}=C . z=T . z$

### 4.8.1 Under- Reinforced Beams (Tension Failure)

When $\rho_{\text {act. }} \leq \rho_{\mathrm{b}}$ or $\rho_{\text {max. }}$ the tension steel will reach yielding $\left(f_{s} \geq f_{y}\right)$ before crushing of concrete. The stress resultant in tension ( $\mathrm{T}=A_{s} \times f_{y}$ ). Increase in the external loads will result in extension and elongation of the tension steel and increase in deflection and cracks width, decrease in the depth of the compression area, increase in the compression strain. The strength of the crosss-section is attained when the compression strain reaches a value of 0.003 . At this stage, the the compression force equal to ( $\mathrm{C}=0.85 \mathrm{f}_{\mathrm{c}}$ ' $\mathrm{a} \times \mathrm{b}$ ).
Using the equation of equilibrium in the horizontal direction ( $\sum \mathrm{F}_{\mathrm{x}}=0$ ):
$\mathrm{C}=\mathrm{T}$
$a=A_{s} f_{y} /\left(0.85 f_{c}^{\prime} b\right)$
$z=d-a / 2$
$M_{n}=A_{s} \cdot f_{y}(d-a / 2)=0.85 f_{c}^{\prime} a \cdot b(d-a / 2)$
By substituting, $a=A_{s} f_{y} /\left(0.85 f_{c}^{\prime} b\right)$ andin the above equation, the $A_{s}=\rho . b . d$ equation becomes:
$M_{n}=f_{c}^{\prime} \cdot b . d^{2} . \omega(1-0.59 \omega)$
Where $\omega=\rho f_{y} / f_{c}^{\prime}$.

## EXAMPLE (4.1)

A single reinforced concrete rectangular beam with $\mathrm{b}=300 \mathrm{~mm}, \mathrm{~h}=450 \mathrm{~mm}$, $\mathrm{d}=390 \mathrm{~mm}, \mathrm{~A}_{\mathrm{s}}=4 \# 20=1256 \mathrm{~mm}^{2}$. If $f_{\mathrm{c}}^{\prime}=20 \mathrm{MPa}$ and $f_{\mathrm{y}}=276 \mathrm{MPa}$. Find the design flexural strength $\varphi \mathrm{M}_{\mathrm{n}}$.

## SOLUTION

The actual reinforcement ratio $\rho_{\text {act. }}$ equal to
$\rho_{a c t}=\frac{A_{S}}{b . d}=\frac{1256}{300 \times 390}=0.0107$
The minimum reinforcement ratio $\rho_{\text {min. }}$ equal to:
$\rho_{\text {min }}=\frac{1.4}{f_{y}}=\frac{1.4}{276}=0.005$
$\rho_{\text {min }}=\frac{\sqrt{f_{c}^{\prime}}}{4 f_{y}}=\frac{\sqrt{20}}{4 \times 276}=0.004$


Therefore $\rho_{\min }=0.005<\rho_{\text {act. }}$ (O.K.).
From Tables (4.2) $\rho_{\max }=0.0224\left(\epsilon_{\mathrm{t}}=0.004\right)$ and from Table (4.3) $\rho_{\max }=0.0196$ $\left(\epsilon_{t}=0.005\right)$, therefore the beam is under reinforced and $\epsilon_{t}>0.005$.
$a=\frac{A_{s} \cdot f_{y}}{0.85 f_{c}^{\prime} \cdot b}=\frac{1256 \times 276}{0.85 \times 20 \times 300}=68 \mathrm{~mm}<a_{\max }=0.32 \times 390=125 \mathrm{~mm}$
When calculating (a), use either m or mm so that the units are consistent,

$$
\begin{aligned}
& a=\frac{A_{s} \cdot f_{y}}{0.85 f_{c}^{\prime} \cdot b}=\frac{1256 \times 10^{-6} \times 276}{0.85 \times 20 \times 0.30}=0.068 \mathrm{~m}=68 \mathrm{~mm} \\
& z=d-a / 2=390-68 / 2=356 \mathrm{~mm} \\
& M_{n}=A_{s} \cdot f_{y} \cdot z=1256 \times 10^{-6} \times 276 \times 0.356=0.1234 M N . \mathrm{m}=123.4 \mathrm{kN} . \mathrm{m}
\end{aligned}
$$

Since $a_{\max .}>a$ and $\rho_{\max }>\rho_{\text {act. }}$, therefore $0.005<\varepsilon_{t}$ (tension controlled) and $\phi=0.9$. The value of $\epsilon_{\mathrm{t}}$ can be calculated from the strain triangles after finding the value of c :
$c=a / \beta_{1}=68 / 0.85=80 \mathrm{~mm}$
$\varepsilon_{t}=\frac{d-c}{c} \times 0.003=\frac{390-80}{80} 0.003=0.011625>0.005$, that is $\phi=0.9$;
$\phi M_{n}=0.9 \times 123.4=111.1 \mathrm{kN} . \mathrm{m}$
The value of $M_{n}$ can be calculated in another way, by finding $\omega=\rho f_{y} / f_{c}^{\prime}$ using equation (4.20):

$$
\begin{aligned}
& \omega=\rho f_{y} / f_{c}^{\prime}=0.0107 \times 276 / 20=0.148 \\
& M_{n}=20 \times 0.3(0.39)^{2} \times 0.148(1-0.59 \times 0.148)=0.1233 M N . m
\end{aligned}
$$

### 4.8.2 Analysis of Over- Reinforced Beams (Compression Failure)

When the area of tension steel is relatively large, the concrete in compression may reach its strength before yielding of steel. In such case the depth of the neutral axis is relatively large resulting in large compression force (C) to equalize the large tension force ( T ). The section will fail when the strain in compression reaches a value of 0.003 , failure in such cases is brittle without visual warnings such as large deflection and wide cracks since $f_{s}<f_{y}$.
The stress in the tension steel can be found in terms of the neutral axis depth, from similar strain triangles:

$$
\frac{\varepsilon_{S}}{d-c}=\frac{0.003}{c} \therefore \quad \varepsilon_{S}=\frac{d-c}{c} 0.003
$$

From the equilibrium equation $(\mathrm{C}=\mathrm{T})$ :

$$
0.85 f_{c}^{\prime} \beta 1 . c . b=E_{s} \varepsilon_{s} . A_{s}=200000 \times 0.003 \frac{d-c}{c} A_{s}=600 \frac{d-c}{c} A_{s}
$$

By substituting $A_{s}=\rho . b . d$ and $c=k_{u} \cdot d$, the above equation becomes:

$$
\begin{equation*}
k_{u}^{2}+m \cdot \rho \cdot k_{u}-m \cdot \rho=0 \tag{4.21}
\end{equation*}
$$

where $m$ equal to:
$m=\frac{600}{0.85 f_{c}^{\prime} \cdot \beta 1}$
Solving equation (4.21) for $k_{u}$ :
$k_{u}=\sqrt{m . \rho+(m . \rho / 2)^{2}}-m . \rho / 2$
After that find the value of $c=k_{u} \cdot d, a=\beta_{1} \times c$, and $f_{s}$ as follow
$f_{s}=600 \frac{(d-c)}{c}$
The nominal flexural strength equal to:
$M_{n}=0.85 f_{c}^{\prime} \cdot a \cdot b(d-a / 2)=A_{s} \cdot f_{s}(d-a / 2)$
It is not possible to use the equation $M_{n}=A_{s} \cdot f_{y}(d-a / 2)$ to calculate $\mathrm{M}_{\mathrm{n}}$, since $\mathrm{f}_{\mathrm{s}}<\mathrm{f}_{\mathrm{y}}$.

## EXAMPLE 4.2

Recalculate the design flexural strength of the beam of example (4.1) if $\mathrm{A}_{\mathrm{s}}=$ $5000 \mathrm{~mm}^{2}$.

## SOLUTION

The reinforcement ratio $\rho$ equal to
$\rho=\frac{5000}{300 \times 390}=0.0427>\rho_{\text {min. }}=0.005$
Refer to Table (4.2), this means that the $\rho_{b}=0.0359$ and $>\rho>\rho_{\text {max }}=0.0224$
section is over reinforced and failure will be in compression.
$m=\frac{600}{0.85 \times 20 \times 0.85}=41.5225$
$m . \rho=41.225 \times 0.0427=1.773$
$k_{u}=\sqrt{1.773+(1.773 / 2)^{2}}-1.773 / 2=0.713$
$c=0.713 * 390=278 \mathrm{~mm}$
$a=0.85 \times 278=236 \mathrm{~mm}>a_{b}=227 \mathrm{~mm}$
$f_{s}=\frac{390-278}{278} 600=242 \mathrm{MPa}<f_{y}=276 \mathrm{MPa}$
$\varepsilon_{s}=\frac{390-278}{278} 0.003=0.001209<0.002$
The section is in the compression controlled zone and $\phi=0.65$

$$
\begin{aligned}
& M_{n}=0.85 \times 20 \times 0.236 \times 0.3(0.39-0.236 / 2)=0.3274 M \mathrm{~N} . \mathrm{m}=327.4 \mathrm{kN} . \mathrm{m} \\
& \phi M_{n}=0.65 \times 327.4=212.8 \mathrm{kN} . \mathrm{m}
\end{aligned}
$$

The following table can be used to find the value of $k_{u}$ and $f_{s}$ against the value of m. $\rho$

Table (4.4) Values of $\mathrm{k}_{\mathrm{u}}$ and $\mathrm{f}_{\mathrm{s}}$ against m. $\rho$

| $m . \rho$ | 1.0 | 1.25 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $k_{u}$ | 0.618 | 0.686 | 0.686 | 0.732 | 0.766 | 0.791 | 0.812 | 0.828 |
| $f_{s}$ | 371 | 315 | 274 | 220 | 184 | 158 | 139 | 124 |

The above steps give exact values for the neutral axis depth and the design flexural strength. The ACI Code recommends that $\rho=\rho_{\text {max }}=0.0224$ to ensure ductile failure. The solution of Example (4.2) is as follow:
$\rho=\rho_{\text {max }}$ and $a=a_{\text {max }}=0.36 d=0.36 \times 390=140 \mathrm{~mm}$
$M_{n}=0.85 f_{c}^{\prime} \cdot a_{\text {max. }} . b\left(d-a_{\text {max. }} / 2\right)=A_{s} \cdot f_{s}\left(d-a_{\text {max. }} / 2\right)$
$=0.85 \times 20 \times 0.140 \times 0.3(0.39-0.14 / 2)=0.2285 M N . m=228.5 \mathrm{kN} . \mathrm{m}$
Or the value of max. $k_{n}=5.06 \mathrm{MPa}$ to calculate $\mathrm{M}_{\mathrm{n}}$

## EXAMPLE 4.3

Recalculate the design flexural strength of the beam of example (4.2) according to the ACI recommendation.

## SOLUTION

The maximum area of steel that can be used in this case so that ( $\mathrm{f}_{\mathrm{s}}=\mathrm{f}_{\mathrm{y}}$ ) according to the ACI recommendation Max. $A_{s}=\rho_{\max } b d=390 \times 300 \times 0.0224=2621 \mathrm{~mm}^{2}$
The maximum design flexural strength equal to:
$\phi M_{n}=k_{m} \cdot b . d^{2}=4.14 \times 0.3 \times 0.39^{2}=188.9 \mathrm{kN} . \mathrm{m}$

### 4.8.3 Analysis of One- Way Slabs

The one way slab is a slab supported on two opposite sides or a slab with length to width ratio $\geq 2.0$. The load in such cases is transferred in the span direction or the short direction in case the slab is supported on four sides. The behavior of the slab is such case is similar to that of beams. For the sake of analysis a strip of 1.0 m wide is considered as a beam and the equivalent reinforcement in this strip is calculated.

## EXAMPLE 4.4

A one way simply supported reinforced concrete slab has overall thickness of $150 \mathrm{~mm}, \mathrm{c} / \mathrm{c}$ span $=3.5 \mathrm{~m}$, and reinforced in tension with 12 mm bars on 150 $\mathrm{mm} \mathrm{c} / \mathrm{c}$. the slab is subjected to a finishing dead load of 2 kPa (in addition to its self-weight) and a live load of 3.5 kPa . Show whether the slab can resist the ultimate load. $f_{c}^{\prime}=20 \mathrm{MPa}$ and $f_{y}=276 \mathrm{MPa}$.

## SOLUTION

i. Required strength $=$ External ultimate bending moment

Calculate the weight of $1.0 \mathrm{~m}^{2}$ of the slab as a dead load:
$1 \times 1 \times 0.15 \times 24=3.6 \mathrm{kPa}$
Ultimate Loads or Factored Loads

$$
w_{u}=1.2 D L+1.6 L L=1.2(3.6+2)+1.6(3.5)=12.32 \mathrm{kPa}
$$


(a)
(b)

Figure (4.5) Example (4.4) (a) cross-section of the slab, (b) one meter strip

Maximum external bending moment $\mathrm{M}_{\mathrm{u}}$
$M_{u}=12.32(3.5)^{2} / 8=18.87 \mathrm{kN} . \mathrm{m} / \mathrm{m}$
ii. Design flexural strength $\varphi \mathrm{M}_{\mathrm{n}}$ :

Area of one bar with diameter of $12 \mathrm{~mm}=113 \mathrm{~mm}^{2}$ with a spacing of 150 mm ,
Area of reinforcement $\left(\mathrm{A}_{\mathrm{s}}\right)$ in a strip 1.0 m width $=(1000 / 150) \times 113=753$ $\mathrm{mm}^{2} / \mathrm{m}$ :
$\rho_{g}=\frac{A_{s}}{b . h}=\frac{753}{1000 \times 150}=0.005>$ Min. $\rho_{g}=0.002 \quad$ (O.K.)
$\rho_{g}$ is calculated on the basis of the gross-section (b.h)
$d=h-($ clear cover $)-d_{b} / 2=150-20-12 / 2=124 m m$
$\rho=\frac{753}{1000 \times 124}=0.006<\rho_{\text {max. }}=0.0196$ (therefore the section is under reinforced)

$$
\begin{aligned}
& a=\frac{753 \times 276}{0.85 \times 20 \times 1000}=12 \mathrm{~mm} \quad \quad z=124-12 / 2=118 \mathrm{~mm} \\
& \begin{aligned}
M_{n}= & 0.9 A_{S} \cdot f_{y} \cdot z=0.9 \times 753 \times 10^{-6} \times 276 \times 0.118=0.0221 M N . m \\
\quad=22.1 \mathrm{kN} . \mathrm{m} & >18.87 \mathrm{kN} . \mathrm{m}
\end{aligned}
\end{aligned}
$$

Therefore the slab can resist the ultimate load.

### 4.9 Analysis of Doubly Reinforced Rectangular Beams

In some cases, the size of the beam is limited due to its position in a specific place or due to aesthetic requirements. If the required flexural strength is more than the maximum flexural strength, an increase in the flexural strength in this case is necessary. This can be accomplished by using extra steel in tension and in compression. The steel in the compression zone is used for three purpose:
i. Increase in the flexural strength together with additional tension steel,
ii. Decrease of the long term deflection due to creep and shrinkage of concrete,
iii. Stirrups hanger, and
iv. In beams subjected to moment reversal.

The method of analysis is similar to that of single reinforced sections, with the compression force divided into two parts, the first one is that of the concrete of area $(\mathrm{a} \times \mathrm{b})$ and the second in the compression steel. The stress in the compression steel may or may not reach yielding when the section attains its strength depending the neutral axis depth and the area of the tension and compression steel. The stress in the compression steel is calculated from the strain triangles at failure.

The flexural strength $M_{n}$ assumed to be composed of two parts, $M_{n 1}$ and $\mathrm{M}_{\mathrm{n} 2}$, Figure (4.6). The forces creating $M_{n 1}$ are that in the tension steel ( $T_{1=} A_{s l} \times f_{y}$ ) and that in the concrete in compression ( $C_{I}=0.85 f_{c}^{\prime} a \times b$ ). The forces creating $M_{n 2}$ are that in the tension steel $\left(T_{2}=A_{s 2} \times f_{y}\right)$ and that in the compression steel $\left[C_{2}=A_{s}\right.$ ( $f_{s}^{\prime}-0.85 f_{c}$ )].
The ACI Code limits $\rho_{1 .}=A_{s 1} /(b \times d) \leq \rho_{\max }$ for double reinforced sections as that for single reinforced section. The area $A_{s 2}$ that equalize or neutralize $A_{s}$ is not controlled by this limitation.

To check whether the compression steel reaches yielding or not, refer to Figure (4.7) and from the similar strain triangles compute the value of c :

$M_{n}=M n_{1}+M n_{2}$
Design Moment $=\boldsymbol{\Phi} \boldsymbol{M}_{n}$


$$
\begin{aligned}
& A_{s 2}=A_{s}{ }^{\prime} \\
& M_{n 2}=A_{s} f_{y}\left(d-d^{`}\right)
\end{aligned}
$$

$$
\text { a to make } N_{c l}=N_{t l}
$$

$$
\text { i.e., } a=A_{s 1} f_{y} /(0.85 f c b)
$$

$$
M_{n 1}=A_{s l} f_{y}(d-a / 2)
$$

Figure (4.6) Analysis of Double Reinforced Sections
$\frac{0.003}{c}=\frac{\varepsilon_{s}^{\prime}\left(\varepsilon_{y}\right)}{c-d^{\prime}} \quad$ Solving for $\mathrm{c}:$

$$
\begin{equation*}
c=\frac{0.003}{0.003-\varepsilon_{y}} d \tag{4.23}
\end{equation*}
$$

Using the equilibrium equation $N_{t 1}=N_{c 1}$ to derive the least tension reinforcement ( $\rho_{\text {lim. }}$ ) ratio that ensure yielding of the compression steel when the section attains its strength.

$$
\begin{equation*}
0.85 f_{c}^{\prime} \beta 1 . c . b=\left(A_{s}-A_{s}^{\prime}\right) f_{y} \tag{4.24}
\end{equation*}
$$

Substituting $A_{s}=\rho_{\lim .}^{\prime}$ b.d, $A_{s}^{\prime}=\rho^{\prime}$.b.d, and C from equation (4.23) and $\varepsilon_{y}=f_{y} / E_{s}$ in the above equation, the value of $\rho_{\text {lim }}$. become:

$$
\begin{equation*}
\rho_{\text {lim. }}^{\prime}=0.85 \beta 1 \frac{f_{c}^{\prime}}{f_{y}^{\prime}} \frac{d^{\prime}}{d} \frac{600}{600-f_{y}}+\rho^{\prime} \tag{4.25}
\end{equation*}
$$

If $\rho_{\text {lim }}^{\prime}<\rho$, the compression steel will reach yielding. Table (4.5) shows the depth that ensure yielding of the compression steel


Figure (4.7) Strain distribution in the compression zone when the compression steel yield


Figure (4.8) Stain distribution in a balanced double reinforced concrete section

Table (4.5) Minimum depth of beams to ensure yielding of compression steel

| $f_{y}$ | $\epsilon_{\mathrm{t}}=0.004$ |  | $\epsilon_{\mathrm{t}}=0.005$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Min. d |  | Min. d |  |
|  | Max. d'/d | $\mathrm{d}^{\prime}=63 \mathrm{~mm}$ | Max. d'/d | $\mathrm{d}^{\prime}=63 \mathrm{~mm}$ |
| 276 | 0.231 | 275 | 0.203 | 310 |
| 345 | 0.182 | 345 | 0.159 | 395 |
| 400 | 0.143 | 440 | 0.125 | 505 |
| 520 | 0.057 | 1105 | 0.050 | 1260 |

The maximum reinforcement ratio to ensure tension failure equal to:

$$
\begin{equation*}
\rho_{\text {max. }}^{\prime}=\rho_{\text {max. }}+\rho^{\prime} \tag{4.26}
\end{equation*}
$$

In some cases, the compression steel may not reach yielding when the section attains its strength, like beams with shallow depth or when $\rho_{\text {act. }}<\rho_{\text {lim }}^{\prime}$, the stress
in the compression steel can be calculated from the strain triangles, Figure (4.8) above, if the section is in a balanced strain condition:

$$
\begin{aligned}
& \frac{d}{0.003+\varepsilon_{y}}=\frac{d-d^{\prime}}{\varepsilon_{y}+\varepsilon_{s}^{\prime}} \text { Or } \\
& \varepsilon_{s}^{\prime}=0.003 \frac{d^{\prime}}{d}\left(0.003+\varepsilon_{y}\right)
\end{aligned}
$$

The balanced reinforcement ratio $\rho_{b}$ equal to:

$$
\rho_{b}^{\prime}=\rho_{b}+\rho^{\prime} \frac{f_{s}^{\prime}}{f_{y}}
$$

Where $f_{s}^{\prime}$ equal to:

$$
f_{S}^{\prime}=E_{S}\left[0.003-\frac{d^{\prime}}{d}\left(0.003+\varepsilon_{y}\right)\right]
$$

The maximum reinforcement ratio equal to:

$$
\begin{equation*}
\rho_{\text {max. }}^{\prime}=\rho_{\text {max. }}+\rho^{\prime} f_{s}^{\prime} / f_{y} \tag{4.27}
\end{equation*}
$$

The value of $f_{s}$ can be calculated also from the strain tringles in the compression zone:

$$
f_{S}^{\prime}=0.003 E_{S} \frac{c-d^{\prime}}{c}=600 \frac{c-d^{\prime}}{c}
$$

From the equilibrium equation $T=C_{1}+C_{2}$

$$
\begin{equation*}
0.85 f_{c}^{\prime} \cdot \beta_{1} \cdot c \cdot b+A_{S}^{\prime}\left(f_{s}^{\prime}-0.85 f_{c}^{\prime}\right)=A_{s} . f_{y} \tag{4.28}
\end{equation*}
$$

Substituting $f_{s}^{\prime}$ in the above equation becomes a second degree equation in terms of c . after calculating the value of c , the values of $a$ and $f_{s}^{\prime}$ can be calculated as follow:

$$
\begin{equation*}
M_{n}=0.85 f_{c}^{\prime} \cdot a \cdot b(d-a / 2)+A_{s}^{\prime} \cdot f_{S}^{\prime}\left(d-d^{\prime}\right) \tag{4.29}
\end{equation*}
$$

The ACI Code recommends that transverse steel should be provided (stirrups or equivalent welded wire fabric) in places where there is a compression steel to avoid bucking of the compression steel.

## EXAMPLE (4.5)

A rectangular section with $b=350 \mathrm{~mm}, \mathrm{~d}=680 \mathrm{~mm}$, reinforced in tension with 6 bars \#28( $\left.A_{s}=3696 \mathrm{~mm}^{2}\right)$, two bars \# 25 in compression $\left(\mathrm{A}_{\mathrm{s}}=982 \mathrm{~mm}^{2}\right)$, d'=63
mm from the compression face. $f_{c}^{\prime}=20 M P a$ and $f_{y}=400 M P a$. Calculate the nominal and design flexural strength for the section with and without compression steel.

## SOLUTION

Calculate $\rho_{\text {lim. }}$.from Equation (4.25), to check whether the compression steel will reach yielding or not:

$$
\begin{aligned}
& \rho=\frac{3696}{350 \times 680}=0.0155 \\
& \rho^{\prime}=\frac{982}{350 \times 680}=0.00413 \\
& \rho_{\text {lim. }}^{\prime}=0.85 \times 0.85 \times \frac{20}{400} \times \frac{63}{680} \times \frac{600}{600-400}+0.00413=0.0142<\rho=0.0155
\end{aligned}
$$

Therefore the compression steel will reach yielding. Compare $\rho$ with $\rho_{\text {max }}^{\prime}$ to check whether it is under (T.F.) or over reinforced (C.F.):

$$
\rho_{\text {max. }}^{\prime}=\rho_{\text {max. }}+\rho^{\prime}=0.0135+0.00413=0.01763>\rho=0.0155
$$

therefore the beam is under reinforced, $\rho_{\max }=0.0135$ is taken from Table (4.3) for $\varepsilon_{t}>0.005$ and $\phi=0.9$
$T=A_{s} \cdot f_{y}=3696 \times 10^{-6} \times 400=1.478 M N$
$C_{1}=0.85 \times 20 \times a \times 0.35=5.95 a$
$C_{2}=982 \times 10^{-6}(400-0.85 \times 20)=0.376 M N$
$a=\frac{1.478-0.376}{5.95}=0.185 \mathrm{~m}=185 \mathrm{~mm}<a_{\max } .=0.32 \times 680=218 \mathrm{~mm}$
$c=185 / 0.85=218 \mathrm{~mm}$
$\varepsilon_{t}=\frac{680-218}{218}(0.003)=0.0064>0.005 \quad, \quad \phi=0.9$
$M_{n 1}=0.85 \times 20 \times 0.185 \times 0.35(0.68-0.185 / 2)=0.6467 M N . m$
$M_{n 2}=982 \times 10^{-6}(400-0.85 \times 20)(0.68-0.063)=0.226 M N . m$
$M_{n}=646.7+226=872.7 \mathrm{kN} . \mathrm{m}$
$\phi M_{n}=0.9 \times 872.7=785.4 \mathrm{kN} . \mathrm{m}$
If the compression steel is omitted $\rho=0.0155>\rho_{\max }=0.0135$ for $\left(\varepsilon_{t}=0.005\right)$, but for $\varepsilon_{t}=0.004, \rho_{\max }=0.0155$, the nominal and design flexural strength equal to:
$M_{n}=k_{, n \max } b . d^{2}=5.06 \times 0.35 \times 0.68^{2}=818.9 \mathrm{kN} . \mathrm{m}$
$\phi M_{n}=k_{m, \text { max }} b . d^{2}=4.14 \times 0.35 \times 0.68^{2}=670 \mathrm{kN} . \mathrm{m}$

The values of $k_{n, \max }$ and $k_{m, \max }$ are taken from Table (4.2) for $\left(\varepsilon_{t}=0.004\right)$ and $\varphi=0.817$.
The increase in the nominal flexural strength with the compression steel equal to:

$$
\frac{872.7-818.9}{818.9}=6.6 \%
$$

While the total area of steel (tension and compression) is increased by $26.6 \%$, this shows that providing compression steel only will not increase the design flexural strength in the same proportion, but additional tension steel must be provided also.

## EXAMPLE (4.6)

Resolve Example (4.5) if $f_{c}^{\prime}=30 M P a$.

## SOLUTION

Calculate $\rho_{\text {lim. }}$ from Equation (4.25) to check whether the compression steel will reach yielding or not:

$$
\rho_{\text {lim. }}^{\prime}=0.85 \times 0.836 \times \frac{30}{400} \times \frac{63}{680} \times \frac{600}{600-400}+0.00413=0.0189>\rho=0.0155
$$

Therefore the compression steel will not reach yielding when the section attains its strength, Equation (4.28) to find the value of c :

$$
0.85 * 30 \times 0.836(c) 0.35+982 \times 10^{-6}\left[600 \frac{c-0.063}{c}-0.85 \times 30\right]=3696 \times 10^{-6} \times 400
$$

$c^{2}-0.1225 c-0.00498=0$
$c=155 \mathrm{~mm}, a=0.836 \times 155=130 \mathrm{~mm}<a_{\text {max. }}=0.31 \times 680=211 \mathrm{~mm}$
This means that the section is under reinforced and $\varepsilon_{t}>0.005$ and $\phi=0.9$. The value of a is less than that of the example (4.5) because of the increase in $f_{c}^{\prime}$.
$f_{s}^{\prime}=600 \frac{155-63}{155}=356.1 \mathrm{MPa}<f_{y}=400 \mathrm{MPa}$
$\rho_{\text {max. }}^{\prime}=0.02+0.00413 \frac{356.1}{400}=0.0237>\rho$
Which mean that the section is under reinforced as mentioned previously.

$$
\begin{aligned}
& M_{n 1}=0.85 \times 30 \times 0.130 \times 0.35(0.68-0.130 / 2)=0.7136 M N . m \\
& M_{n 2}=982 \times 10^{-6}(356.1-0.85 \times 20)(0.68-0.063)=0.2003 \mathrm{MN} . \mathrm{m} \\
& M_{n}=713.6+200.3=913.9 \mathrm{kN} . \mathrm{m} \\
& \phi M_{n}=0.9 \times 913.9=822.5 \mathrm{kN} . \mathrm{m}
\end{aligned}
$$

If the compression steel is removed, $M_{n}=882.9 \mathrm{kN} . \mathrm{m}$. The increase in the nominal flexural strength in the presence of compression steel equal to:
$\frac{913.9-882.9}{882.9}=3.5 \%$
This mean, that adding compression steel only will not increase the flexural strength significantly, but additional tension and compression steel must be provided.
Comparing the results of this example with the previous one, shows that increasing $f_{c}^{\prime}$ by $50 \%$ from ( 20 to 30 MPa ), increased the nominal flexural strength by $4.7 \%$ and $7.8 \%$ for the double and single reinforced sections respectively. The compression strength of concrete doesn't have great influence on the flexural strength of beams like slabs and beams, but in columns it have a great influence especially those failing in compression.

### 4.10 Analysis of T-Beams

In constructing slabs and roofs, the beams and slabs are cast together, and if it is cast separately (as in bridges) they are bonded together (by shear connectors). The stirrups and top bars of the beams are extended to the slabs, and the slabs top and bottom reinforcement are extended to the beam and therefore both become one unit as a T-shape, Figure (4.9). The slab part is called flange and the lower part called web or (stem). When the beam is subjected to positive bending moment (compression at the top and tension at the bottom) part of the slab adjacent to the beam will be subjected to compression stresses to balance the tensile force in the web. These compression stresses will decrease with the distance from the web, Figure (4.9). At the ultimate stage, the distribution of the longitudinal compression stresses becomes more uniform.

For beams with slabs on both sides, the ACI code recommends that the width of the beam flange is the smaller of the following:
i. Span / 4,
ii. $16 h_{f}+b_{w}$, and
iii. Web width + the clear distance to the next beam (Center to center of the beams).

For beams with slab on one side (edge beams), the width of effective slab that is part of the beam, is the smaller of the following:
i. Six times the slab thickness $\left(6 h_{f}\right)$,
ii. Span / 12, and
iii. Half the clear distance to the next beam.

If the beam is subjected to negative bending moment (tension at the top and compression at the bottom) the beam is designed as a rectangular beam with dimensions ( $\mathrm{b}_{\mathrm{w}} \cdot \mathrm{d}$ ).

Depending on the beam and slab dimensions, tension steel area, and the compression strength of concrete, there are three possible locations of the depth of the stress block (a):


Figure (4.9) Distribution of the compression stresses on the flange
i. In the flange $\left(a<h_{f}\right)$, and the beam is considered as a rectangular beam with dimensions ( $\mathrm{b} \times \mathrm{d}$ ),
ii. Equal to the flange thickness $\left(a=h_{f}\right)$ and the beam is considered as a rectangular beam with dimensions ( $b \times d$ ) or,
iii. Below the bottom of the flange $\left(\mathrm{a}>\mathrm{h}_{\mathrm{f}}\right)$ and the beam is considered as a T beam.

In the third case, the tension steel is divided into two parts, one equalizing the compression force in the two wings of the flange $\left(A_{s f}\right)$, and the other equalizing the compression force in the web $\left(A_{s w}\right)$, Figure (4.10) and computed as follow:

$$
\begin{align*}
& A_{s f} \cdot f_{y}=0.85 f_{c}^{\prime} \cdot h_{f}\left(b-b_{w}\right)  \tag{4.30a}\\
& M_{n 1}=A_{s f} \cdot f_{y}\left(d-h_{f} / 2\right)  \tag{4.30b}\\
& A_{s w}=A_{s}-A_{s f} \tag{4.31a}
\end{align*}
$$

$$
\begin{align*}
& A_{s w} \cdot f_{y}=0.85 f_{c}^{\prime} \cdot a \cdot b_{w}  \tag{4.31b}\\
& M_{n 2}=A_{s w} \cdot f_{y}(d-a / 2) \tag{4.31c}
\end{align*}
$$

the total nominal flexural strength equal to:

$$
\begin{equation*}
M_{n}=M_{n 1}+M_{n 2} \tag{4.32}
\end{equation*}
$$

To ensure a tension failure, the reinforcement ratio should be limited as for rectangular beam:

$$
\begin{align*}
& \rho_{w}=\frac{A_{S}}{b_{w} \cdot d} \leq \rho_{w, \max } \\
& \rho_{w, \max }=\rho_{\max .}+\rho_{f}  \tag{4.33}\\
& \rho_{f}=\frac{A_{s f}}{b_{w} d} \tag{4.34}
\end{align*}
$$



Figure (4.10) Calculation of the flexural strength of a T section
$\rho_{\text {max } . ~ i s ~ c a l c u l a t e d ~ a s ~ f o r ~ s i n g l e ~ r e i n f o r c e d ~ r e c t a n g u l a r ~ b e a m . ~ T h e ~ r e i n f o r c e m e n t ~}^{\text {b }}$ ratio ( $\rho_{w}$ ) should be compared also with the minimum reinforcement ratio ( $\rho_{\text {min. }}$ ).

## EXAMPLE (4.7)

A reinforced concrete floor composed of continuous slab supported on parallel beams spaced 3.0 m on centers and 6.0 m span. $f_{c}^{\prime}=20 \mathrm{MPa}$ and $f_{y}=400 \mathrm{MPa}$. The beams are reinforced in tension with six bars 35 mm diameter ( $5772 \mathrm{~mm}^{2}$ ). Calculate the design flexural strength of an intermediate beam.


Figure (4.11) Beam slab system of Example (4.7)

## SOLUTION

The effective width of the flange equal to:
$b \leq$ span $/ 4=6000 / 4=1500 \mathrm{~mm}$
$b \leq b_{w}+16 h_{f}=300+16 \times 150=2700 \mathrm{~mm}$
$b \leq c / c$ distance between beams $=3000 \mathrm{~mm}$
therefore, $b=1500 \mathrm{~mm}$

$$
\begin{aligned}
& A_{s f}=\frac{0.85 \times 20 \times 0.15(1.5-0.3)}{400}=7.65 \times 10^{-6} \mathrm{~m}^{2}=7650 \mathrm{~mm}^{2} \\
& \rho_{f}=\frac{A_{s f}}{b_{w} \cdot d}=\frac{7650}{300 \times 600}=0.0425 \\
& \rho_{w}=\frac{5772}{b_{w} \cdot d}=\frac{5772}{300 \times 600}=0.032 \\
& \rho_{w, \text { max. }}=\rho_{\text {max. }}+\rho_{f}=0.0135+0.0425=0.056>\rho_{w}=0.032
\end{aligned}
$$

The value of $\rho_{\text {max }}$ is taken from Table (4.3), $\varepsilon_{t}=0.005$ and $\phi=0.9$. If $\rho_{w}>\rho_{w, \text { max }}$ the value of $\rho_{\max }$. can be taken from Table (4.2) and substituted in the above equation. If $\rho_{w}$ still $>\rho_{w, \text { max. }}$ the section is either in the transition or the compression controlled zone.
Therefore the section is under reinforced. To check the value of (a), calculate T and $\mathrm{C}_{\mathrm{f}}$ :
$N_{t}=A_{s} \cdot f_{y}=5772 \times 10^{-6} \times 400=2.309 M N$
$N_{c f}=0.85 f_{c}^{\prime} b . h_{f}=0.85 \times 20 \times 1.5 \times 0.15=3.825 M N>N_{t}$
Therefore $\mathrm{a}<\mathrm{h}_{\mathrm{f}}$, the section is considered as a rectangular section with dimensions (b.d $=1500 \times 600 \mathrm{~mm}$ ):

$$
\begin{aligned}
& a=\frac{2.309}{0.85 \times 20 \times 1.5}=0.09 \mathrm{~m}=90 \mathrm{~mm} \\
& M_{n}=N_{t}(d-a / 2)=2.309(0.6-0.09 / 2)=1.281 M N . m \\
& \phi M_{n}=0.9 \times 1281=1153 \mathrm{kN} . \mathrm{m}
\end{aligned}
$$

## EXAMPLE (4.8)

Calculate the design flexural strength for the edge beam of Example (4.7).

## SOLUTION

Calculate the value of the effective flange width $b$ :
$b \leq b_{w}+\operatorname{span} / 12=300+6000 / 12=800 \mathrm{~mm}$
$b \leq b_{w}+6 h_{f}=300+6 \times 150=1200 \mathrm{~mm}$
clear distance between beams $=300+2700 / 2=1650 \mathrm{~mm} b \leq b_{w}+\frac{1}{2}$
therefore $b=800 \mathrm{~mm}$
$A_{s f}=\frac{0.85 \times 20 \times 0.15(0.8-0.3)}{400}=3.188 \times 10^{-6} \mathrm{~m}^{2}=3188 \mathrm{~mm}^{2}$
$\rho_{f}=\frac{A_{s f}}{b_{w} \cdot d}=\frac{3188}{300 \times 600}=0.0177$
$\rho_{w}=\frac{5772}{b_{w} \cdot d}=\frac{5772}{300 \times 600}=0.032$
$\rho_{w, \text { max. }}=\rho_{\text {max. }}+\rho_{f}=0.0135+0.0177=0.0312<\rho_{w}=0.032$
From Table (4.3) for $\varepsilon_{t}=0.005, \rho_{\text {max. }}=0.0155$
$\rho_{w, \max .}=\rho_{\max .}+\rho_{f}=0.0155+0.0177=0.0332>\rho_{w}=0.32$
That is $0.005>\varepsilon_{t}>0.004$,
Check the location of the depth of the stress block (a);
$T=A_{s} \cdot f_{y}=5772 \times 10^{-6} \times 400=2.309 \mathrm{MN}$
$C_{f}=0.85 f_{c}^{\prime} b . h_{f}=0.85 \times 20 \times 0.8 \times 0.15=2.045 M N<T_{t}$
Therefor $\mathrm{a}>\mathrm{h}_{\mathrm{f}}$, and the beam is considered as a T beam and divided into two sections:

$$
\begin{aligned}
& M_{n 1}=A_{s f} f_{y}\left(d-h_{f} / 2\right)=3.188 \times 10^{-6} \times 400(0.6-0.15 / 2)=0.6695 M N . m \\
& A_{s w}=A_{s}-A_{s f}=5772-3188=2584 \mathrm{~mm}^{2} \\
& a=\frac{A_{s w} \cdot f_{y}}{0.85 f_{c}^{\prime} \cdot b_{w}}=\frac{2584 \times 400}{0.85 \times 20 \times 300}=203 \mathrm{~mm} \\
& c=203 / 0.85=239 \mathrm{~mm} \\
& \varepsilon_{t}=\frac{600-239}{239}(0.003)=0.00453 \\
& \phi=0.65+\frac{\varepsilon_{t}-0.002}{3}(250)=0.861 \\
& M_{n 2}=2584 \times 10^{-6}(0.6-0.203 / 2)=0.5152 M N . m \\
& \phi M_{n}=0.861(669.5+515.2)=1020 \mathrm{kN} . \mathrm{m}
\end{aligned}
$$

### 4.11 Analysis of Special Beam Shapes

Special shapes mean shapes other than rectangular or T shapes. The method of analyzing these sections is the same as that for T shapes, and can be summarized as follow:
i. Calculate the tensile force $T=A_{s} \cdot f_{y}$, which should be equalized by a compression force C ,
ii. The compression force $\mathrm{C}=0.85 f_{c}^{\prime}\left(\mathrm{A}_{\mathrm{c}}=\right.$ area under compression $)$ which is not rectangular, from this area the value of (a) can be found,
iii. Calculate the centroid of this area $\left(\mathrm{y}_{\mathrm{c}}\right)$ by taking the moment of areas about the compression face,
iv. The lever $\operatorname{arm} \mathrm{z}=\mathrm{d}-\mathrm{y}_{\mathrm{c}}$,
v. The nominal flexural strength $M_{n}=T . z=C . z$
vi. Check the values of $\mathrm{c}, \varepsilon_{t}$, and $\phi$.

## EXAMPLE 4.9

Calculate the design flexural strength for the section in Figure (4.12). $A_{s}=4 \# 25=1964 \mathrm{~mm}^{2}$ $f_{c}^{\prime}=30 \mathrm{MPa}$ and $f_{y}=400 \mathrm{MPa}$.

## SOLUTION



Figure (4.12) Example (4.9)
$\rho_{\text {min. }}=1.4 / 400=0.0035$
$\rho_{\text {min. }}=\sqrt{30} /(4 \times 400)=0.00342$
Therefore $\rho_{\text {min. }}=0.0035$
$\rho_{w}=1964 /(400 \times 540)=0.0091>\rho_{\text {min }} . \quad($ O.K. $)$
$T=0.001964 \times 400=0.7856 \mathrm{MN}$
$A_{c}=0.7856 /(0.85 \times 30)=0.0308078 \mathrm{~m}^{2}=30808 \mathrm{~mm}^{2}$
Area of the two parts in the compression zone $=2 \times 150 \times 100=30000 \mathrm{~mm}^{2}<\mathrm{A}_{c}$
Therefore a > 100
$30808=30000+400(\mathrm{a}-100)$
$\mathrm{a}=102 \mathrm{~mm}$
$\beta_{1}=0.85-0.00725(30-28)=0.836$
$c=102 / 0.836=122 \mathrm{~mm}$
$\varepsilon_{t}=\frac{d-c}{c} 0.003=\frac{540-122}{122} 0.003=0.01027>0.005$
Therefore $\phi=0.9$
$y_{c}=\frac{400 \times 102 \times 51-100 \times 100 \times 50}{30808}=51 \mathrm{~mm}$
$z=d-y_{c}=540-51=489 \mathrm{~mm}$
$M_{n}=0.7856 \times 0.489=0.3816 M N . m=381.6 \mathrm{kN} . \mathrm{m}$
$\phi M_{n}=0.9 \times 381.6=345.7 \mathrm{kN} . \mathrm{m}$

ALNOOR UNIVERSITY COLLEGE

Dept. of Building Engineering and Projects Management

# REINFORCED CONCRETE (I) 

C.E. 3234

FIRST SEMESTER 2023-2024

## CHATER FIVE

## STRENGTH DESIGN METHOD FOR BEAMS AND ONE-WAY SLABS

LECTURER<br>Dr. SA'AD A. AL-TA'AN

## Chapter Five Strength Design for Flexure

### 5.1 Introduction

In chapter four, analysis of various reinforced concrete sections were discussed using the strength design method. In all cases, the dimensions of the crosssection, area of steel, and materials strength $f_{c}^{\prime}$ and $f_{y}$ are given or known and the design flexural strength $\left(\varphi M_{n}\right)$ is required.
In this chapter, the strength design for flexure will be discussed. The loads (Ultimate loads), required moments (Ultimate moments), and materials strength are given or known and the required information are part or all of the crosssection dimensions and the area of steel.

### 5.2 Design of Single Reinforced Rectangular Sections

The design of rectangular sections may include determining the dimensions $b$ or $d$ or both of them and the area of steel. The dimensions and area of steel determine the mode of failure. Compression failure may be dangerous because it is brittle and occur suddenly without visual warnings. While tension failure, gives visual warnings like large deflection and wide cracks. To ensure such failure, the reinforcement ratio should not exceed $\rho_{\max }$. which is less than the balanced reinforcement ratio $\rho_{\mathrm{b}}$. The reinforcement ratio also, should be more than the minimum reinforcement ratio $\rho_{\text {min }}$., so that the section will not lose its strength at cracking unless the area of steel provided is $(4 / 3)$ of that required.
There are many sections that satisfy this requirement ( $\rho_{\min .} \leq \rho \leq \rho_{\text {max. }}$ ). If it is required to use small concrete section it means that $\rho \approx \rho_{\text {max }}$. but such sections are not economical because of the relatively large area of steel and the probability of large deflection. It is possible to use Table (5.1) to find the minimum depth for beams and one-way slabs:

Table (5.1) Minimum Depth of non-Prestressed beams and one-way slabs unless deflection is computed

|  | Minimum thickness h |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Simply <br> supported | One end <br> continuous | Both ends <br> continuous | Cantilever |
| Member | Members not supporting or attached to partitions or other <br> construction likely to be effected by large deflection |  |  |  |
| Solid one-way slabs | $\mathrm{L} / 20$ | $\mathrm{~L} / 24$ | $\mathrm{~L} / 28$ | $\mathrm{~L} / 10$ |
| Beams or ribbed one- <br> way slabs | $\mathrm{L} / 16$ | $\mathrm{~L} / 18.5$ | $\mathrm{~L} / 21$ | $\mathrm{~L} / 8$ |

## Notes:

The values in the above table are for cast in place normal concrete and $f_{y}=400 \mathrm{MPa}$. For other cases, the values should be multiplied by the following factors:
a- For lightweight concrete with density between ( 1440 to $1920 \mathrm{~kg} / \mathrm{m}^{3}$ ) multiply by $\left(1.65-0.0003 \mathrm{w}_{c}\right) \geq 1.09$
b- For $f_{y} \neq 400 \mathrm{MPa}$, multiply by $\left(0.42+f_{y} / 689\right)$.
At the beginning calculate the required $M_{n} \geq M_{u} / \phi, M_{u}$ required external flexural strength, and $\phi=$ strength reduction factor $=0.65-0.9$, depending on the value $\epsilon_{\mathrm{t}}$. If:
Max. $\mathrm{M}_{\mathrm{n}}=$ max. $\mathrm{k}_{\mathrm{n}} \mathrm{bd}^{2}>$ required $\mathrm{M}_{\mathrm{n}}$ (the beam is single under reinforced, and $\rho \leq \rho_{\max }$ ), and if:
Max. $\mathrm{M}_{\mathrm{n}}=$ max. $\mathrm{k}_{\mathrm{n}} \mathrm{bd}^{2}$ < required $\mathrm{M}_{\mathrm{n}}$
The beam is double reinforced.

### 5.2.1Determination of the Tension Reinforcement

If the dimensions of the cross-section are given or known, the following procedure is followed to find the necessary reinforcement area:
i. From the equilibrium equation

$$
\begin{gather*}
C=0.85 f_{c}^{\prime} a \cdot b=T=\rho \cdot b \cdot d \cdot f_{y}  \tag{5.1}\\
a=\rho \cdot d\left(f_{y} / 0.85 f_{c}^{\prime}\right)=\rho \cdot d \cdot m \tag{5.2}
\end{gather*}
$$

Where $m=f_{y} /\left(0.85 f_{c}^{\prime}\right)$
Using the moment equation:
$M_{n}=T . z=\rho . b . d . f_{y}(d-a / 2)$
Substituting (a) from Equation (5.2), the equation becomes:

$$
\begin{equation*}
M_{n}=\rho \cdot b \cdot d \cdot f_{y}\left(d-\frac{\rho \cdot f_{y} \cdot d}{2 \times 0.85 f_{c}^{\prime}}\right) \tag{5.3}
\end{equation*}
$$

Substituting ( $m=f_{y} / 0.85 f_{c}^{\prime}$ ) and dividing both sides of equation (5.3) by ( $\mathrm{bd}^{2}$ ), Equation (5.3) becomes:

$$
\begin{equation*}
M_{n} /\left(b d^{2}\right)=k_{n}=\rho \cdot f_{y}(1-\rho m / 2) \tag{5.4}
\end{equation*}
$$

Solving Equation (5.4) for the reinforcement ratio $\rho$ :

$$
\begin{equation*}
\rho=\frac{1}{m}\left[1-\sqrt{1-\frac{2 k_{n} \cdot m}{f_{y}}}\right] \tag{5.5}
\end{equation*}
$$

It is possible to use Equation (4.20) in chapter four in the following form to calculate the reinforcement ratio

$$
\begin{align*}
& M_{n} /\left(f_{c}^{\prime} b d^{2}\right)=\omega(1-0.59 \omega)  \tag{5.6}\\
& \omega=\rho f_{y} / f_{c}^{\prime}
\end{align*}
$$

The following Table is a solution of the above Equation.

## Table (5.2) Solution of Equation (5.6)

$$
\begin{array}{cccccccccc}
\omega=\frac{\rho f_{y}}{f_{c}^{\prime}} 0.000 & 0.001 & 0.002 & 0.003 & 0.004 & 0.005 & 0.006 & 0.007 & 0.008 & 0.009 \\
M_{n} /\left(f_{c}^{\prime} b d^{2}\right)=\omega(1-0.59 \omega)
\end{array}
$$

| . 00 | . 0000 | . 0010 | . 0020 | . 0030 | . 0040 | . 0050 | . 0060 | . 0070 | . 0080 | . 0090 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| . 01 | . 0099 | . 0109 | . 0119 | . 0129 | . 0139 | . 0149 | . 0159 | . 0168 | . 0178 | . 0188 |
| . 02 | . 0197 | . 0207 | . 0217 | . 0226 | . 0236 | . 0246 | . 0256 | . 0266 | . 0275 | . 0285 |
| . 03 | . 0295 | . 0304 | . 0314 | . 0324 | . 0333 | . 0343 | . 0352 | . 0362 | . 0372 | 0381 |
| . 04 | . 0391 | . 0400 | . 0410 | . 0419 | . 0429 | . 0438 | . 0448 | . 0457 | . 0466 | . 0476 |
| . 05 | . 0485 | . 0495 | . 0504 | . 0513 | . 0523 | . 0532 | . 0541 | . 0551 | . 0560 | . 0569 |
| . 06 | . 0579 | . 0588 | . 0597 | . 0607 | . 0616 | . 0625 | . 0634 | . 0643 | . 0653 | . 0662 |
| . 07 | . 0671 | . 0680 | . 0689 | . 0699 | . 0708 | . 0717 | . 0726 | . 0735 | . 0744 | 0753 |
| . 08 | . 0762 | . 0771 | . 0780 | . 0789 | . 0798 | . 0807 | . 0816 | . 0825 | . 0834 | . 0843 |
| . 09 | . 0852 | . 0861 | . 0807 | . 0879 | . 0888 | . 0897 | . 0906 | . 0915 | . 0923 | . 0932 |
| . 10 | . 0941 | . 0950 | . 0959 | . 0967 | . 0976 | . 0985 | . 0994 | . 1002 | . 1011 | . 1020 |
| . 11 | . 1029 | . 1037 | . 1046 | . 1055 | . 1063 | . 1072 | . 1081 | . 1089 | . 1098 | . 1106 |
| . 12 | . 1115 | . 1124 | . 1133 | . 1141 | . 1149 | . 1158 | . 1166 | . 1175 | . 1183 | . 1192 |
| . 13 | . 1200 | . 1209 | . 1217 | . 1226 | . 1234 | . 1243 | . 1251 | . 1259 | . 1268 | . 1276 |
| . 14 | . 1284 | . 1293 | . 1301 | . 1309 | . 1318 | . 1326 | . 1334 | . 1342 | . 1351 | . 1359 |
| . 15 | . 1367 | . 1375 | . 1384 | . 1392 | . 1400 | . 1408 | . 1416 | . 1425 | . 1433 | 1441 |
| . 16 | . 1449 | . 1457 | . 1465 | . 1473 | . 1481 | . 1489 | . 1497 | . 1506 | . 1514 | . 1522 |
| . 17 | . 1529 | . 1537 | . 1545 | . 1553 | . 1561 | . 1569 | . 1577 | . 1585 | . 1593 | . 1601 |
| . 18 | . 1609 | . 1617 | . 1624 | . 1632 | . 1640 | . 1648 | . 1656 | . 1664 | . 1671 | . 1679 |
| . 19 | . 1687 | . 1695 | . 1703 | . 1710 | . 1718 | . 1726 | . 1733 | . 1741 | . 1749 | . 1756 |
| . 20 | . 1764 | . 1772 | . 1779 | . 1787 | . 1794 | . 1802 | . 1810 | . 1817 | . 1825 | 1832 |
| . 21 | . 1840 | . 1847 | . 1855 | . 1862 | . 1870 | . 1877 | . 1885 | . 1892 | . 1900 | . 1907 |
| . 22 | . 1914 | . 1922 | . 1929 | . 1937 | . 1944 | . 1951 | . 1959 | . 1966 | . 1973 | . 1981 |
| . 23 | . 1988 | . 1995 | . 2002 | . 2010 | . 2017 | . 2024 | . 2031 | . 2039 | . 2046 | . 2053 |
| . 24 | . 2060 | . 2067 | . 2075 | . 2082 | . 2089 | . 2096 | . 2103 | . 2110 | . 2117 | . 2124 |
| . 25 | . 2131 | . 2138 | . 2145 | . 2152 | . 2159 | . 2166 | . 2173 | . 2180 | . 2187 | . 2194 |
| . 26 | . 2201 | . 2208 | . 2215 | . 2222 | . 2229 | . 2236 | . 2243 | . 2249 | . 2256 | . 2263 |
| . 27 | . 2270 | . 2277 | . 2284 | . 2290 | . 2297 | . 2304 | . 2311 | . 2317 | . 2324 | . 2331 |
| . 28 | . 2337 | . 2344 | . 2351 | . 2357 | . 2364 | . 2371 | . 2377 | . 2384 | . 2391 | . 2397 |
| . 29 | . 2404 | . 2410 | . 2417 | . 2423 | . 2430 | . 2437 | . 2443 | . 2450 | . 2456 | . 2463 |
| . 30 | . 2469 | . 2475 | . 2482 | . 2488 | . 2495 | . 2501 | . 2508 | . 2514 | . 2520 | . 2527 |
| . 31 | . 2533 | . 2539 | . 2546 | . 2552 | . 2558 | . 2565 | . 2571 | . 2577 | . 2583 | 2590 |
| . 32 | . 2596 | . 2602 | . 2608 | . 2614 | . 2621 | . 2627 | . 2633 | . 2639 | . 2645 | . 2651 |
| . 33 | . 2657 | . 2664 | . 2670 | . 2676 | . 2682 | . 2688 | . 2694 | . 2700 | . 2706 | 2712 |
| . 34 | . 2718 | . 2724 | . 2730 | . 2736 | . 2742 | . 2748 | . 2754 | . 2760 | . 2766 | . 2771 |
| . 35 | . 2777 | . 2783 | . 2789 | . 2795 | . 2801 | . 2807 | . 2812 | . 2818 | . 2824 | . 2830 |
| . 36 | . 2835 | . 2841 | . 2847 | . 2853 | . 2858 | . 2864 | . 2870 | . 2875 | . 2881 | . 2887 |
| . 37 | . 2892 | . 2898 | . 2904 | . 2909 | . 2915 | . 2920 | . 2926 | . 2931 | . 2937 | . 2943 |
| . 38 | . 2948 | . 2954 | . 2959 | . 2965 | . 2970 | . 2975 | . 2981 | . 2986 | . 2992 | 2997 |
| . 39 | . 3003 | . 3008 | . 3013 | . 3019 | . 3024 | . 3029 | . 3035 | . 3040 | . 3045 | . 3051 |
| . 40 | . 3056 | . 3061 | . 3067 | . 3072 | . 3077 | . 3082 | . 3087 | . 3093 | . 3098 | 3103 |

## Example (5.1)

A reinforced concrete beam with $\mathrm{b}=350 \mathrm{~mm}$, and $\mathrm{d}=540 \mathrm{~mm}$ is subjected to a factored moment of $450 \mathrm{kN} . \mathrm{m}$. If $f_{c}^{\prime}=25 M P a$ and $f_{y}=276 \mathrm{MPa}$, calculate the necessary area of steel.

## Solution

To calculate the required nominal moment $\left(M_{n}\right)$ the value of $\varphi$ must be known or assume a certain mode of failure subjected to a later check. Assume $\varphi=0.9$ :
$M_{n} \geq 450 / 0.9=500 \mathrm{kN} . \mathrm{m}$
Referring to Table (4.3) to find the value of $\operatorname{Max} \cdot k_{n}=5.69 \mathrm{MPa}$, the nominal maximum moment equal to:
$M a x . M_{n}=5.69 \times 0.35 \times 0.54^{2}=0.5807 M N . m=580.7 \mathrm{kN} . m>M_{n}=500 \mathrm{kN} . \mathrm{m}$ Therefore the section is single under reinforced ( $\rho<\rho_{\text {max }}$.)

$$
\begin{aligned}
& m=\frac{f_{y}}{0.85 f_{c}^{\prime}}=\frac{276}{0.85 \times 25}=12.99 \\
& k_{n}=\frac{M_{n}}{b . d^{2}}=\frac{0.5}{0.35 \times 0.54^{2}}=4.9 \mathrm{MPa}<{\operatorname{Max} . k_{n}}=5.69 \mathrm{MPa} \\
& \rho=\frac{1}{12.99}\left[1-\sqrt{1-\frac{2 \times 12.99 \times 4.9}{276}}\right]=0.0205<\rho_{\text {max. }}=0.0245
\end{aligned}
$$

Using Table (5.1) to find $\omega$ and then $\rho$ :
$\frac{M_{n}}{f_{c}^{\prime} \cdot b \cdot d^{2}}=\frac{0.5}{25 \times 0.35 \times 0.54^{2}}=0.196$
$\omega=0.226=\frac{\rho f_{y}}{f_{c}^{\prime}}$, then $\rho$ equal to $\rho=0.226 \times 25 / 276=0.0205$ :
The required area of steel equal to:

$$
A_{s}=0.0205 \times 350 \times 540=3875 \mathrm{~mm}^{2}
$$

To determine the number and diameter of bars, it is preferred to satisfy the following provisions:
i. Arrange the bars symmetrical about the vertical axis,
ii. Use at least two bars, one in each corner,
iii. For beams with normal dimensions, use bars with diameter $\leq 35 \mathrm{~mm}$ starting with the small sizes,
iv. Use no more than two diameters with one size difference to avoid mistakes during construction, e.g., $(22,28),(16,20)$ and not $(22,35)$ or (12, 25),
v. Arrange the bars in layer wherever possible,
vi. Satisfy the provisions for bars spacing,
vii. When using more than one layer of steel or more than one size of bars, put the larger sizes next to the tension face, and
viii. In large beams and columns, sometimes it is necessary to put the bars in bundles (two, three, or four) as shown below. These bundles should be surrounded by stirrups or ties.


Figure (5.1) arrangement of bars in bundles

Referring to Table (5.3) for bars sizes and diameter, there are many choices, but it is preferable to start with small sizes because it is to transport, cut and shape it. Try $8 \# 25=3928 \mathrm{~mm}^{2}$ and put it into two layers as shown.


Figure (5.2) Arrangement of bars for Example (5.1)
or it is possible to choose $4 \# 28$ in the bottom layer $=2646 \mathrm{~mm}^{2}$ and $4 \# 22=$ $1520 \mathrm{~mm}^{2}$ in the layer above, the sum $=4166 \mathrm{~mm}^{2}$.

Table (5.3) Area of group of bars

|  | Number of bars |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Dia. |  |  |  |  |  |  |  |  |  |  |  |
| Mm | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |
| 10 | 79 | 158 | 237 | 316 | 395 | 474 | 553 | 632 | 711 | 790 |  |
| 12 | 113 | 226 | 339 | 452 | 565 | 678 | 791 | 904 | 1017 | 1130 |  |
| 16 | 201 | 402 | 603 | 804 | 1005 | 1206 | 1407 | 1608 | 1809 | 2010 |  |
| 19 | 284 | 568 | 852 | 1136 | 1420 | 1704 | 1988 | 2272 | 2556 | 2840 |  |
| 22 | 380 | 760 | 1140 | 1520 | 1900 | 2280 | 2660 | 3040 | 3420 | 3800 |  |
| 25 | 491 | 982 | 1473 | 1964 | 2455 | 2946 | 3437 | 3928 | 4419 | 4910 |  |
| 28 | 616 | 1232 | 1848 | 2464 | 3080 | 3696 | 4312 | 4928 | 5544 | 6160 |  |
| 32 | 804 | 1608 | 2412 | 3216 | 4020 | 4824 | 5628 | 6432 | 7236 | 8040 |  |
| 35 | 962 | 1924 | 2886 | 3848 | 4810 | 5772 | 6734 | 7696 | 8658 | 9620 |  |

Bars \#44 and 57 mm are used in columns and rarely used in beams.
If the beam is not exposed to weather or in contact with soil, the reinforcement requires 40 mm clear cover. This cover protects the reinforcement from rusting,
fire, and integrates the bars with the other parts of the beam. The clear spacing between bars must be checked and should be not less than the following:
i. 25 mm ,
ii. Bar diameter, and
iii. (4/3) maximum aggregate size.

Table (5.4) shows:
i. The minimum width for different bar sizes using \#10 mm stirrups, when using larger bar size the difference is added to the two sides,
ii. Add the last figure in the last column for every additional bar,
iii. If the diameters are different, the width is limited to the small bars and the difference in the last column is added to the large bars, and
iv. Maximum coarse aggregate size should not exceed (3/4) clear spacing between bars.

Table 20.6.1.3.1- Specified concrete cover for cast in-place nonprestressed concrete members

| Concrete exposure | Member | Reinforcement | Specified cover |
| :--- | :---: | :---: | :---: |
| Cast against and <br> permanently in contact <br> with ground | All | All | 75 |
| Exposed to weather or in <br> contact with ground | All | No. 19 through No. 57 | 50 |
|  |  | No. 16 bar, W31 or <br> D31 wire, and smaller | 40 |
| Not exposed to weather or <br> in contact with ground | Slabs, joists, and <br> walls | No. 45 and No. 57 | 40 |
|  | Neams, columns, bar and smaller <br> pedestals, and <br> tension ties | Primary reinforcement, <br> stirrups, ties, spirals, <br> and hoops | 20 |

Table (5.4) Minimum width for beams according to ACI

| Dia. | Number of bars in single layer |  |  |  |  |  |  | For each <br> additional <br> bar |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Mm | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 37 |
| 12 | 177 | 214 | 251 | 288 | 325 | 362 | 399 | 41 |
| 16 | 181 | 222 | 263 | 304 | 345 | 389 | 427 | 47 |
| 19 | 184 | 228 | 272 | 316 | 360 | 404 | 448 | 44 |
| 22 | 187 | 234 | 281 | 328 | 375 | 422 | 469 | 47 |
| 25 | 190 | 240 | 290 | 340 | 390 | 440 | 490 | 50 |
| 28 | 196 | 252 | 308 | 364 | 420 | 476 | 532 | 56 |
| 32 | 204 | 268 | 332 | 396 | 460 | 524 | 588 | 64 |
| 35 | 210 | 280 | 350 | 420 | 490 | 560 | 630 | 70 |
| 44 | 232 | 320 | 408 | 496 | 584 | 672 | 760 | 88 |
| 57 | 271 | 385 | 499 | 613 | 727 | 841 | 955 | 114 |

A $=40 \mathrm{~mm}$ (clear cover to the stirrups),
B $=10 \mathrm{~mm}$ (diameter of stirrups),
$\mathrm{C}=20 \mathrm{~mm}$ for bars with diameter $\leq 35 \mathrm{~mm}$, for bars with dia. 44 and 57 mm $\mathrm{C}=\mathrm{d}_{\mathrm{b}} / 2$
$\mathrm{D}=$ clear distance between bars and equal to whichever is greater of the following limits:
i. 25 mm ,
ii. Bar diameter,
iii. (4/3) of the maximum coarse aggregate size.


Figure (5.3) Bars arrangement

Table 20.6.1.3.1- Specified concrete cover for cast in-place nonprestressed concrete members

| Concrete exposure | Member | Reinforcement | Specified cover |
| :--- | :---: | :---: | :---: |
| Cast against and <br> permanently in contact <br> with ground | All | All | 75 |
| Exposed to weather or in <br> contact with ground | All | No. 19 through No. <br> 57 | 50 |
|  |  | No. 16 bar, W31 or <br> D31 wire, and <br> smaller | 40 |
| Not exposed to weather or <br> in contact with ground | Slabs, joists, and <br> walls | No. 45 and No. 57 <br>  | No. 35 bar and <br> smaller |

### 5.2.2 Determination of Cross Section Dimensions and Steel Area

When the cross-section dimensions and area of steel are unknown, the steps below may be followed:
i. Assume a certain value for the reinforcement ratio ( $\rho_{\max .} \geq \rho \geq \rho_{\min .}$.),
ii. Calculate the shape factor $\mathrm{k}_{\mathrm{n}}$ from Equation (5.4)

$$
k_{n}=\rho . f_{y}(1-\rho m / 2)
$$

iii. Calculate $\left(\mathrm{bd}^{2}\right)$ from the following Equation:

$$
\text { b. } \cdot d^{2}=M_{n} / k_{n}
$$

iv. Choose a suitable dimensions for $b$ and $d$, and usually $b$ is fixed first and then d is calculated from ( $\mathrm{d}=\mathrm{Vbd}^{2} / \mathrm{b}$ ). to the value of d add the clear cover, stirrup diameter, half bar diameter to get the value of $h$ :
$\mathrm{h}=\mathrm{d}+$ clear cover + stirrup diameter $+\mathrm{d}_{\mathrm{b}} / 2$. Refer to ACI Code (20.6.1) for the concrete cover.
v. The value of h then rounded to the nearest ( 25 or 50 mm ),
vi. Calculate a new value of $d=h$ - clear cover - stirrup dia. $-d_{b} / 2$
vii. Calculate a new value of $k_{n}$ :
$k_{n}=\frac{M_{n}}{b d^{2}}$
viii. Calculate the value of $\rho$ as shown in the previous section or from Equation $M_{n} /\left(f_{c}^{\prime} b d^{2}\right)=\omega(1-0.59 \omega), \omega=\frac{\rho \cdot f_{y}}{f_{c}^{\prime}}$.

## Example (5.2)

Find the dimensions $\mathrm{b}, \mathrm{h}, \mathrm{d}$, and area of steel for a reinforced concrete rectangular beam to carry a factored (ultimate) $M_{u}=360 \mathrm{kN} . \mathrm{m} . f_{c}^{\prime}=30 \mathrm{MPa}$ and $f_{y}=400 \mathrm{MPa}$.

## Solution

Assume $\phi=0.9$ and it later on after determining the dimensions and area of steel.

$$
M_{n} \geq 360 / 0.9=400 \mathrm{kN} . \mathrm{m}
$$

Choose a suitable value for $\rho$ so that ( $\left.\rho_{\max .} \geq \rho \geq \rho_{\min }.\right),(0.020 \geq \rho \geq 0.0035)$
For example try $\rho=0.012$ (about $60 \%$ of $\rho_{\text {max }}$ )

$$
m=\frac{400}{0.85 \times 30}=15.69
$$

$$
k_{n}=0.012 \times 400(1-0.012 \times 15.69 / 2)=4.35 \mathrm{MPa} \prec{\mathrm{Max} . k_{n}}=6.74 \mathrm{MPa}
$$

$$
\text { b. } . d^{2}=\frac{M_{n}}{k_{n}}=\frac{0.4}{4.35}=0.091954 \mathrm{~m}^{3}=91954000 \mathrm{~mm}^{3}
$$

If $b$ is assumed equal to 300 mm , then d equal to:

$$
d=\sqrt{91954000 / 300}=554 \mathrm{~mm}
$$

to calculate h assume $\mathrm{d}_{\mathrm{s}}($ diameter of stirrups $)=10 \mathrm{~mm}$, clear concrete cover $=$ 40 mm , and $\mathrm{d}_{\mathrm{b}}$ ( 20 to 30 mm )
$h=554+40+10+25 / 2=617 \mathrm{~mm}$

This dimension is not practical, then use $h=600$ or 650 mm , with $\mathrm{h}=650 \mathrm{~mm} \mathrm{~d}$ will equal to:
$d=650-40-10-25 / 2=587 \mathrm{~mm}$ recalculate $\mathrm{k}_{\mathrm{n}}$ since d is changed
$k_{n}=\frac{0.4}{0.3 \times 0.587^{2}}=3.87 \mathrm{MPa}$
$\rho=\frac{1}{15.69}\left[1-\sqrt{1-\frac{2 \times 3.87 \times 15.69}{400}}\right]=0.01055$
Or use $\frac{\rho \cdot f_{y}}{f_{c}^{\prime}}=\omega$ after calculating $\frac{M_{n}}{f_{c}^{\prime} \cdot b \cdot d^{2}}=\frac{0.4}{30 \times 0.3 \times 0.587^{2}}=0.129$
From Table (5.2) $\omega=\frac{\rho \cdot f_{y}}{f_{c}{ }^{\prime}}=0.1405, \rho=0.01054$
$\rho=0.01055 \times 300 \times 587=1858 \mathrm{~mm}^{2}$
Referring to Tables (5.3 and 5.4), use $4 \# 25=1964 \mathrm{~mm}^{2}$, which requires 290 mm width.

Figure (5.4) Arrangement Of bars for Example (5.2)

## Example (5.3)



A one-way slab is simply supported on an effective span of $4.0 \mathrm{~m}(\mathrm{c} / \mathrm{c})$ and carries a working live load of 5.0 kPa , and a finishing DL of 2.0 kPa .
$f_{c}^{\prime}=20 \mathrm{MPa}$ and $f_{y}=400 \mathrm{MPa}$. Find the necessary thickness and area of steel.

## Solution

To find the required thickness, refer to Table (5.2), the minimum thickness $h \geq \frac{\text { span }}{20}=\frac{4000}{20}=200 \mathrm{~mm}$ to avoid excessive deflection. Take a strip equal to 1.0 width for the design sake.

The ultimate load on the slab $=w_{u}=1.2(2+4.8)+1.6 \times 5=16.16 \mathrm{kPa}$
External factored (ultimate moment) equal to:
$M_{u}=16.16(4)^{2} / 8=32.32 \mathrm{kN} . \mathrm{m} / \mathrm{m}$
$M_{n} \geq 32.32 / 0.9=35.91 \mathrm{kN} . \mathrm{m} / \mathrm{m}$
The effective depth of the slab $d=200-20-d_{b} / 2=200-20-12 / 2=174 \mathrm{~mm}$
$\operatorname{Max} . M_{n}=4.56 \times 1(0.174)^{2}=138.1 \mathrm{kN} . \mathrm{m} / \mathrm{m}>M_{n}=35.91 \mathrm{kN} . \mathrm{m} / \mathrm{m}$
The section is under reinforced $\rho<\rho_{\text {max }}$.

$$
\begin{aligned}
& m=400 /(0.85 \times 20)=23.53 \\
& k_{n}=\frac{0.03591}{1 \times 0.174^{2}}=1.19 M P a \\
& \rho=\frac{1}{23.53}\left[1-\sqrt{1-\frac{2 \times 1.19 \times 23.53}{400}}\right]=0.00309
\end{aligned}
$$

$$
A_{s}=0.00309 \times 1000 \times 174=538 \mathrm{~mm}^{2} / \mathrm{m}
$$

Referring to Table (5.5) for groups of bars in slabs,
Table (5.5) Area of groups of Bars in Slabs one meter width

| $\mathrm{d}_{\mathrm{b}}$ | Spacing of bars (mm) |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 50 | 75 | 100 | 125 | 150 | 175 | 200 | 225 | 250 | 275 | 300 |
| 6 | 560 | 373 | 280 | 224 | 187 | 160 | 140 | 124 | 112 | 102 | 93 |
| 8 | 1000 | 667 | 500 | 400 | 333 | 286 | 250 | 222 | 200 | 182 | 167 |
| 10 | 1571 | 1047 | 785 | 628 | 523 | 449 | 393 | 349 | 314 | 285 | 262 |
| 12 | 2260 | 1507 | 1130 | 904 | 753 | 646 | 565 | 502 | 452 | 411 | 377 |
| 16 | 4020 | 2680 | 2010 | 1608 | 1340 | 1149 | 1005 | 893 | 804 | 731 | 670 |
| 20 | 6280 | 4187 | 3140 | 2512 | 2093 | 1794 | 1570 | 1396 | 1256 | 1142 | 1047 |
| 22 | 7600 | 5067 | 3800 | 3040 | 2533 | 2171 | 1900 | 1689 | 1520 | 1382 | 1267 |
| 25 | ---- | 6547 | 4910 | 3928 | 3273 | 2806 | 2455 | 2182 | 1964 | 1785 | 1637 |
| 28 | --- | 8213 | 6160 | 4928 | 4107 | 3520 | 3080 | 2738 | 2464 | 2240 | 2059 |
| 32 | ---- | 10720 | 8040 | 6432 | 5360 | 4594 | 4020 | 3573 | 3216 | 2924 | 2680 |
| 35 | --- | 12827 | 9620 | 7696 | 6413 | 5497 | 4810 | 4276 | 3848 | 3498 | 3207 |

(\#10@ $125 \mathrm{~mm} \mathrm{c} / \mathrm{c}=628 \mathrm{~mm}^{2} / \mathrm{m}$, or \#12 @ $200 \mathrm{~mm} \mathrm{c} / \mathrm{c}=565 \mathrm{~mm}^{2} / \mathrm{m}$ ).
Or the spacing can be calculated manually as shown below:
No. of bars $=538 / 113=4.76$
Sps. $=1000 / 4.76=210 \mathrm{~mm}$
Use 200 mm spacing c /c
Provided $A_{s}=(1000 / 200) 113=565 \mathrm{~mm}^{2} / \mathrm{m}$

$$
\rho_{g}=\frac{565}{1000 \times 200}=0.00283>\text { Min. } \rho_{g}=0.0018
$$

Since the slab is one-way (load transferred in the span direction) shrinkage and temperature steel must be provided in the perpendicular direction:

$$
0.0018 \times 1000 \times 200=360 \mathrm{~mm}^{2} / \mathrm{m}
$$

$\# 10$ bars every $200 \mathrm{~mm} \mathrm{c} / \mathrm{c}$ provide an area $=(1000 / 200) 79=395 \mathrm{~mm}^{2}$ as shown in the figure below.


Figure (5.5) Arrangement of steel for Example (5.3)

## Ribbed slabs

Ribbed slabs is a slab with small ribs spaced uniformly and covered with a thin slab with thickness ranging between $50-75 \mathrm{~mm}$. A ribbed slab must satisfy the following three limitations:
i. Rib width $\mathrm{b}_{\mathrm{w}} \geq 100 \mathrm{~mm}$,
ii. Rib depth $\leq 3.5 b_{w}$, and
iii. Clear spacing between ribs should not exceed 750 mm .

A slab not satisfying the above three limitations has to be considered as a beamslab system.

## Example (5.4)

Resolve Example (5.3) using one-way ribbed slab.

## Solution

To find the required thickness, refer to Table (5.1), the minimum thickness $h \geq \frac{\operatorname{span}}{16}=\frac{4000}{16}=250 \mathrm{~mm}$ to avoid excessive deflection.
Self-weight of one rib $/ \mathrm{m}=[0.2(0.1+0.13) / 2] 24+0.6 \times 0.05 \times 24=1.27 \mathrm{kN} / \mathrm{m}$
Self-weight $/ \mathrm{m}^{2}=1.27 / 0.6=2.12 \mathrm{kPa}$


The ultimate load on the slab $=w_{u}=1.2(2+2.12)+1.6 \times 5=12.94 \mathrm{kPa}$
External factored (ultimate moment) equal to:
$M_{u}=12.94(4)^{2} / 8=25.88 \mathrm{kN} . \mathrm{m} / \mathrm{m}$
$M_{n} \geq 25.88 / 0.9=28.76 \mathrm{kN} . \mathrm{m} / \mathrm{m}$
The effective depth of the slab $\mathrm{d}=250-20-\mathrm{d}_{\mathrm{b}} / 2=250-20-12 / 2=224 \mathrm{~mm}$ Max. $M_{n}=4.56 \times 1(0.224)^{2}=228.8 \mathrm{kN} . \mathrm{m} / \mathrm{m}>M_{n}=28.76 \mathrm{kN} . \mathrm{m} / \mathrm{m}$
The section is under reinforced $\rho<\rho_{\text {max }}$.

$$
\begin{aligned}
& m=400 /(0.85 \times 20)=23.53 \\
& k_{n}=0.02876 /\left(1 \times 0.224^{2}\right)=0.57 M P a \\
& \rho=\frac{1}{23.53}\left[1-\sqrt{1-\frac{2 \times 0.57 \times 23.53}{400}}\right]=0.00145 \\
& A_{s}=0.00145 \times 1000 \times 224=325 \mathrm{~mm}^{2} / \mathrm{m}
\end{aligned}
$$

$$
\operatorname{Min} . A_{s}=0.0018 \times 1000 \times 250=450 \mathrm{~mm}^{2} / \mathrm{m}
$$

$$
A_{s} / \mathrm{rib}=0.6 \times 450=270 \mathrm{~mm}^{2} / \mathrm{rib} \text {, use } 1 \# 20 \text { for each rib }=314 \mathrm{~mm}^{2} .
$$

The top slab has to be reinforced using shrinkage and temperature reinforcement,
$A_{s}=0.0018 \times 1000 \times 50=90 \mathrm{~mm}^{2} / \mathrm{m}$, max. spacing $=5 \times 50=250 \mathrm{~mm}$ or 450 mm , therefore max. spacing $=250 \mathrm{~mm}$, refer to Table (5.4) above, use \# 6 @ 250 mm c/c both directions.

### 5.3 Design of Double Reinforced Rectangular Sections

When the beams dimensions are fixed for architectural reasons or because of its existence in a certain location, and the maximum nominal moment ( $\mathrm{Max}_{\mathrm{n}} \mathrm{M}_{\mathrm{n}}$ ) as a single reinforced section is less than the required nominal flexural strength $\left(\mathrm{M}_{\mathrm{n}}\right)$, additional steel in tension and compression must be used to increase the moment capacity. The compression steel as mentioned previously may be used to act as:
i. Stirrups hanger,
ii. To decrease the long-term deflection due to creep and shrinkage, and iii. in zones where moment reversal may occur.

The design method starts by comparing (Max. $\mathrm{M}_{\mathrm{n}}$ ) with the required nominal flexural strength $\left(M_{n}\right)$. If $\left(M_{n}>\operatorname{Max}_{n}\right)$, the beam must be designed as a double reinforced section, otherwise it is a single reinforced section, i.e.; ( $\rho<\rho_{\text {max. }}$ ). The required flexural strength is divided into two parts, the first one $\left(\mathrm{M}_{\mathrm{n} 1}\right)$ equal to (Max. $\mathrm{M}_{\mathrm{n}}$ ), Figure (5.6)

$$
\begin{align*}
& M_{n 1}=\text { Max. } M_{n}=\text { Max. } k_{n} \cdot b \cdot d^{2}  \tag{5.7}\\
& A_{s 1}=\rho_{\max } \cdot b \cdot d \tag{5.8}
\end{align*}
$$

Or $\mathrm{A}_{\mathrm{s} 1}$ may be calculated as follow:

$$
\begin{equation*}
A_{s 1}=\frac{M_{n 1}}{f_{y}\left(d-a_{\max .} / 2\right)} \tag{5.9}
\end{equation*}
$$

The other part of the moment $\left(\mathrm{M}_{\mathrm{n} 2}\right)$ equal to:
$M_{n 2}=M_{n}-M_{n 1}$
The two forces of the couple $\left(\mathrm{M}_{\mathrm{n} 2}\right)$ equal to:

$$
\begin{align*}
& C_{2}=T_{2}=M_{n 2} /\left(d-d^{\prime}\right)  \tag{5.11}\\
& A_{s 2}=T_{2} / f_{y}  \tag{5.12}\\
& A_{S}=A_{s 1}+A_{s 2} \tag{5.13}
\end{align*}
$$

Check the stress in the compression steel, $c=a_{\text {max }} / / \beta_{1}$

$$
f_{s}^{\prime}=600\left(\frac{c-d^{\prime}}{c}\right)
$$


$M_{n 1}=T_{1} \cdot z=C_{1} \cdot z$ $=k_{n} \cdot b \cdot d^{2}$

$T_{1}=A_{s 1} \cdot f_{y}$


Figure (5.6) Design of double reinforced sections
If $f_{s}^{\prime} \geq f_{y}$ this means the compression steel will reach yielding, and to calculate the area of the comp
ed first:
$f_{s}^{\prime \prime}=f_{y}-0.85 f_{c}^{\prime}$
If $f_{s}^{\prime}<f_{y}$ then the effective stress equal to:
$f_{s}^{\prime \prime}=f_{s}^{\prime}-0.85 f_{c}^{\prime}$
The area of the compression steel equal to:

$$
A_{s}^{\prime}=C_{2} / f_{s c}^{\prime \prime \prime}
$$

The ACI recommend that transverse reinforcement (stirrups, ties, or equivalent welded wire fabric) should be used where there is compression steel to avoid the probability of bucking.

## EXAMPLE 5.5

A rectangular beam with $\mathrm{b}=250 \mathrm{~mm}, \mathrm{~h}=500 \mathrm{~mm}$ is simply supported on an effective span of 5.0 m . The beam carries a uniformly distributed working live
load of $30 \mathrm{kN} / \mathrm{m}$ and a uniformly distributed working dead load of $10 \mathrm{kN} / \mathrm{m}$ (excluding the beam weight). Calculate the necessary area of steel. $f_{c}^{\prime}=20 \mathrm{MPa}$ and $f_{y}=400 \mathrm{MPa}$.

## SOLUTION

Calculate the beam weight and add it to the DL,
Beam weight $=0.25 \times 0.5 \times 1 \times 24=3.0 \mathrm{kN} / \mathrm{m}$
Total dead load $=10+2=13 \mathrm{kN} / \mathrm{m}$
The factored (ultimate) load $w_{u}=1.2 \times 13+1.6 \times 30=63.6 \mathrm{kN} / \mathrm{m}$
The factored (ultimate) external moment $M_{u}=63.6(5)^{2} / 8=198.75 \mathrm{kN} . \mathrm{m}$
Assume a value for $\phi$ to calculate the required $\mathrm{M}_{\mathrm{n}}$, a minimum value of 0.817 can be assumed with $\left(\varepsilon_{t}=0.004, \rho_{\text {max. }}=0.0155\right)$, or $\left(\phi=0.9,0.005=\varepsilon_{t}\right.$,

$$
\begin{aligned}
\rho_{\text {max. }} & \left.=0.0135, \operatorname{Max} . k_{n}=4.56 \mathrm{MPa}\right), \text { then } \mathrm{M}_{\mathrm{n}} \text { equal to: } \\
M_{n} & >198.75 / 0.9=220.8 \mathrm{kN} . \mathrm{m}
\end{aligned}
$$

Assume an effective depth $\mathrm{d}=410 \mathrm{~mm}$, since the reinforcement will be arranged in two layers. The maximum nominal flexural strength equal to:

Max. $M_{n}=4.56 \times 0.25 \times 0.41^{2}=0.191 .6 M N . m=191.6 \mathrm{kN} . \mathrm{m}<$ required $M_{n}=220.8 \mathrm{kN} . \mathrm{m}$
Therefore the beam must be designed as a double reinforced section.
$M_{n 1}=\operatorname{Max} . M_{n}=191.6 \mathrm{kN} . \mathrm{m}$
$A_{s 1}=0.0135 \times 250 \times 410=1384 \mathrm{~mm}^{2}$
$a=0.32 d=0.32 \times 410=131 \mathrm{~mm}$
$c=0.375 d=0.375 \times 410=154 \mathrm{~mm}$.
Assume d' $=60 \mathrm{~mm}$,

$$
\begin{aligned}
& f_{S}^{\prime}=\frac{154-60}{154}(600)=366.2 \mathrm{MPa} \\
& f_{s}^{\prime \prime}=f_{S}^{\prime}-0.85 f_{c}^{\prime}=366.2-0.85 \times 20=349.2 \mathrm{MPa} \\
& M_{n 2}=M_{n}-M_{n 1}=220.8-191.6=29.2 \mathrm{kN} . \mathrm{m} \\
& C_{2}=T_{2}=M_{n 2} /\left(d-d^{\prime}\right)=0.0292 /(0.41-0.06)=0.0834 \mathrm{MN}
\end{aligned}
$$

$\mathrm{A}_{\mathrm{s} 2}=0.0834 / 400=209 \mathrm{~mm}^{2}$
$\mathrm{A}_{\mathrm{s}}=1384+209=1593 \mathrm{~mm}^{2}$
$\mathrm{A}_{\mathrm{s}}{ }^{\prime}=0.0834 / 349.2=239 \mathrm{~mm}^{2}$
Refer to Tables (5.2) and (5.3) to choose the bars, there are many choices to provide the reinforcement, $3 \# 22+3 \# 16=1743 \mathrm{~mm}^{2}$, in two layers in the tension zone, and $2 \# 16$ in the compression zone, Figure (5.7). Check the assumed effective depth:
$\mathrm{d}=500-40-10-22-25 / 2=415 \mathrm{~mm}$

Which is greater than the assumed value by $1.2 \%$, if the calculation is repeated for this value of (d) the new area of steel will not differ from too much from the final value.


## Figure (5.7) Arrangement of the

 bars for Example (5.5)EXAMPLE (5.6)
A rectangular beam with $\mathrm{b}=300 \mathrm{~mm}, \mathrm{~h}=600 \mathrm{~mm}$ carries a working DLM of $100 \mathrm{kN} . \mathrm{m}$ and a LLM of $150 \mathrm{kN} . \mathrm{m} . f_{c}^{\prime}=30 \mathrm{MPa}$ and $f_{y}=345 \mathrm{MPa}$.calculate the required area of steel so that $\rho_{1}=\left[A_{s 1} /(b . d)\right]=0.35 \rho_{b}$ to limit the deflection according to the recommendation of the ACI Committee 435 .

## SOLUTION

The external factored (ultimate) moment equal to:

$$
\begin{aligned}
& M_{u}=1.2 \times 100+1.6 \times 150=360 \mathrm{kN} . \mathrm{m} \\
& \rho_{b}=0.85 \times 0.836 \frac{30}{345} \frac{600}{600+345}=0.0392 \\
& \rho_{1}=0.35 \times 0.0392=0.0137
\end{aligned}
$$

Form the equilibrium equation $C=T$
$0.85 \times 30 \times a \times b=0.0137 b . d \times 345$
$a=0.185 d$
This value is less than $0.31 d=a$ for $\left(\varepsilon_{t}=0.005, \rho_{\text {max. }}=0.0232\right)$ and $(\phi=0.9)$, the total required nominal flexural strength equal to:

$$
M_{n} \geq 360 / 0.9=400 k N . m
$$

The required nominal flexural strength of the single reinforced beam $\left(M_{n 1}\right)$ equal to:
$M_{n 1}=0.85 \times 30 \times 0.185 d . b(d-0.185 / 2)=4.28 b . d^{2}$
It is possible to use Equation (5.4) to find $\mathrm{k}_{\mathrm{n}}, m=345 /(0.85 \times 30)=13.53$ :

$$
k_{n}=\rho \cdot f_{y}(1-\rho m / 2)=0.0137 \times 345(1-0.137 \times 13.53 / 2)=4.28 M P a
$$

Assume d=510 mm,

$$
M_{n 1}=4.28 b \cdot d^{2}=4.28 \times 0.3 \times 0.51^{2}=334 \mathrm{kN} \cdot \mathrm{~m}<M_{n}=400 \mathrm{kN} \cdot \mathrm{~m}
$$

Therefore the beam must be double reinforced, and the moment must be divided into two parts:
$A_{s 1}=0.0137 \times 300 \times 510=2096 \mathrm{~mm}^{2}$
$a=0.185 \times 510=94 \mathrm{~mm} \quad c=94 / 0.836=112 \mathrm{~mm}$
$M_{n 2}=M_{n}-M_{n 1}=400-334=66 \mathrm{kN} . \mathrm{m}$
Assume $d^{\prime}=60 \mathrm{~mm}$,
$C_{2}=T_{2}=M_{n 2} /\left(d-d^{\prime}\right)=0.066 /(0.51-0.06)=0.147 M N$
$A_{s 2}=0.147 / 345=426 \mathrm{~mm}^{2}$
$A_{s}=2096+426=2522 \mathrm{~mm}^{2}$
$f_{S}^{\prime}=600 \frac{112-60}{112}=278.6 \mathrm{MPa}$
$f_{s}^{\prime \prime}=f_{s}^{\prime}-0.85 f_{c}^{\prime}=278.6-0.85 \times 30=253.1 \mathrm{MPa}$
$A_{s}^{\prime}=0.147 / 253.1=581 \mathrm{~mm}^{2}$
Use $4 \# 25+2 \# 20=2592 \mathrm{~mm}^{2}$, four in the bottom layer and two in the upper layer in the tension zone and $2 \# 20$
$=628 \mathrm{~mm}^{2}$ in the compression zone as in Figure (5.8). check the value of d :

$$
d=600-40-10-25-25 / 2=513 \mathrm{~mm} \approx 510 \mathrm{~mm}
$$



Figure (5.8) Arrangement of steel for Example (5.6)

This is approximately equal to the assumed value.
If the value of $\rho_{1}$ is not assigned to limit the deflection, it is possible to design the section as single reinforced one with $\left(A_{s}=2565 \mathrm{~mm}^{2}\right)$, but the difference between the two beams is that the double reinforced beam will have less deflection than the single reinforced especially the long-term deflection.

### 5.4 Design of T-beams

The design of T-beams include determination of the flange thickness $\left(h_{f}\right)$, effective flange width (b), web width ( $\mathrm{b}_{\mathrm{w}}$ ), total depth (h), and area of steel ( $\mathrm{A}_{\mathrm{s}}$ ). The thickness of the flange $\left(\mathrm{h}_{\mathrm{f}}\right)$ is determined before designing the beam. The effective width (b) depends on the span and distance between the beams which determined during preparing the preliminary drawings, and therefore the web dimensions and area of steel are unknowns. When choosing the web dimensions, the following conditions must be satisfied if possible:
i. Keeping the reinforcement ratio low to avoid excessive deflection,
ii. Keeping the shear strength that depends on $\left(b_{w} . d\right)$ and $\left(f_{c}^{\prime}\right)$ more than the external factored shear force,
iii. In continuous beams the dimensions $\left(b_{w}\right)$ and (d) are compatible with the requirements of the negative moment region, where the section is designed as a rectangular section with dimensions $\left(b_{w} \cdot d\right)$.
In addition to the longitudinal (main) reinforcement, the ACI code recommends that the flange must be reinforced perpendicular to the longitudinal axis of the beam in locations where the main slab reinforcement is parallel to the beam. The transverse reinforcement should be designed to carry the factored load on the cantilever part of the flange with a span $\left[\left(b-b_{w}\right) / 2\right]$ as shown in the Figure below. The maximum spacing for this reinforcement should not exceed $5 h_{f}$ or 450 mm .


If the flange is in tension (Negative moment region), the reinforcement is placed at the top and part of it should be distributed in the effective flange width (b) or (span/10) whichever is smaller. If (b) > (span/10) longitudinal reinforcement should be placed on the exterior part of the flange between the limits of (b) and (span/10) to limit the cracks that may occur outside the web.
To find the area of steel, calculate the nominal flexural strength $\left(\mathrm{M}_{\mathrm{nf}}\right)$ assuming that the whole flange is carrying compression $\left(a=h_{f}\right)$ :
$M_{n f}=0.85 f_{c}^{\prime} \cdot h_{f} \cdot b\left(d-h_{f} / 2\right)$
This value is compared with the required $M_{n}$, if:
i. $\mathrm{M}_{\mathrm{n}}<\mathrm{M}_{\mathrm{nf}}$, therefore $\mathrm{a}<\mathrm{h}_{\mathrm{f}}$, and the beam is treated as a rectangular beam with dimensions ( $b \times d$ ),
ii. $\mathrm{M}_{\mathrm{n}}=\mathrm{M}_{\mathrm{nf}}, \mathrm{a}=\mathrm{h}_{\mathrm{f}}$, and the beam is treated as a rectangular beam with dimensions ( $b \times d$ ),
iii. $\quad M_{n}>M_{n f}, a>h_{f}$, and the beam is a T-beam.

In the last case, the area of steel is composed of two parts, the first one equalizing the compression on the flange (outside the web sides) $\mathrm{A}_{\mathrm{sf}}$ :

$$
\begin{equation*}
A_{s f}=0.85 f_{c}^{\prime} h_{f}\left(b-b_{w}\right) / f_{y} \tag{5.18}
\end{equation*}
$$

The nominal strength for this part equal to:
$M_{n 1}=A_{s f} \cdot f_{y}\left(d-h_{f} / 2\right)$
The second part of the moment $\mathrm{M}_{\mathrm{n} 2}$, is carried by a rectangular section of dimension ( $b_{w}$ d).The area of steel for this part ( $A_{s w}$ ) is calculated as for single reinforced sections.

## EXAMPLE 5.7

A reinforced concrete floor composed of a slab 100 mm thick and supported on a series of beams 1.30 m center to center and a span of 6.0 m , Figure ( 5.10 ). $\mathrm{b}_{\mathrm{w}}=300 \mathrm{~mm}, \mathrm{~h}=600 \mathrm{~mm}, \mathrm{M}_{\mathrm{u}}=720 \mathrm{kN} . \mathrm{m}$. $f_{c}^{\prime}=20 \mathrm{MPa}$ and $f_{y}=400 \mathrm{MPa}$. Find the necessary area of steel.

## SOLUTION

Calculate the effective flange width (b):
$b \leq$ span $/ 4=6.0 / 4=1.5 \mathrm{~m}$


Figure (5.10) Reinforced concrete floor of Example (5.7)
$b \leq b_{w}+16 h_{f}=300+16 \times 100=1900 \mathrm{~mm}$
$b \leq b_{w}+$ clear distance between beams $=300+1000=1300 \mathrm{~mm}$
Therefore $\mathrm{b}=1.30 \mathrm{~m}$.
Assume $\mathrm{d}=510 \mathrm{~mm}$, since the reinforcement may be arranged in two layers, assume also $\varphi=0.9$ and its value will be checked later.
The required flexural strength $M_{n}$ equal to:
$M_{n} \geq 720 / 0.9=800 \mathrm{kN} . \mathrm{m}$
If the whole flange is under compression, the nominal flexural strength $\mathrm{M}_{\mathrm{nf}}$ equal to:
$M_{n f}=0.85 \times 20 \times 0.1 \times 1.3(0.51-0.1 / 2)=1.017 M N . m=1017 \mathrm{kN} . \mathrm{m}>800 \mathrm{kN} . \mathrm{m}$
Therefore $\mathrm{a}<\mathrm{h}_{\mathrm{f}}$ and the beam can be considered as a rectangular beam with dimensions ( $b \times d$ ). to calculate the required area of tension steel, use Equations (5.4. and 5.5)

$$
k_{n}=\frac{0.8}{1.3 \times 0.51^{2}}=2.37 \mathrm{MPa}
$$

$m=\frac{400}{0.85 \times 20} 23.53$
$\rho=\frac{1}{23.53}\left[1-\sqrt{1-\frac{2 \times 2.37 \times 23.53}{400}}\right]=0.0064$
$A_{s}=0.0064 \times 1300 \times 510=4243 \mathrm{~mm}^{2}$
Compare $\rho_{w}$ with $\rho_{w, \text { max }}$ to check whether the beam is under or over reinforced:
$\rho_{w}=\frac{4243}{300 \times 510}=0.0277$
$A_{s f}=\frac{0.85 \times 20 \times 0.10(1.3-0.3)}{400}=0.00425 \mathrm{~m}^{2}=4250 \mathrm{~mm}^{2}$
$\rho_{f}=\frac{4250}{300 \times 510}=0.0278$
From Table (4.3), $\left(\varepsilon_{t}=0.005,0.0135=\rho_{\text {max }}\right.$, and $\left.\rho_{\max }=0.0135\right)$, therefore $\rho_{w, \text { max }}$ equal to:
$\rho_{w, \text { max }}=0.0135+0.0278=0.0413>\rho_{w}=0.0277$
the beam is under reinforced, use $3 \# 35$ and $3 \# 25=4359 \mathrm{~mm}^{2}$, the actual value of $d$ equal to:
$y_{c}=\frac{3 \times 962 \times 67.5+3 \times 491 \times 122.5}{4356}=86 \mathrm{~mm}$
$\mathrm{d}=600-86=514 \mathrm{~mm}$, a small difference of $0.78 \%$. The value of a is calculated for this area of steel,
$a=\frac{4356 \times 400}{0.85 \times 20 \times 1300}=79 \mathrm{~mm} \quad, c=77 / 0.85=93 \mathrm{~mm}$
$\varepsilon_{t}=(514-93) 0.003 / 93=0.01358>0.005$, therefore $\varphi=0.9$ as assumed before and the beam is under reinforced.

Figure (5.11) Arrangement of bars for Example (5.7)


## EXAMPLE 5.8

In example (5.7) if the moment $\mathrm{M}_{\mathrm{u}}$ is increased to $960 \mathrm{kN} . \mathrm{m}$, determine the required area of steel.

## SOLUTION

Assume a value of 0.9 for $\phi$ subjected to a later check. The required nominal flexural strength $\mathrm{M}_{\mathrm{n}}$ equal to:
$M_{n} \geq 960 / 0.9=1067 \mathrm{kN} . \mathrm{m}>M_{n f}=1017 \mathrm{kN} . \mathrm{m}$
Therefore $\mathrm{a}>\mathrm{h}_{\mathrm{f}}$ and it is a T beam. First the area of steel required to equalize the compression in the wings of the flange is calculated, from the previous example it is $\mathrm{A}_{\mathrm{sf}}=4250 \mathrm{~mm}^{2}$. The nominal flexural strength for this area of steel equal to:
$M_{n 1}=A_{s f} \cdot f_{y}\left(d-h_{f} / 2\right)$
$=0.004250 \times 400(0.51-0.15 / 2)=0.739 .5 M \mathrm{~N} . \mathrm{m}=739.5 \mathrm{kN} . \mathrm{m}$
The remaining moment $\mathrm{M}_{\mathrm{n} 2}$ equal to:
$M_{n 2}=1067-739.5=327.5 \mathrm{kN} . \mathrm{m}$
$k_{n}=\frac{0.3275}{0.3 \times 0.51^{2}}=4.2 \mathrm{MPa}$
$\rho=\frac{1}{23.53}\left[1-\sqrt{1-\frac{2 \times 4.2 \times 23.53}{400}}\right]=0.0123$
$A_{s w}=0.0123 \times 300 \times 510=1882 \mathrm{~mm}^{2}$
$A_{s}=A_{s f}+A_{s w}=4250+1882=6132 \mathrm{~mm}^{2}$
$\rho_{w}=\frac{6132}{300 \times 510}=0.0401$
Compare $\rho_{w}=0.0382$ with $\rho_{w, \text { max }}=0.0413$, therefore the section is under reinforced and $\varepsilon_{t}>0.005$ and $\varphi=0.9$.
Use $3 \# 35+6 \# 28=6582 \mathrm{~mm}^{2}$ can be used in three layers as shown below. The value of $d=600-40-10-35-25-28 / 2=476 \mathrm{~mm}$, or can be calculated exactly as:

$$
y_{T}=\frac{3 \times 962 \times 67.5+3 \times 616 \times 124+3 \times 616 \times 177}{6582}=114 \mathrm{~mm}
$$

$\mathrm{d}=600-114=486 \mathrm{~mm}$, the difference is $4.7 \%$ while the provided area of steel is already $7.3 \%$ more than required. However, if the area of steel is recalculated for this new depth of ( 486 mm ), the area of steel $=6584 \mathrm{~mm}^{2}$ which is approximately equal to the provided area of $6582 \mathrm{~mm}^{2}$.


## Figure (5.12) Arrangement of bars

for Example (5.8)

### 5.6 DESIGN OF BEAMS WITH SPECIAL SHAPES

When the area under compression is not rectangular, the steps below may be followed to find the required area of steel:
i. Assume a value of the lever arm $(z=0.8-0.9 \mathrm{~d})$,
ii. Assume a value of $\varphi$ subjected to later check,
iii. From the first equilibrium equation, calculate C and $T: C=T=M n / z$,
iv. Calculate the required area under compression: $A c=C /\left(0.85 f c^{\prime}\right)$,
v. Divide the area into components (triangles and rectangles) and calculate the centroid of this area (yc), $\mathrm{z}=\mathrm{d}-\mathrm{yc}$
vi. Repeat steps 3 to 5 until the values converge.
vii. The required area of steel then equal to $A_{s}=0.85 f_{c}^{\prime}\left(A_{c}\right) / f_{y}$,
viii. Compare this area of steel with maximum $\mathrm{A}_{\mathrm{s}}$, to check whether the section is under or over reinforced,
ix. Calculate c to find $\varepsilon_{t}, \phi$ and compare with assumed value at the beginning.

## EXAMPLE 5.9

The section in Figure (5.13) is subjected to a factored moment $\mathrm{M}_{\mathrm{u}}=360 \mathrm{kN} . \mathrm{m}$. $f_{c}^{\prime}=20 \mathrm{MPa}$ and $f_{y}=276 \mathrm{MPa}$, find the required area of steel.

## SOLUTION

$$
\begin{aligned}
& \text { Assume } \phi=0.9 \\
& M_{n} \geq 360 / 0.9=400 \mathrm{kN} . \mathrm{m}
\end{aligned}
$$

Assume $\mathrm{z}=500 \mathrm{~mm}(0.77 \mathrm{~d})$, since the area at the top is a triangle,
$C=T=M_{n} / z=0.4 / 0.5=0.8 M N$

$$
A_{c}=\frac{C}{0.85_{c}^{\prime}}=\frac{0.8}{0.85 \times 20}=0.047059 \mathrm{~m}^{2}=47059 \mathrm{~mm}^{2}
$$

This area is more than that of the triangle $\left(22500 \mathrm{~mm}^{2}\right)$ at the top, therefore a > 150 mm :
$47059=22500+300(a-150)$
$a=232 \mathrm{~mm}$

$$
\begin{aligned}
& c=232 / 0.85=273 \mathrm{~mm} \\
& \varepsilon_{t}=\frac{650-273}{273} 0.003=0.004143
\end{aligned}
$$

The section is in the transition zone, and $\phi$ equal to:

$$
\phi=0.65+(0.00413-0.002)(250 / 3)=0.829
$$

Assume $\phi=0.8$

$$
\begin{aligned}
& M_{n} \geq 360 / 0.8=450 \mathrm{kN} . \mathrm{m} \\
& C=T=M_{n} / z=0.45 / 0.5=0.9 \mathrm{MN} \\
& A_{c}=\frac{C}{0.85_{c}^{\prime}}=\frac{0.9}{0.85 \times 20}=0.052941 \mathrm{~m}^{2}=52941 \mathrm{~mm}^{2} \\
& 52941=22500+300(a-150)
\end{aligned}
$$

$$
a=251 \mathrm{~mm}
$$

$$
c=251 / 0.85=295 \mathrm{~mm}
$$

$$
\varepsilon_{t}=\frac{650-295}{295} 0.003=0.00361<0.004
$$

The section should be designed as a double reinforced section, the required nominal flexural strength equal to:
$M_{n}=360 / 0.817=440.6 \mathrm{kN} . \mathrm{m}$
Take the value of $a=a_{\text {max. }}=0.36 d=234 m m$ and calculate the maximum area under compression:
Max. $A_{c}=22500+(234-150) 300=47700 \mathrm{~mm}^{2}$

The centroid of this area:
$y_{c}=\frac{22500(100)+300(234-150)(150+42)}{47700}=149 \mathrm{~mm}$
$z=650-149=501 \mathrm{~mm}$
The maximum nominal flexural strength equal to:
$\operatorname{Max} . M_{n}=M_{n}=0.85 f_{c}^{\prime}\left(\right.$ Max. $\left._{c}\right) z=$
$0.85 \times 20 \times 0.477 \times 0.501=406.3 \mathrm{kN} . \mathrm{m}$
$A_{s 1}=\frac{0.85 f_{c}^{\prime} \cdot M a x . A_{c}}{f y}=\frac{0.85 \times 20 \times 0.0477}{276}=2938 \mathrm{~mm}^{2}$
$M_{n 2}=440.6-406.3=34.3 \mathrm{kN} . \mathrm{m}$

$$
\begin{aligned}
& C_{2}=T_{2}=0.0343 /(0.65-0.06)=0.0581 M N \\
& A_{S 2}=0.0581 / 276=211 \mathrm{~mm}^{2}
\end{aligned}
$$

$$
A_{s}=2938+211=3149 \mathrm{~mm}^{2} \quad \text { use } 3 \# 32+2 \# 25=3394 \mathrm{~mm}^{2}
$$

$$
\mathrm{c}=234 / 0.85=275 \mathrm{~mm}, \quad f_{s}^{\prime}=600 \frac{275-60}{275}=469 \mathrm{MPa}>f_{y}=276 \mathrm{MPa}
$$

$$
A_{s}^{\prime}=0.0581 /(276-0.85 \times 20)=224 \mathrm{~mm}^{2}, \text { use } 2 \# 12=226 \mathrm{~mm}^{2}
$$

### 5.6 REINFORCING VERTICAL FACES FOR DEEP BEAMS

Deep beams require longitudinal reinforcement in addition to the main tension reinforcement to limit the width of cracks that extend from the tension face upward (in case of positive moment), or downward (in case of negative moment). The ACI recommend that for beams with total depth more than 900 mm , additional steel should be provided on the vertical faces between the mid depth and the tension face, the spacing of this reinforcement should not exceed:

$$
\begin{equation*}
s=375\left(276 / f_{s}\right)-2.5 C_{c} \leq 305\left(276 / f_{s}\right) \tag{5.20}
\end{equation*}
$$

Where $\mathrm{C}_{\mathrm{c}}=$ clear concrete cover to the vertical face of the beam, and $f_{s}=$ working steel stress which may be taken $=\left(2 f_{y} / 3\right)$. The used bars diameter ranges between 10 to 16 mm . Such additional reinforcement may be included in the flexural strength calculation.

ALNOOR UNIVERSITY COLLEGE Dept. of Building Engineering and
Projects Management

## REINFORCED CONCRETE (I)

C.E. 3234

FIRST SEMESTER 2023-2024
CHAPTER SIX
DESIGN OF TWO-WAY SLABS BY THE APPROXIMATE METHOD

LECTURER<br>Dr. SA'AD A. AL-TA'AN

## Introduction

## TYPES OF SLABS

In reinforced concrete construction, slabs are used to provide flat, useful surfaces. A reinforced concrete slab is a broad, flat plate, usually horizontal, with top and bottom surfaces parallel or nearly so. It may be supported by reinforced concrete beams (and usually cast monolithically with such beams), by masonry or reinforced concrete walls, by structural steel members, directly by columns, or continuously by the ground.



Fig. (6.1) Types of Slabs, (a) One-way slab, (b) Two-way slab, (c) One-way slab, (d) Flat plate, (e) Flat slab with drop panel, (f) Two-way ribbed slab.

Approximate thickness of a two-way slab supported on beams or walls can be found by the following equation:

$$
h=\frac{\text { Perimeter }}{180}
$$

The coefficients in Tables (1-4) can be used to bending moments in the x and y directions. The values in the Tables are based on the assumptions that the supports are rigid.

The deflection for the two perpendicular strips are equal at the intersection point:
$\Delta_{a}=\frac{5 w_{a} L_{a}^{4}}{384 E I}=\Delta_{b}=\frac{5 w_{b} L_{b}^{4}}{384 E I}$
$w_{a} L_{a}^{4}=w_{b} L_{b}^{4}$
$w_{a} / w_{b}=\left(L_{b}^{4} / L_{a}^{4}\right)=1 / m^{4} \quad m=L_{a} / L_{b}$
$w_{a}=w_{b} / m^{4}$
$w_{T}=w_{a}+w_{b}=w_{b} / m^{4}+w_{b}=w_{b}\left(\frac{m^{4}+1}{m^{4}}\right)$
$w_{b}=w_{T}\left(\frac{m^{4}}{m^{4}+1}\right) \quad w_{a}=w_{T}\left(\frac{1}{m^{4}+1}\right)$


Percentage of load transferred in each direction

| $\mathrm{L}_{\mathrm{a}} / \mathrm{L}_{\mathrm{b}}$ | 1.0 | $2 / 3=0.667$ | $1 / 2=0.5$ | $1 / 3=0.333$ | $1 / 4=0.25$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{~W}_{\mathrm{a}} / \mathrm{w}_{\mathrm{T}}$ | $1 / 2=.5$ | $81 / 97=.835$ | $16 / 17=.941$ | $81 / 82=.988$ | $256 / 257=.996$ |
| $\mathrm{~W}_{\mathrm{b}} / \mathrm{w}_{\mathrm{T}}$ | $1 / 2=.5$ | $16 / 97=.165$ | $1 / 17=.059$ | $1 / 82=.012$ | $1 / 257=.004$ |

## EXAMPLE 1:

A two-way reinforced concrete building floor system is composed of slab panels measuring $6.0 \times 7.5 \mathrm{~m}(\mathrm{c} / \mathrm{c})$ in plan, supported by shallow column-line beams cast monolithically with the slab ( $350 \times 500 \mathrm{~mm}$ ). Using $f_{c}^{\prime}=30 \mathrm{MPa}$ and $f_{y}=400$ MPa , design a typical exterior panel to carry a service finishing $\mathrm{DL}=2.67 \mathrm{kPa}$ and live load of 5 kPa in addition to the self-weight of the floor.

## SOLUTION:

$\mathrm{L}_{\mathrm{a}}=6-0.35=5.65 \mathrm{~m}$
$\mathrm{L}_{\mathrm{b}}=7.5-0.35=7.15 \mathrm{~m}$
$h=\frac{\text { Perimeter }}{180}=\frac{2(5.65+7.15)}{180}=0.142 \mathrm{~m}=142 \mathrm{~mm}$
Use $\mathrm{h}=175 \mathrm{~mm}$,
Self-weight of the slab $=0.175 \times 1 \times 1 \times 24=4.2 \mathrm{kPa}$,


Total $\mathrm{DL}=4.2+2.67=6.87 \mathrm{kPa}$,
Ultimate dead load on the slab $\mathrm{w}_{\mathrm{ud}}=1.2 \times 6.87=8.24 \mathrm{kPa}$.
Ultimate live load on the slab $\mathrm{w}_{\mathrm{uL}}=1.6 \times 5=8 \mathrm{kPa}$.
Total ultimate load on the slab $\mathrm{w}_{\mathrm{us}}=8.24+8=16.24 \mathrm{kPa}$.

## Moments in the short direction:

$\mathrm{m}=\mathrm{L}_{\mathrm{a}} / \mathrm{L}_{\mathrm{b}}=5.65 / 7.15=0.79$
referring to Tables (1-4), the slab is case (9).
Negative moment:

From Table 1, the coefficient equal to: 0.075;
$-v e M_{u 1}=0.075 \times 16.24(5.65)^{2}=38.79 \mathrm{kN} . \mathrm{m} / \mathrm{m}$
$-v e M_{u 2}=38.79 \mathrm{kN} . \mathrm{m} / \mathrm{m}$

## Positive moment:

$+v e M_{u d}=0.029 \times 8.24(5.65)^{2}=7.63 \mathrm{kN} . \mathrm{m} / \mathrm{m}$
$+v e M_{u L}=0.042 \times 8(5.65)^{2}=10.73 \mathrm{kN} . \mathrm{m} / \mathrm{m}$
$+v e M_{u a}=7.63+10.73=18.36 \mathrm{kN} . \mathrm{m} / \mathrm{m}$
$\underline{\mathrm{d}}_{\text {short }}=175-20-12 / 2=149 \mathrm{~mm}$

## Moments in the Long direction:

## Negative moment:

From Table 1, the coefficient equal to: 0.055;
$-v e M_{u 1}=0.017 \times 16.24(7.15)^{2}=14.11 \mathrm{kN} . \mathrm{m} / \mathrm{m}$
$-v e M_{u 2}=0 \mathrm{kN} . \mathrm{m} / \mathrm{m}$

## Positive moment:

$$
\begin{aligned}
& +v e M_{u a d}=0.01 \times 8.24(7.15)^{2}=4.21 \mathrm{kN} . \mathrm{m} / \mathrm{m} \\
& +v e M_{u a L}=0.017 \times 8(7.15)^{2}=7.16 \mathrm{kN} . \mathrm{m} / \mathrm{m} \\
& +v e M_{u a}=4.21+7.16=11.37 \mathrm{kN} . \mathrm{m} / \mathrm{m} \\
& \mathrm{~d}_{\text {long }}=149-12=137 \mathrm{~mm}
\end{aligned}
$$

Summary of two-way slab design

|  | Short direction |  |  | Long direction |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | -ve $\mathrm{M}_{\mathrm{u}}$ | +ve $\mathrm{M}_{\mathrm{u}}$ | -ve $\mathrm{M}_{\mathrm{u}}$ | -ve $\mathrm{M}_{\mathrm{u}}$ | +ve $\mathrm{M}_{\mathrm{u}}$ | -ve Mu |
| $\mathrm{M}_{\mathrm{u}}$ | 38.79 | 18.36 | 38.79 | 14.11 | 11.37 | 0 |
| $\mathrm{M}_{\mathrm{n}}$ | 43.1 | 20.4 | 43.1 | 15.67 | 12.63 | 0 |
| d | 149 | 149 | 149 | 137 | 137 | 137 |
| $\mathrm{k}_{\mathrm{n}}$ (MPa) | 1.94 | 0.92 | 1.94 | 0.83 | 0.67 | 0 |
| $\mathrm{f}_{\mathrm{y}} /\left(0.85 \mathrm{f}_{\mathrm{c}}{ }^{\text {c }}\right.$ ) | 15.69 | 15.69 | 15.69 | 15.69 | 15.69 | 15.69 |
| $\rho$ | 0.00505 | 0.002343 | 0.00505 | 0.00211 | 0.0017 | 0 |
| $\mathrm{A}_{\text {s }}$ | 752 | 349 | 752 | 289 | 233 | 0 |
| Min. $\mathrm{A}_{\text {s }}$ | $0.0018 \times 1000 \times 175=315 \mathrm{~mm}^{2} / \mathrm{m}$ |  |  |  |  |  |
| Req. $\mathrm{A}_{\text {s }}$ | 752 | 349 | 752 | 315 | 315 | 315 |
| Prov. $\mathrm{A}_{\text {s }}$ | $\begin{gathered} \# 12 \\ @ 150 \\ \text { mmc /c } \end{gathered}$ | $\begin{gathered} \text { \#10@ } \\ 200 \mathrm{~mm} \\ \mathrm{c} / \mathrm{c} \end{gathered}$ | $\begin{gathered} \# 12 \\ @ 150 \mathrm{~mm} \\ \mathrm{c} / \mathrm{c} \end{gathered}$ | $\begin{gathered} \# 10 \\ @ 250 \mathrm{~mm} \\ \mathrm{c} / \mathrm{c} \\ \hline \end{gathered}$ | $\begin{gathered} \# 10 \\ @ 250 \\ \mathrm{mmc} / \mathrm{c} \end{gathered}$ | $\begin{gathered} \# 10 \\ @ 250 \\ \mathrm{mmc} / \mathrm{c} \end{gathered}$ |

## Load transferred in both directions:

## Short direction:

$\mathrm{w}_{\mathrm{a}}=0.83 \times 16.24=13.48 \mathrm{kN} / \mathrm{m} \quad(13.48 / 2=6.74 \mathrm{kN} / \mathrm{m})$ on each support

## Long direction:

$\mathrm{w}_{\mathrm{b}}=0.17 \times 16.24=2.76 \mathrm{kN} / \mathrm{m} \quad(2.76 / 2=1.38 \mathrm{kN} / \mathrm{m})$ on each support

Table 1
Coefficients for negative moments in slabs
$M_{a}^{-}=C_{a}^{-} w L_{a}^{2}$
$M_{b}^{-}=C_{b}^{-} w L_{b}^{2}$
$w=$ Factored uniform dead load + live load

|  |  | Boundary Conditions |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\stackrel{\text { ®f }}{\text { fis }}$ | $\begin{aligned} & 8 \\ & \text { B } \\ & \text { B } \\ & \text { D } \\ & \hline \end{aligned}$ | Case 1 $\square$ | Case 2 $\square$ | Case 3 | Case 4 $\square$ | Case 5 $\square$ | Case 6 $\square$ | Case 7 $\square$ | Case 8 $\square$ | Case 9 $\square$ |
| 1.00 | $\begin{aligned} & C_{a}^{-} \\ & C_{b}^{-} \end{aligned}$ |  | $\begin{aligned} & 0.045 \\ & 0.045 \end{aligned}$ | 0.076 | $\begin{aligned} & 0.050 \\ & 0.050 \end{aligned}$ | 0.075 | 0.071 | 0.071 | $\begin{aligned} & 0.033 \\ & 0.061 \end{aligned}$ | $\begin{aligned} & 0.061 \\ & 0.033 \end{aligned}$ |
| 0.95 | $\begin{aligned} & C_{a}^{-} \\ & C_{b}^{-} \end{aligned}$ |  | $\begin{aligned} & 0.050 \\ & 0.041 \end{aligned}$ | 0.072 | $\begin{aligned} & 0.055 \\ & 0.045 \end{aligned}$ | 0.079 | 0.075 | 0.067 | $\begin{aligned} & 0.038 \\ & 0.056 \end{aligned}$ | $\begin{aligned} & 0.065 \\ & 0.029 \end{aligned}$ |
| 0.90 | $\begin{aligned} & C_{a}^{-} \\ & C_{b}^{-} \end{aligned}$ |  | $\begin{aligned} & 0.055 \\ & 0.037 \end{aligned}$ | 0.070 | $\begin{aligned} & 0.060 \\ & 0.040 \end{aligned}$ | 0.080 | 0.079 | 0.062 | $\begin{aligned} & 0.043 \\ & 0.052 \end{aligned}$ | $\begin{aligned} & 0.068 \\ & 0.025 \end{aligned}$ |
| 0.85 | $\begin{aligned} & C_{a}^{-} \\ & C_{b}^{-} \\ & \hline \end{aligned}$ |  | $\begin{aligned} & 0.060 \\ & 0.031 \end{aligned}$ | 0.065 | $\begin{aligned} & 0.066 \\ & 0.034 \end{aligned}$ | 0.082 | 0.083 | 0.570 | $\begin{aligned} & 0.049 \\ & 0.046 \end{aligned}$ | $\begin{aligned} & 0.072 \\ & 0.021 \end{aligned}$ |
| 0.80 | $\begin{aligned} & C_{a}^{-} \\ & C_{b}^{-} \end{aligned}$ |  | $\begin{aligned} & 0.056 \\ & 0.027 \end{aligned}$ | 0.061 | $\begin{aligned} & 0.071 \\ & 0.029 \end{aligned}$ | 0.083 | 0.086 | 0.051 | $\begin{aligned} & 0.055 \\ & 0.041 \end{aligned}$ | $\begin{aligned} & 0.075 \\ & 0.017 \end{aligned}$ |
| 0.75 | $\begin{aligned} & C_{a}^{-} \\ & C_{b}^{-} \end{aligned}$ |  | $\begin{aligned} & 0.069 \\ & 0.022 \end{aligned}$ | 0.056 | $\begin{aligned} & 0.076 \\ & 0.024 \end{aligned}$ | 0.085 | 0.088 | 0.044 | $\begin{aligned} & 0.061 \\ & 0.036 \end{aligned}$ | $\begin{aligned} & 0.078 \\ & 0.014 \end{aligned}$ |
| 0.70 | $\begin{aligned} & C_{a}^{-} \\ & C_{b}^{-} \end{aligned}$ |  | $\begin{aligned} & 0.074 \\ & 0.017 \end{aligned}$ | 0.050 | $\begin{aligned} & 0.081 \\ & 0.019 \end{aligned}$ | 0.086 | 0.091 | 0.038 | $\begin{aligned} & 0.068 \\ & 0.029 \end{aligned}$ | $\begin{aligned} & 0.081 \\ & 0.011 \end{aligned}$ |
| 0.65 | $\begin{aligned} & C_{a}^{-} \\ & C_{b}^{-} \\ & \hline \end{aligned}$ |  | $\begin{aligned} & 0.077 \\ & 0.014 \end{aligned}$ | 0.043 | $\begin{aligned} & 0.085 \\ & 0.015 \end{aligned}$ | 0.087 | 0.093 | 0.031 | $\begin{aligned} & 0.074 \\ & 0.024 \end{aligned}$ | $\begin{aligned} & 0.083 \\ & 0.008 \end{aligned}$ |
| 0.60 | $\begin{aligned} & C_{a}^{-} \\ & C_{b}^{-} \end{aligned}$ |  | $\begin{aligned} & 0.081 \\ & 0.010 \end{aligned}$ | 0.035 | $\begin{aligned} & 0.089 \\ & 0.011 \end{aligned}$ | 0.088 | 0.095 | 0.024 | $\begin{aligned} & 0.080 \\ & 0.018 \end{aligned}$ | $\begin{aligned} & 0.085 \\ & 0.006 \end{aligned}$ |
| 0.55 | $\begin{aligned} & C_{a}^{-} \\ & C_{b}^{-} \end{aligned}$ |  | $\begin{aligned} & 0.084 \\ & 0.007 \end{aligned}$ | 0.028 | $\begin{aligned} & 0.092 \\ & 0.008 \end{aligned}$ | 0.089 | 0.096 | 0.019 | $\begin{aligned} & 0.085 \\ & 0.014 \end{aligned}$ | $\begin{aligned} & 0.086 \\ & 0.005 \end{aligned}$ |
| 0.50 | $\begin{aligned} & C_{a}^{-} \\ & C_{b}^{-} \end{aligned}$ |  | $\begin{aligned} & 0.086 \\ & 0.006 \end{aligned}$ | 0.022 | $\begin{aligned} & 0.094 \\ & 0.006 \end{aligned}$ | 0.090 | 0.097 | 0.014 | $\begin{aligned} & 0.089 \\ & 0.010 \end{aligned}$ | $\begin{aligned} & 0.088 \\ & 0.003 \end{aligned}$ |

Table 2
Coefficients for dead load positive moments in slabs
$M_{a}^{+}=C_{a}^{+} w_{d} L_{a}^{2}$
$M^{+}=C^{+} w^{2} I^{2}$
$w_{d l}=$ Factored uniform dead load

| $\underset{\sim}{n}$ |  | Boundary Conditions |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Case 1 $\square$ | Case 2 $\square$ | Case 3 | Case 4 $\square$ | Case 5 $\square$ | Case 6 $\square$ | Case 7 | Case 8 $\square$ | Case 9 $\square$ |
| 1.00 | $C_{a}^{+}$ | 0.036 | 0.018 | 0.018 | 0.027 | 0.027 | 0.033 | 0.027 | 0.020 | 0.023 |
|  | $C_{b}^{+}$ | 0.036 | 0.018 | 0.027 | 0.027 | 0.018 | 0.027 | 0.033 | 0.023 | 0.020 |
| 0.95 | $C_{a}^{+}$ | 0.040 | 0.020 | 0.021 | 0.030 | 0.028 | 0.036 | 0.031 | 0.022 | 0.024 |
|  | $C_{b}^{+}$ | 0.033 | 0.016 | 0.025 | 0.024 | 0.015 | 0.024 | 0.031 | 0.021 | 0.017 |
| 0.90 | C | 0.045 | 0.022 | 0.025 | 0.033 | 0.029 | 0.039 | 0.035 | 0.025 | 0.026 |
|  | $C_{b}^{+}$ | 0.029 | 0.014 | 0.024 | 0.022 | 0.013 | 0.021 | 0.028 | 0.019 | 0.015 |
| 0.85 | C | 0.050 | 0.024 | 0.029 | 0.036 | 0.031 | 0.042 | 0.040 | 0.029 | 0.028 |
|  | $C_{b}^{+}$ | 0.026 | 0.012 | 0.022 | 0.019 | 0.011 | 0.017 | 0.025 | 0.017 | 0.013 |
| 0.80 | C | 0.056 | 0.026 | 0.034 | 0.039 | 0.032 | 0.045 | 0.045 | 0.032 | 0.029 |
|  | $C_{b}^{+}$ | 0.023 | 0.011 | 0.020 | 0.016 | 0.009 | 0.015 | 0.022 | 0.015 | 0.010 |
| 0.75 | $C_{a}^{+}$ | 0.061 | 0.028 | 0.040 | 0.043 | 0.033 | 0.048 | 0.051 | 0.036 | 0.031 |
|  | $C_{b}^{+}$ | 0.019 | 0.009 | 0.018 | 0.013 | 0.007 | 0.012 | 0.020 | 0.013 | 0.007 |
| 0.70 | $C_{a}^{+}$ | 0.068 | 0.030 | 0.046 | 0.046 | 0.035 | 0.051 | 0.058 | 0.040 | 0.033 |
|  | $C_{b}^{+}$ | 0.016 | 0.007 | 0.016 | 0.011 | 0.005 | 0.009 | 0.017 | 0.011 | 0.006 |
| 0.65 | $C_{a}^{+}$ | 0.074 | 0.032 | 0.054 | 0.050 | 0.036 | 0.054 | 0.065 | 0.044 | 0.034 |
|  | $C_{b}^{+}$ | 0.013 | 0.006 | 0.014 | 0.009 | 0.004 | 0.007 | 0.014 | 0.009 | 0.005 |
| 0.60 | $C_{a}^{+}$ | 0.081 | 0.034 | 0.062 | 0.053 | 0.037 | 0.056 | 0.073 | 0.048 | 0.036 |
|  | $C_{b}^{+}$ | 0.010 | 0.004 | 0.011 | 0.007 | 0.003 | 0.006 | 0.012 | 0.007 | 0.004 |
| 0.55 | $C_{a}^{+}$ | 0.088 | 0.035 | 0.071 | 0.056 | 0.038 | 0.058 | 0.081 | 0.052 | 0.037 |
|  | $C_{b}^{+}$ | 0.008 | 0.003 | 0.009 | 0.005 | 0.002 | 0.004 | 0.009 | 0.005 | 0.003 |
| 0.50 | $C_{a}^{+}$ | 0.095 | 0.037 | 0.080 | 0.059 | 0.039 | 0.061 | 0.089 | 0.056 | 0.038 |
|  | $C_{b}^{+}$ | 0.006 | 0.002 | 0.007 | 0.004 | 0.001 | 0.003 | 0.007 | 0.004 | 0.002 |

Table 3
Coefficients for live load positive moments in slabs
$M_{a}^{+}=C_{a}^{+} w_{l l} L_{a}^{2}$ $M_{b}^{+}=C_{b}^{+} w_{l l} L_{b}^{2}$
$w_{l l}=$ Factored uniform live load

|  |  | Boundary Conditions |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\underbrace{n}_{n}$ | $\begin{aligned} & \text { 8 } \\ & \text { y } \\ & \text { E } \\ & \text { E } \end{aligned}$ | Case 1 $\square$ | Case 2 $\square$ | Case 3 $\square$ | Case 4 | Case 5 $\square$ | Case 6 $\square$ | Case 7 $\square$ | Case 8 $\square$ | Case 9 $\square$ |
| 1.00 | $\begin{aligned} & C_{a}^{+} \\ & C_{b}^{+} \end{aligned}$ | $\begin{aligned} & 0.036 \\ & 0.036 \end{aligned}$ | $\begin{aligned} & 0.027 \\ & 0.027 \end{aligned}$ | $\begin{aligned} & 0.027 \\ & 0.032 \end{aligned}$ | $\begin{aligned} & 0.032 \\ & 0.032 \end{aligned}$ | $\begin{aligned} & 0.032 \\ & 0.027 \end{aligned}$ | $\begin{aligned} & 0.035 \\ & 0.032 \end{aligned}$ | $\begin{aligned} & 0.032 \\ & 0.035 \end{aligned}$ | $\begin{aligned} & 0.028 \\ & 0.030 \end{aligned}$ | $\begin{aligned} & 0.030 \\ & 0.028 \end{aligned}$ |
| 0.95 | $\begin{aligned} & C_{a}^{+} \\ & C_{b}^{+} \end{aligned}$ | $\begin{aligned} & 0.040 \\ & 0.033 \end{aligned}$ | $\begin{aligned} & 0.030 \\ & 0.025 \end{aligned}$ | $\begin{aligned} & 0.031 \\ & 0.029 \end{aligned}$ | $\begin{aligned} & 0.035 \\ & 0.029 \end{aligned}$ | $\begin{aligned} & 0.034 \\ & 0.024 \end{aligned}$ | $\begin{aligned} & 0.038 \\ & 0.029 \end{aligned}$ | $\begin{aligned} & 0.036 \\ & 0.032 \end{aligned}$ | $\begin{aligned} & 0.031 \\ & 0.027 \end{aligned}$ | $\begin{aligned} & 0.032 \\ & 0.025 \end{aligned}$ |
| 0.90 | $\begin{aligned} & C_{a}^{+} \\ & C_{b}^{+} \end{aligned}$ | $\begin{aligned} & 0.045 \\ & 0.029 \end{aligned}$ | $\begin{aligned} & 0.034 \\ & 0.022 \end{aligned}$ | $\begin{aligned} & 0.035 \\ & 0.027 \end{aligned}$ | $\begin{aligned} & 0.039 \\ & 0.026 \end{aligned}$ | $\begin{aligned} & 0.037 \\ & 0.021 \end{aligned}$ | $\begin{aligned} & 0.042 \\ & 0.025 \end{aligned}$ | $\begin{aligned} & 0.040 \\ & 0.029 \end{aligned}$ | $\begin{aligned} & 0.035 \\ & 0.024 \end{aligned}$ | $\begin{aligned} & 0.036 \\ & 0.022 \end{aligned}$ |
| 0.85 | $\begin{aligned} & C_{a}^{+} \\ & C_{b}^{+} \end{aligned}$ | $\begin{aligned} & 0.050 \\ & 0.026 \end{aligned}$ | $\begin{aligned} & 0.037 \\ & 0.019 \end{aligned}$ | $\begin{aligned} & 0.040 \\ & 0.024 \end{aligned}$ | $\begin{aligned} & 0.043 \\ & 0.023 \end{aligned}$ | $\begin{aligned} & 0.041 \\ & 0.019 \end{aligned}$ | $\begin{aligned} & 0.046 \\ & 0.022 \end{aligned}$ | $\begin{aligned} & 0.045 \\ & 0.026 \end{aligned}$ | $\begin{aligned} & 0.040 \\ & 0.022 \end{aligned}$ | $\begin{aligned} & 0.039 \\ & 0.029 \end{aligned}$ |
| 0.80 | $\begin{aligned} & C_{a}^{+} \\ & C_{b}^{+} \end{aligned}$ | $\begin{aligned} & 0.056 \\ & 0.023 \end{aligned}$ | $\begin{aligned} & 0.041 \\ & 0.017 \end{aligned}$ | $\begin{aligned} & 0.045 \\ & 0.022 \end{aligned}$ | $\begin{aligned} & 0.048 \\ & 0.020 \end{aligned}$ | $\begin{aligned} & 0.044 \\ & 0.016 \end{aligned}$ | $\begin{aligned} & 0.051 \\ & 0.019 \end{aligned}$ | $\begin{aligned} & 0.051 \\ & 0.023 \end{aligned}$ | $\begin{aligned} & 0.044 \\ & 0.019 \end{aligned}$ | $\begin{aligned} & 0.042 \\ & 0.017 \end{aligned}$ |
| 0.75 | $\begin{aligned} & C_{a}^{+} \\ & C_{b}^{+} \end{aligned}$ | $\begin{aligned} & 0.061 \\ & 0.019 \end{aligned}$ | $\begin{aligned} & 0.045 \\ & 0.014 \end{aligned}$ | $\begin{aligned} & 0.051 \\ & 0.019 \end{aligned}$ | $\begin{aligned} & 0.052 \\ & 0.016 \end{aligned}$ | $\begin{aligned} & 0.047 \\ & 0.013 \end{aligned}$ | $\begin{aligned} & 0.055 \\ & 0.016 \end{aligned}$ | $\begin{aligned} & 0.056 \\ & 0.020 \end{aligned}$ | $\begin{aligned} & 0.049 \\ & 0.016 \end{aligned}$ | $\begin{aligned} & 0.046 \\ & 0.013 \end{aligned}$ |
| 0.70 | $\begin{aligned} & C_{a}^{+} \\ & C_{b}^{+} \end{aligned}$ | $\begin{aligned} & 0.068 \\ & 0.016 \end{aligned}$ | $\begin{aligned} & 0.049 \\ & 0.012 \end{aligned}$ | $\begin{aligned} & 0.057 \\ & 0.016 \end{aligned}$ | $\begin{aligned} & 0.057 \\ & 0.014 \end{aligned}$ | $\begin{aligned} & 0.051 \\ & 0.011 \end{aligned}$ | $\begin{aligned} & 0.060 \\ & 0.013 \end{aligned}$ | $\begin{aligned} & 0.063 \\ & 0.017 \end{aligned}$ | $\begin{aligned} & 0.054 \\ & 0.014 \end{aligned}$ | $\begin{aligned} & 0.050 \\ & 0.011 \end{aligned}$ |
| 0.65 | $\begin{aligned} & C_{a}^{+} \\ & C_{b}^{+} \end{aligned}$ | $\begin{aligned} & 0.074 \\ & 0.013 \end{aligned}$ | $\begin{aligned} & 0.053 \\ & 0.010 \end{aligned}$ | $\begin{aligned} & 0.064 \\ & 0.014 \end{aligned}$ | $\begin{aligned} & 0.062 \\ & 0.011 \end{aligned}$ | $\begin{aligned} & 0.055 \\ & 0.009 \end{aligned}$ | $\begin{aligned} & 0.064 \\ & 0.010 \end{aligned}$ | $\begin{aligned} & 0.700 \\ & 0.014 \end{aligned}$ | $\begin{aligned} & 0.059 \\ & 0.011 \end{aligned}$ | $\begin{aligned} & 0.054 \\ & 0.009 \end{aligned}$ |
| 0.60 | $\begin{aligned} & C_{a}^{+} \\ & C_{b}^{+} \end{aligned}$ | $\begin{aligned} & 0.081 \\ & 0.010 \end{aligned}$ | $\begin{aligned} & 0.058 \\ & 0.007 \end{aligned}$ | $\begin{aligned} & 0.071 \\ & 0.011 \end{aligned}$ | $\begin{aligned} & 0.067 \\ & 0.009 \end{aligned}$ | $\begin{aligned} & 0.059 \\ & 0.007 \end{aligned}$ | $\begin{aligned} & 0.068 \\ & 0.008 \end{aligned}$ | $\begin{aligned} & 0.077 \\ & 0.011 \end{aligned}$ | $\begin{aligned} & 0.065 \\ & 0.009 \end{aligned}$ | $\begin{aligned} & 0.059 \\ & 0.007 \end{aligned}$ |
| 0.55 | $\begin{aligned} & C_{a}^{+} \\ & C_{b}^{+} \end{aligned}$ | $\begin{aligned} & 0.088 \\ & 0.008 \end{aligned}$ | $\begin{aligned} & 0.062 \\ & 0.006 \end{aligned}$ | $\begin{aligned} & 0.080 \\ & 0.009 \end{aligned}$ | $\begin{aligned} & 0.072 \\ & 0.007 \end{aligned}$ | $\begin{aligned} & 0.063 \\ & 0.005 \end{aligned}$ | $\begin{aligned} & 0.073 \\ & 0.006 \end{aligned}$ | $\begin{aligned} & 0.085 \\ & 0.009 \end{aligned}$ | $\begin{aligned} & 0.070 \\ & 0.007 \end{aligned}$ | $\begin{aligned} & 0.063 \\ & 0.006 \end{aligned}$ |
| 0.50 | $C_{a}^{+}$ $C_{b}^{+}$ | $\begin{aligned} & 0.095 \\ & 0.006 \end{aligned}$ | $\begin{aligned} & 0.066 \\ & 0.004 \end{aligned}$ | $\begin{aligned} & 0.088 \\ & 0.007 \end{aligned}$ | 0.077 0.005 | 0.067 0.004 | $\begin{aligned} & 0.078 \\ & 0.005 \end{aligned}$ | $\begin{aligned} & 0.092 \\ & 0.007 \end{aligned}$ | $\begin{aligned} & 0.076 \\ & 0.005 \end{aligned}$ | $\begin{aligned} & 0.067 \\ & 0.004 \end{aligned}$ |

Table 4
Ratio of load in (a and b) direction for shear in slab and load on support

| $\underset{i}{i}$ |  | Boundary Conditions |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Case 1 $\square$ | Case 2 $\square$ | Case 3 $\square$ | Case 4 $\square$ | Case 5 $\square$ | Case 6 $\square$ | Case 7 $\square$ | Case 8 $\square$ | Case 9 $\square$ |
| 1.00 | $\begin{aligned} & W_{a} \\ & W_{b} \end{aligned}$ | $\begin{aligned} & 0.50 \\ & 0.50 \end{aligned}$ | $\begin{aligned} & 0.50 \\ & 0.50 \end{aligned}$ | $\begin{aligned} & 0.17 \\ & 0.83 \end{aligned}$ | $\begin{aligned} & 0.50 \\ & 0.50 \end{aligned}$ | $\begin{aligned} & 0.83 \\ & 0.17 \end{aligned}$ | $\begin{aligned} & 0.71 \\ & 0.29 \end{aligned}$ | $\begin{aligned} & 0.29 \\ & 0.71 \end{aligned}$ | $\begin{aligned} & 0.33 \\ & 0.67 \end{aligned}$ | $\begin{aligned} & 0.67 \\ & 0.33 \end{aligned}$ |
| 0.95 | $\begin{aligned} & W_{a} \\ & W_{b} \end{aligned}$ | $\begin{aligned} & 0.55 \\ & 0.45 \end{aligned}$ | $\begin{aligned} & 0.55 \\ & 0.45 \end{aligned}$ | $\begin{aligned} & 0.20 \\ & 0.80 \end{aligned}$ | $\begin{aligned} & 0.55 \\ & 0.45 \end{aligned}$ | $\begin{aligned} & 0.86 \\ & 0.14 \end{aligned}$ | $\begin{aligned} & 0.75 \\ & 0.25 \end{aligned}$ | $\begin{aligned} & 0.33 \\ & 0.67 \end{aligned}$ | $\begin{aligned} & 0.38 \\ & 0.62 \end{aligned}$ | $\begin{aligned} & 0.71 \\ & 0.29 \end{aligned}$ |
| 0.90 | $\begin{aligned} & W_{a} \\ & W_{b} \end{aligned}$ | $\begin{aligned} & 0.60 \\ & 0.40 \end{aligned}$ | $\begin{aligned} & 0.60 \\ & 0.40 \end{aligned}$ | $\begin{aligned} & 0.23 \\ & 0.77 \end{aligned}$ | $\begin{aligned} & 0.60 \\ & 0.40 \end{aligned}$ | $\begin{aligned} & 0.88 \\ & 0.12 \end{aligned}$ | $\begin{aligned} & 0.79 \\ & 0.21 \end{aligned}$ | $\begin{aligned} & 0.38 \\ & 0.62 \end{aligned}$ | $\begin{aligned} & 0.43 \\ & 0.57 \end{aligned}$ | $\begin{aligned} & 0.75 \\ & 0.25 \end{aligned}$ |
| 0.85 | $\begin{aligned} & W_{a} \\ & W_{b} \end{aligned}$ | $\begin{aligned} & 0.66 \\ & 0.34 \end{aligned}$ | $\begin{aligned} & 0.66 \\ & 0.34 \end{aligned}$ | $\begin{aligned} & 0.28 \\ & 0.72 \end{aligned}$ | $\begin{aligned} & 0.66 \\ & 0.34 \end{aligned}$ | $\begin{aligned} & 0.90 \\ & 0.10 \end{aligned}$ | $\begin{aligned} & 0.83 \\ & 0.17 \end{aligned}$ | $\begin{aligned} & 0.43 \\ & 0.57 \end{aligned}$ | $\begin{aligned} & 0.49 \\ & 0.51 \end{aligned}$ | $\begin{aligned} & 0.79 \\ & 0.21 \end{aligned}$ |
| 0.80 | $\begin{aligned} & W_{a} \\ & W_{b} \end{aligned}$ | $\begin{aligned} & 0.71 \\ & 0.29 \end{aligned}$ | $\begin{aligned} & 0.71 \\ & 0.29 \end{aligned}$ | $\begin{aligned} & 0.33 \\ & 0.67 \end{aligned}$ | $\begin{aligned} & 0.71 \\ & 0.29 \end{aligned}$ | $\begin{aligned} & 0.92 \\ & 0.08 \end{aligned}$ | $\begin{aligned} & 0.86 \\ & 0.14 \end{aligned}$ | $\begin{aligned} & 0.49 \\ & 0.51 \end{aligned}$ | $\begin{aligned} & 0.55 \\ & 0.45 \end{aligned}$ | $\begin{aligned} & 0.83 \\ & 0.17 \end{aligned}$ |
| 0.75 | $\begin{aligned} & W_{a} \\ & W_{b} \end{aligned}$ | $\begin{aligned} & 0.76 \\ & 0.24 \end{aligned}$ | $\begin{aligned} & 0.76 \\ & 0.24 \end{aligned}$ | $\begin{aligned} & 0.39 \\ & 0.61 \end{aligned}$ | $\begin{aligned} & 0.76 \\ & 0.24 \end{aligned}$ | $\begin{aligned} & 0.94 \\ & 0.06 \end{aligned}$ | $\begin{aligned} & 0.88 \\ & 0.12 \end{aligned}$ | $\begin{aligned} & 0.56 \\ & 0.44 \end{aligned}$ | $\begin{aligned} & 0.61 \\ & 0.39 \end{aligned}$ | $\begin{aligned} & 0.86 \\ & 0.14 \end{aligned}$ |
| 0.70 | $\begin{aligned} & W_{a} \\ & W_{b} \end{aligned}$ | $\begin{aligned} & 0.81 \\ & 0.19 \end{aligned}$ | $\begin{aligned} & 0.81 \\ & 0.19 \end{aligned}$ | $\begin{aligned} & 0.45 \\ & 0.55 \end{aligned}$ | $\begin{aligned} & 0.81 \\ & 0.19 \end{aligned}$ | $\begin{aligned} & 0.95 \\ & 0.05 \end{aligned}$ | $\begin{aligned} & 0.91 \\ & 0.09 \end{aligned}$ | $\begin{aligned} & 0.62 \\ & 0.38 \end{aligned}$ | $\begin{aligned} & 0.68 \\ & 0.32 \end{aligned}$ | $\begin{aligned} & 0.89 \\ & 0.11 \end{aligned}$ |
| 0.65 | $\begin{aligned} & W_{a} \\ & W_{b} \end{aligned}$ | $\begin{aligned} & 0.85 \\ & 0.15 \end{aligned}$ | $\begin{aligned} & 0.85 \\ & 0.15 \end{aligned}$ | $\begin{aligned} & 0.53 \\ & 0.47 \end{aligned}$ | $\begin{aligned} & 0.85 \\ & 0.15 \end{aligned}$ | $\begin{aligned} & 0.96 \\ & 0.04 \end{aligned}$ | $\begin{aligned} & 0.93 \\ & 0.07 \end{aligned}$ | $\begin{aligned} & 0.69 \\ & 0.31 \end{aligned}$ | $\begin{aligned} & 0.74 \\ & 0.26 \end{aligned}$ | $\begin{aligned} & 0.92 \\ & 0.08 \end{aligned}$ |
| 0.60 | $\begin{aligned} & W_{a} \\ & W_{b} \end{aligned}$ | $\begin{aligned} & 0.89 \\ & 0.11 \end{aligned}$ | $\begin{aligned} & 0.89 \\ & 0.11 \end{aligned}$ | $\begin{aligned} & 0.61 \\ & 0.39 \end{aligned}$ | $\begin{aligned} & 0.89 \\ & 0.11 \end{aligned}$ | $\begin{aligned} & 0.97 \\ & 0.03 \end{aligned}$ | $\begin{aligned} & 0.95 \\ & 0.05 \end{aligned}$ | $\begin{aligned} & 0.76 \\ & 0.24 \end{aligned}$ | $\begin{aligned} & 0.80 \\ & 0.20 \end{aligned}$ | $\begin{aligned} & 0.94 \\ & 0.06 \end{aligned}$ |
| 0.55 | $\begin{aligned} & W_{a} \\ & W_{b} \end{aligned}$ | $\begin{aligned} & 0.92 \\ & 0.08 \end{aligned}$ | $\begin{aligned} & 0.92 \\ & 0.08 \end{aligned}$ | $\begin{aligned} & 0.69 \\ & 0.31 \end{aligned}$ | $\begin{aligned} & 0.92 \\ & 0.08 \end{aligned}$ | $\begin{aligned} & 0.98 \\ & 0.02 \end{aligned}$ | $\begin{aligned} & 0.96 \\ & 0.04 \end{aligned}$ | $\begin{aligned} & 0.81 \\ & 0.19 \end{aligned}$ | $\begin{aligned} & 0.85 \\ & 0.15 \end{aligned}$ | $\begin{aligned} & 0.95 \\ & 0.05 \end{aligned}$ |
|  | ${ }^{\text {m }}$ | n 0. | $\cdots \mathrm{m}$ | n | - $\quad$. | n mm | n mo | $\ldots$ | - ... | ~. |



# ALNOOR UNIVERSITY COLLEGE 

## Dept. of Building Engineering and Projects Management

## REINFORCED CONCRETE (I)

$$
\text { C.E. } 3234
$$

FIRST SEMESTER 2023-2024

## CHATER SEVEN

# SHEAR STRENGTH OF BEAMS AND ONE WAY SLABS 

LECTURER<br>Dr. SA'AD A. AL-TA'AN

## Shear Strength of Beams

### 7.1 INTRODUCTION

Reinforced concrete members should resist the shear forces that seldom act alone but with bending moments, axial forces, and sometimes torsion. The shear transfer in reinforced concrete members depends on the tensile and compression strength of concrete. When shear failure occurs, the deflection is usually small and seldom ductile, and this type of failure must be avoided in practice, and to achieve this, the shear strength of the member should be more than its flexural strength.
The tensile strength of concrete is very small compared with its compression strength and the shear strength is between the two. Most shear failures are basically a diagonal tension or diagonal compression failure.
Reinforced concrete composed of two materials (concrete and steel) and the equations used for the analysis of homogeneous members can be used for reinforced concrete members to predict the initiation of diagonal tension cracking and shear strength.

### 7.2 Shear Stresses in Homogeneous Beams

The horizontal shear stresses in homogeneous beams are useful to know the stresses created in the beams web:

$$
\begin{equation*}
v=\frac{V \cdot Q}{I \cdot b} \tag{7.1}
\end{equation*}
$$

Where $\mathrm{V}=$ shear force acting on the cross-section, $\mathrm{I}=$ moment of inertia, b $=$ width of cross-section, and $\mathrm{Q}=$ moment of the area between the level considered and the nearest face about the neutral axis.
It is possible to imagine the role of shear stresses for a beam composed of two rectangular strips under vertical loads, Figure (7.1). If the bond between the two strips is perfect, the deformation is as shown in Figure (7.1a). If the bond is weak, the two strips will separate and slide over each other as shown in Figure (7.1b). If the bond between the two strips is perfect, there are horizontal stresses at the common interface between the two faces to prevent sliding as shown in Figure (7.1c). Such stresses are created in horizontal planes of beams composed of one part and its magnitude varies with the distance from the neutral axis. Figure (7.1d) show a small part of a beam with dimensions ( $\mathrm{b} \times \mathrm{h}$ ) subjected to a shear force V . Vertical equilibrium is
provided by vertical shear stresses. These stresses vary parabolically from zero at the top and bottom faces to maximum at the neutral axis. The average of theses stresses equal to $\mathrm{v}_{\text {avg. }}=[\mathrm{V} /(\mathrm{b} \times \mathrm{h})]$. For shapes other than rectangular sections, it is possible to use Equation (7.1) to predict the magnitude and distribution of shear stresses.


Figure (7.1) Vertical and horizontal shear stresses in homogeneous beams

In Figure (7.2) show the distribution of shear and longitudinal flexural stresses on three elements, at the neutral axis where there are shear stresses only (longitudinal flexural stresses $=0$ ) Figure (7.2b) diagonal tensile and compression are created on the element diagonals. In the compression zone where there are both shear $(v)$ and longitudinal flexural compression stresses (c) Figure (7.2c) the diagonal tensile stresses on (a-a) decrease and the diagonal compression stresses on (b-b) increases. In the tension zone where there are both shear and longitudinal flexural tension stresses $(t)$ Figure (6.2d) the diagonal tensile stresses on (a-a) increases and the diagonal compression stresses on (b-b) decrease.

Using the relationships of combining stresses (shear and flexural stresses) it is possible to draw stress trajectories as shown in Figure (7.2e):

$$
\begin{equation*}
t, c=f / 2 \pm \sqrt{(f / 2)^{2}+v^{2}} \tag{7.2}
\end{equation*}
$$



Figure (7.2) Diagonal stresses in a homogeneous beam, (a) uniformly loaded beam, (b) Stresses at point A, (c) Stresses at point B, (d) Stresses at point C, (e) Tensile stress trajectories

The resulting stresses composed of superposing the shear and flexural stresses at any point. When the diagonal tensile stresses exceed the tensile
strength of concrete, cracks will initiate perpendicular to the stress trajectory.

### 7.3 Shear Stresses in Reinforced Concrete Beams

Figure (7.3a) shows the shear stress distribution for a single reinforced rectangular beam. The shear stress distribution above the neutral axis is the same as that for a homogeneous beam. Figure (7.3b) shows the increase in the tensile force (dT) that should be equalized by a shear stress (v) multiplied by a horizontal area ( $\mathrm{b} \times \mathrm{dx}$ ), because the tensile stress of concrete vanishes after the appearance of cracks, since ( $\mathrm{dT}=\mathrm{v} \times \mathrm{b} \times \mathrm{dx}$ ) and ( $\mathrm{dT}=\mathrm{dM} / \mathrm{z}$ ), solving the two equations:

$$
\begin{equation*}
v=\frac{d T}{d x}\left[\frac{1}{b}\right]=\frac{d M}{d x}\left[\frac{1}{z . b}\right]=\frac{V}{b . z} \tag{7.3}
\end{equation*}
$$



Figure (7.3) Distribution of shear stresses in reinforced concrete sections

The zone under the neutral axis remains under a state of pure shear and the above equation gives a measure of the diagonal tension in the cracked zone.

All codes of practices uses Equation (7.3) as an index for the shear stress and replace the lever arm (z) by the effective depth (d):

$$
\begin{equation*}
v=\frac{V}{b_{w} \cdot d} \tag{7.4}
\end{equation*}
$$

### 6.4 Behaviour of Beams without Shear Reinforcement

In reinforced concrete beams, inclined cracks appear in the web either without flexural cracks (usually vertical), adjacent to it, or extension of the flexural cracks. Inclined cracks that appear in the web of uncracked beam, Figure (7.4a) called (web shear cracks). Inclined cracks that are extension of vertical flexural cracks called flexure shear cracks, Figure (7.4b).

(a) Fiexural crack.

(b) Inclined crack.


Secondary crack
(b) Flexure shear crack

### 7.4.1 Transfer of Shear Forces in Reinforced Concrete Beams

Shear transfer in reinforced concrete members composed of the following actions, Figure (7.5):
i. Shear strength of concrete in the compression zone, $V_{c z}, 20-40 \%$ of the external shear force
ii. Aggregate interlock on the cracks faces, this action is similar to the irregular interlock of the aggregates on rough surfaces of cracks, 33$50 \%$ of the external shear force
iii. Dowel action, which represent the resistance of the longitudinal reinforcement to the transverse shear forces, $15-25 \%$ of the external shear force
iv. Arch action in deep beams, and
v. Shear strength of stirrups (vertical or inclined).


Figure (7.5) distribution of shear resistance after the appearance of inclined cracks

### 7.4.2 Modes of Failure

The mode of shear failure and shear strength depends on the shear span / effective depth:
i. When $\mathrm{a}_{\mathrm{v}} / \mathrm{d}>6.0$, flexure failure may occurs,
ii. When $\mathrm{a}_{\mathrm{v}} / \mathrm{d}<6.0$, shear failure mostly occurs, and in this case various types of failure may be identified depending on $\mathrm{a}_{\mathrm{v}} / \mathrm{d}$ :
(a) Diagonal tension failure occurs when ( $2.5<\mathrm{a}_{\mathrm{v}} / \mathrm{d}<6.0$ ), Figure (7.6a),

(a) Beam.

(b) Moments at cracking and fallure.

(c) Shear at cracking and failure.
(b) Shear-compression failure when ( $1.0<\mathrm{a}_{\mathrm{v}} / \mathrm{d}<2.5$ ), Figure (7.6b),
(c) Shear-tension failure when ( $1.0<\mathrm{a}_{\mathrm{v}} / \mathrm{d}<2.5$ ), Figure (7.6c), if the bond between the steel and concrete is weak, and
(d) Splitting or true shear failure occurs when ( $\mathrm{a}_{\mathrm{v}} / \mathrm{d}<1.0$ ), Figure (7.6d).

(b) Shear compression failure.

(a) Shear-tension failure.

Figure (7.7) Types of shear failures, (a) diagonal tension failure, (b) shearcompression failure, (c) shear -tension failure, (d) splitting or true shear failure


Typical failure of deep beams


### 7.5 Critical Section for Calculating the Nominal Shear Strength

The shear span has a great effect on the shear strength and shear failure, and the worst location of the (diagonal tension failure) of a concentrated load on a simply supported beam is not near the support but at a certain distance from the support. The ACI Code recommends that the critical section for shear is a distance (d) from the face of support. The zone between the face of support and the critical section is designed for the same shear as that at the critical section.
There are cases where the critical section for shear should be taken at the face of support:
i. When the shear increases in the direction of the face of support, and the support is a beam or girder and there is no compression at the support,
ii. When there is concentrated load at a distance $\leq \mathrm{d}$,
iii. When there is a load may create inclined crack at the face of support and extend into the support.


Fig. R11.1.3.1(b)-Location of critical section for shear in a member loaded near bottom.


Fig. RII.I.3.1(c), (d), (e), (f)-Typical support conditions for locating factored shear force $\mathbf{V}_{\mathbf{u}}$.

Figure (7.7) Critical section for shear for some cases

### 6.6 Shear Strength of Beams without Shear Reinforcement

The shear strength is influenced by the shear span / effective depth, tensile strength of concrete, and the reinforcement ratio. The ACI Code recommends the following Equation for the shear strength of concrete:

$$
\begin{equation*}
V_{c}=\frac{1}{7}\left(\lambda \sqrt{f_{c}^{\prime}}+120 \rho_{w} \frac{V_{u} \cdot d}{M_{u}}\right) b_{w} \cdot d \leq 0.29 \lambda \sqrt{f_{c}^{\prime}} b_{w} \cdot d \tag{7.5a}
\end{equation*}
$$

Or in a simpler form:
$V_{c}=\frac{\lambda \sqrt{f_{c}^{\prime}}}{6} b_{w} \cdot d$
$V_{u}=$ external ultimate or factored shear force acting on the section, $M_{u}=$ external ultimate or factored moment acting on the section, $V_{u} d / M_{u} \leq 1.0$, the reason for this limitation, that the moment at the points of inflection and regions with small bending moment will result in very high shear strength. The width of the section $b_{w}$ in the above Equation is that for the web when the section is T shape.

The factor $(\lambda)$ is for the type of concrete:
i. Equal to 1.0 for normal weight concrete,
ii. Equal to 0.75 for all light weight concrete, and
iii. Equal to 0.85 for sand light weight concrete.

If $f_{c t}$ splitting strength of concrete is known then $\lambda=1.8 f_{c t} / \sqrt{f_{c}^{\prime}} \leq 1.0$

### 7.7 Role of Shear Reinforcement

Figure (7.8) shows the types of shear reinforcement:
i. Vertical stirrups perpendicular to the main tension reinforcement,
ii. Inclined stirrups with an angle $\geq 45^{\circ}$ to the main tension reinforcement,
iii. Longitudinal bent up reinforcement with angle of inclination $\geq 30^{\circ}$, and
iv. Longitudinal bent up reinforcement with vertical or inclined stirrups. Spiral bars and welded wire fabrics may be used as shear reinforcement.


Figure (7.8) Types of shear reinforcement

From the figure above, the stirrups are vertical or inclined reinforcement distributed along the span or part of it and embedded in the compression zone and wraps around the tension reinforcement. Before the inclined cracks initiation there is no stresses in the stirrups, after the cracks initiation which intersects the stirrups and tensile stresses will be developed, and the stirrups increase the shear strength by the following actions:
i. Improve the dowel action by warping the longitudinal bars intersecting the inclined shear cracks,
ii. Retarding the extension of the inclined cracks thus increasing the aggregate interlock,
iii. Preventing bond failure when the longitudinal cracks appear.

### 7.8 Shear Strength of Shear Reinforcement

The vertical stirrups are the most widely used stirrups because it is easy to shape and place. The nominal shear strength $\left(\mathrm{V}_{\mathrm{s}}\right)$ of inclined stirrups equal to:
$V_{s}=A_{v} \cdot f_{y}(\sin \alpha+\cos \alpha) d / s \leq \frac{2}{3} \sqrt{f_{c}^{\prime}} b_{w} d$
$\alpha=$ angle of inclination, if $\alpha=90^{\circ}$ (vertical stirrups), $\mathrm{V}_{\mathrm{s}}$ becomes equal to:

$$
\begin{equation*}
V_{s}=A_{v} \cdot f_{y} d / s \leq \frac{2}{3} \sqrt{f_{c}^{\prime}} b_{w} d \tag{7.7}
\end{equation*}
$$

$A_{v}=$ area of both legs of stirrups $=2 A_{b}(\mathrm{U}$ - stirrups $)$ or $=4 A_{b}(\mathrm{w}$-stirrups $)$.
Longitudinal bent-up bars act also as shear reinforcement like inclined stirrups, and its $V_{s}$ equal to:

$$
\begin{equation*}
V_{S}=A_{v} \cdot f_{y} \sin \alpha \leq \frac{\sqrt{f_{\mathcal{C}}^{\prime}}}{4} b_{w} \cdot d \tag{7.8}
\end{equation*}
$$

The ACI Code limits the yield strength of shear reinforcement to 400 MPa , to limit the width of inclined shear cracks.

### 7.9 Spacing of Shear Reinforcement

The ACI Code limits the spacing of vertical shear reinforcement to ( $\mathrm{d} / 2$ or 600 mm ) to ensure that each $45^{\circ}$ crack may be intersected by a stirrup if
, the limits become ( $\mathrm{d} / 4$ or 300 mm ). $V_{s}>\frac{\sqrt{f_{c}^{\prime}}}{3} b_{w} d$ when $V_{s}<\frac{\sqrt{f_{c}^{\prime}}}{3} b_{w} d$
The spacing of inclined shear reinforcement $[0.75(\mathrm{~d}-\mathrm{d}$ ' $)]$ when
)]. 'd-limits become $\left[0.375\left(\mathrm{~d}\right.\right.$, the $V_{s}>\frac{\sqrt{f_{c}^{\prime}}}{3} b_{w} d$ when $V_{s}<\frac{\sqrt{f_{c}^{\prime}}}{3} b_{w} d$

### 7.10 Minimum and Maximum Limits for Shear Reinforcement

The ACI Code limits the minimum strength of shear reinforcement by:
${\text { Min. } V_{s}}=\frac{1}{3} b_{w} d(\mathbf{M N})$ or Min. $A_{v}=\frac{b_{w} \cdot s}{3 f_{y}} \quad\left(\mathbf{m m}^{2}\right)$
${\text { Min. } V_{s}}=\frac{\sqrt{f_{c}^{\prime}}}{16} b_{w} d(\mathbf{M N}) \quad$ or Min. $A_{v}=\frac{\sqrt{f_{c}^{\prime}}}{16} \frac{b_{w} \cdot s}{f_{y}} \quad\left(\mathbf{m m}^{2}\right)$
The ACI Code recommends that when:
$V_{n} \prec V_{c} / 2$ there is no need to use minimum shear reinforcement.
The ACI Code limits also the maximum shear strength for shear reinforcement and reinforced concrete members by:
Max. $V_{n}=V_{c}+$ Max. $V_{s}=\frac{\sqrt{f_{c}^{\prime}}}{6} b_{w} d+\frac{2 \sqrt{f_{c}^{\prime}}}{3} b_{w} d=\frac{5 \sqrt{f_{c}^{\prime}}}{6} b_{w} d$
Table (6.1) Summary of design for shear reinforcement according to ACI Code

| Zone | Limits of $\mathrm{V}_{\mathrm{n}}$ | $\mathrm{A}_{\mathrm{v}}$ | Spacing |
| :---: | :---: | :---: | :---: |
| 1 | $0 \leq V_{n} \leq \frac{\sqrt{f_{c}^{\prime}}}{12} b_{w} \cdot d$ | $V_{n}<V_{c} / 2$ <br> No shear <br> reinforcement <br> required | --------- |
| 2 | $V_{c}+$ Min. $V_{s} \geq V_{n}>\frac{\sqrt{f_{c}^{\prime}}}{12} b_{w} \cdot d$ | Min. $\frac{b_{w} \cdot s}{3 f_{y}}$ or <br> $\frac{\sqrt{f_{c}^{\prime}} b_{w} \cdot s}{16 f_{y}}$ | $\mathrm{~d} / 2$ or 600 mm |
| 3 | $V_{c}+$ Min. $^{\prime} \cdot V_{s}<V_{n} \leq \frac{\sqrt{f_{c}^{\prime}}}{2} b_{w} \cdot d$ | $\frac{V_{s} \cdot s}{f_{y} \cdot d}$ | $\mathrm{~d} / 2$ or 600 mm |
| 4 | $\left(\frac{5}{6} \sqrt{f_{c}^{\prime}}\right) b_{w} \cdot d \geq V_{n} \geq \frac{\sqrt{f_{c}^{\prime}}}{2} b_{w} \cdot d$ | $\frac{V_{s} \cdot s}{f_{y} \cdot d}$ | $\mathrm{~d} / 4$ or 300 mm |

## EXAMPLE (6.1)

Determine the minimum dimensions of a rectangular beam if the shear is controlling the design without using shear reinforcement. $f_{c}^{\prime}=25 \mathrm{MPa}$ and $V_{u}=180 \mathrm{kN}$.

## SOLUTION

Shear strength of concrete equal to:
$V_{c}=\frac{\sqrt{f_{c}^{\prime}}}{6} b_{w} \cdot d=\frac{\sqrt{25}}{6} b_{w} \cdot d$
The required nominal shear strength equal to:
$V_{n} \geq V_{u} / \varphi=180 / 0.75=240 \mathrm{kN}$
Since there is no shear reinforcement:
$V_{n}=0.24 \geq \frac{V_{c}}{2}=\frac{5}{12} b_{w} \cdot d$
From the above equation:
$b_{w} \cdot d=0.576 m^{2}$
If $d / b_{w} \approx 2$ the above equation becomes:
$b_{w}\left(2 b_{w}\right)=0.576$
$b_{w}=0.537 \mathrm{~m}=537 \mathrm{~mm}$, if $\mathrm{b}_{\mathrm{w}}$ is chosen equal to 550 mm , then $\mathrm{d}=1047 \mathrm{~mm}$, and $\mathrm{h}=1047+63=1110 \mathrm{~mm}$. the final dimensions of the beam are:
$B=500 \mathrm{~mm}$ and $\mathrm{h}=1150 \mathrm{~mm}$.

## EXAMPLE (6.2)

A rectangular beam with $\mathrm{b}=350 \mathrm{~mm}, \mathrm{~d}=637 \mathrm{~mm}, f_{c}^{\prime}=20 \mathrm{MPa}$ and $f_{y}=$ 400MPa . Determine the spacing of \#10 vertical stirrups for the following factored (ultimate) shear forces:
(a) $\mathrm{Vu}=60 \mathrm{kN}$, (b) $\mathrm{Vu}=250 \mathrm{kN}$, © $\mathrm{Vu}=400 \mathrm{kN}$, (d) $\mathrm{Vu}=700 \mathrm{kN}$.

## SOLUTION

(a) $\mathrm{Vu}=60 \mathrm{kN}$

$$
V_{n} \geq V_{u} / \varphi=60 / 0.75=80 \mathrm{kN}
$$

$V_{c}=\frac{\sqrt{20}}{6} \times 0.35 \times 0.637 \times 1000=166.2 \mathrm{kN}>V_{n}=80 \mathrm{kN}$
Compare Vn with Vc/2
$V_{c} / 2=166.2 / 2=83.1 \mathrm{kN}>V_{n}$
Therefore the section doesn't require shear reinforcement.
(b) $\mathrm{Vu}=250 \mathrm{kN}$
$V_{n} \geq V_{u} / \varphi=250 / 0.75=333.3 k N>V_{c}=166.2 k N$
Therefore the section requires shear reinforcement.
$V_{s}=333.3-166.2=167.1 \mathrm{kN}$
Min. $V_{s}=\frac{1}{3} b_{w} \cdot d=\frac{1}{3} \times 0.35 \times 0.637 \times 1000=74.3 \mathrm{kN}$
$\operatorname{Min} . V_{S}=\frac{\sqrt{f_{c}^{\prime}}}{16} b_{w} \cdot d=\frac{1}{16} \times 0.35 \times 0.637 \times 1000=62.3 \mathrm{kN}$
$\operatorname{Re} q \cdot V_{S}=167.1 \mathrm{kN}>\operatorname{Min} . V_{S}=74.3 \mathrm{kN}$
$\frac{\sqrt{f_{c}^{\prime}}}{3} b_{w} \cdot d=\frac{\sqrt{20}}{3} 0.35 \times 0.637 \times 1000=332.4 k N>V_{S}$
Therefore the spacing of stirrups $=\mathrm{d} / 2=637 / 2=319 \mathrm{~mm}$ or 600 mm whichever is smaller.
$\therefore$ Max. $s=d / 2=637 / 2=319 \mathrm{~mm}$
$s=\frac{A_{\nu} \cdot f_{y} \cdot d}{V_{s}}=\frac{158 \times 10^{-6} \times 400 \times 0.637}{0.1671}=0.241 \mathrm{~m}=241 \mathrm{~mm}<\max . s=319 \mathrm{~mm}$
Use \#10 U vertical stirrups @ 225 mm c/c.
(c) $\mathrm{V}_{\mathrm{u}}=400 \mathrm{kN}$
$V_{n} \geq V_{u} / \varphi=400 / 0.75=533.3 \mathrm{kN}>V_{c}=166.2 \mathrm{kN}$
$V_{s}=533.3-166.2=367.1 \mathrm{kN}$
$\frac{\sqrt{f_{c}^{\prime}}}{3} b_{w} \cdot d=\frac{\sqrt{20}}{3} 0.35 \times 0.637 \times 1000=332.4 \mathrm{kN}<V_{s}=367.1 \mathrm{kN}$
Therefore the maximum spacing between stirrups less than $\mathrm{d} / 2=319 \mathrm{~mm}$. compare $\mathrm{V}_{\mathrm{s}}$ with the maximum limit of $\mathrm{V}_{\mathrm{s}}$

Max. $V_{s}=\frac{2}{3} \sqrt{f_{c}^{\prime}} b_{w} \cdot d=\frac{2}{3} \times 20 \times 0.35 \times 0.637 \times 1000=664.8 \mathrm{kN}>V_{s}$
Therefore maximum spacing $=\mathrm{d} / 4=159 \mathrm{~mm}$ or 300 mm whichever is smaller:
$\therefore M a x . s=d / 2=637 / 2=319 \mathrm{~mm}$
$s=\frac{A_{v} \cdot f_{y} \cdot d}{V_{s}}=\frac{158 \times 10^{-6} \times 400 \times 0.637}{0.3671}=0.1097 \mathrm{~m}$
$\approx 110 \mathrm{~mm}<$ Max. $^{s}=159 \mathrm{~mm}$
Use \#10 U stirrups @ 100 mm c/c.
(d) $\mathrm{Vu}=700 \mathrm{kN}$
$V_{n} \geq V_{u} / \varphi=700 / 0.75=933.3 \mathrm{kN}$
$\operatorname{Max} . V_{n}=\frac{5}{6} \sqrt{f_{c}^{\prime}} b_{w} \cdot d=\frac{5}{6} \sqrt{20} \times 0.35 \times 0.637 \times 1000=830.9 \mathrm{kN}<\operatorname{Re} q \cdot V_{n}$
Therefore a larger cross-section or higher compression strength of concrete should be used.

## EXAMPLE 6.3

A cantilever beam with a span of 2.4 m subjected to a uniformly distributed live load of $10 \mathrm{kN} / \mathrm{m}$ and a uniformly distributed dead load of $7 \mathrm{kN} / \mathrm{m}$, in addition to the beam weight. Determine the zones that require shear reinforcement. $f_{c}^{\prime}=20 \mathrm{MPa}$ and $f_{y}=400 \mathrm{MPa}$.

## SOLUTION:

Add the beam weight to the DL ;
Beam weight $=0.25 \times 0.4 \times 1 \times 24=2.4 \mathrm{kN} / \mathrm{m}$
$\mathrm{w}_{\mathrm{u}}=1.2(2.4+7)+1.6 \times 10=27.28 \mathrm{kN} / \mathrm{m}$
$\mathrm{V}_{\mathrm{uf}}=27.28 \times 2.4=65.5 \mathrm{kN}, \quad \mathrm{V}_{\mathrm{nf}}=65.5 / 0.75=87.3 \mathrm{kN}$
$\mathrm{V}_{\text {ud }}=27.28(2.4-0.34)=56.2 \mathrm{kN}, \quad \mathrm{V}_{\mathrm{nd}}=56.2 / 0.75=74.9 \mathrm{kN}$
$V_{c}=\frac{\sqrt{20}}{6} \times 0.25 \times 0.34 \times 1000=63.4 \mathrm{kN}<V_{n d}=74.9 \mathrm{kN}$
Therefore the beam require shear reinforcement at point (d).
$V_{c} / 2=63.4 / 2=31.7 \mathrm{kN}$
Figure (6.9) shows the $V_{u}$ and $V_{n}$ diagrams and the points where $V_{n}=V_{c} / 2$ and $\mathrm{V}_{\mathrm{c}}$ and the zone that does not require shear reinforcement.
$\frac{87.3}{2.4}=\frac{31.7}{x 1}, \quad x 1=0.871 \mathrm{~m}=871 \mathrm{~mm}$ (from the free end to this point the beam does not require shear reinforcement)
$\frac{87.3}{2.4}=\frac{63.4}{x 3} \quad, \quad x 3=1.743 \mathrm{~m}=1743 \mathrm{~mm}$
$\mathrm{x} 2=\mathrm{x} 3-\mathrm{x} 1=1743-871=872 \mathrm{~mm}$

Min. $V_{S}=\frac{1}{3} b_{w} \cdot d=\frac{1}{3} \times 0.25 \times 0.34 \times 1000$

$$
=28.3 \mathrm{kN}
$$

$$
\operatorname{Min} . V_{s}=\frac{\sqrt{f_{c}^{\prime}}}{16} b_{w} \cdot d=\frac{\sqrt{20}}{16} \times 0.25 \times 0.34 \times 1000
$$



$$
=23.8 \mathrm{kN}
$$

$$
V_{c}+{\text { Min. } V_{s}}=63.4+28.3
$$

$$
=91.7 \mathrm{kN}>V_{n d}=74.9 \mathrm{kN}
$$



Therefore the beam requires minimum shear reinforcement from the point (x1) to the face of support. If \#10 U stirrups is used, the spacing will be as shown below:

$$
s=3 A_{v} \cdot f_{y} / b_{w}=3 \times 158 \times 10^{-6} \times 400 / 0.25=0.758 m
$$

$$
s=\frac{16}{\sqrt{f_{c}^{\prime}}} \frac{A_{v} \cdot f_{y}}{b_{w}}=\frac{16}{\sqrt{20}} \frac{158 \times 10^{-6} \times 400}{0.25}=0.904 \mathrm{~m}
$$

Max. $s=d / 2=340 / 2=170 \mathrm{~mm}$
Use \#10 U stirrups @ 150 mm c/c. the through which shear reinforcement should be provided $L_{v}=2400-871=1529 \mathrm{~mm}$, number of stirrups
$=1529 / 150=10.1$,
Use 11 \#10 U stirrups, 1 @ $75 \mathrm{~mm}+10$ @ 150 mm c/c.

## EXAMPLE 6.4

A simply supported with an effective span $=5.0 \mathrm{~m}$ (support width $=300$ mm ) is carrying a working DL $=70 \mathrm{kN} / \mathrm{m}$ (including beam weight) and a working LL $80=\mathrm{kN} / \mathrm{m} . f_{c}^{\prime}=25 \mathrm{MPa}, f_{y}=400 \mathrm{MPa}, \mathrm{b}=300 \mathrm{~mm}$, and $\mathrm{d}=537$
mm .

## SOLUTION

Calculate the factored (ultimate) loads;
$w_{u d}=1.2 \times 70=84 \mathrm{kN} / \mathrm{m}$
$w_{u l}=1.6 \times 80=128 \mathrm{kN} / \mathrm{m}$
Total ultimate load equal to:
$w_{u}=84+128=212 \mathrm{kN} / \mathrm{m}$
Calculate the ultimate shear force at the center of support, by considering the
DL and LL on the whole beam:
$V_{u s}=212 \times 5 / 2=530 \mathrm{kN}$,
$V_{n s}=530 / 0.75=706.7 \mathrm{kN}$
$V_{u c}=128 \times 5 / 8=80 \mathrm{kN}, \quad V_{n c}=80 / 0.75=106.7 \mathrm{kN}$
Connect these two points (center of support and mid span section) by a straight line as shown in the figure below.
$V_{c}=\frac{\sqrt{25}}{6} \times 0.3 \times 0.537 \times 1000=134.3 \mathrm{kN}>V_{c n}$
$V_{c} / 2=67.1 \mathrm{kN}<V_{c n}$
From the similar triangles, calculate the distance from the mid span section to the point where $V_{n}=V_{c}$ and equal to 115 mm ,

$$
\frac{706.7-106.7}{2.5}=\frac{134.3-106.7}{x_{1}} \quad x_{1}=0.115 \mathrm{~m}=115 \mathrm{~mm}
$$

through this distance $V_{c} / 2<V_{n}<V_{c}$ where the minimum shear reinforcement should be provided.
From the similar triangles also, calculate the factored shear force at the critical section (d from the face of support)
$\frac{706.7-106.7}{2.5}=\frac{V_{n d}-106.7}{2.5-0.15-0.537}, \quad V_{n d}=541.8 \mathrm{kN}$
$V_{s d}=541.8-134.3=407.5 \mathrm{kN}>\frac{\sqrt{f_{c}^{\prime}}}{3} b_{w} \cdot d=268.5 \mathrm{kN}$

So the maximum spacing between stirrups is less than $\mathrm{d} / 2$, then calculate the maximum shear strength of the shear reinforcement:
$V_{s d}=407.5 \mathrm{kN}<2 \frac{\sqrt{f_{c}^{\prime}}}{3} b_{w} \cdot d=537 \mathrm{kN}$
So the maximum spacing between stirrups is $\mathrm{d} / 4$ or 300 mm (whichever is smaller)

$$
V_{s d}=407.5 \mathrm{kN}<2 \frac{\sqrt{f_{c}^{\prime}}}{3} b_{w} \cdot d=537 \mathrm{kN}
$$

If \#12 U stirrups is tried, the spacing equal to:

$$
s_{d}=\frac{A_{v} \cdot f_{y}}{V_{s}} d=\frac{226 \times 10^{-6} \times 345}{0.4075} 0.537=0.103 \mathrm{~m}=103 \mathrm{~mm}
$$

The required shear strength of the shear reinforcement $\left(\mathrm{V}_{\mathrm{s}}\right)$ decrease with the distance from the face of support, and in such cases it is preferred to draw a spacing curve (relationship between the spacing and distance along the span). The relationship between (s) and $\left(\mathrm{V}_{\mathrm{s}}\right)$ is not linear but curvilinear, and at least three points have to be determined.
The distance between point (e) and the critical section $=2350-537-115=$ 1698 mm . The $\mathrm{V}_{\mathrm{s}}$ diagram is a triangle whose base $=1698 \mathrm{~mm}$ and height $=$ 407.5 kN . To draw a curve along the span divide tis distance into equal distances, e.g. (four divisions), Table (6.2) shows the nominal shear strength $\left(\mathrm{V}_{\mathrm{n}}\right)$, shear strength of the shear reinforcement $\left(\mathrm{V}_{\mathrm{s}}\right)$, and spacing of the stirrups.

Table (6.2) Spacing of stirrups versus the distance along the span

| Point | d | a | b | c | e | Beam <br> CL |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Dist.(mm) | 537 | 962 | 1386 | 1811 | 2235 | 2350 |
| $V_{n}$ | 541.8 | 439.0 | 338.0 | 236.1 | 134.3 | 106.7 |
| $V_{S}$ | 407.6 | 305.7 | 203.8 | 101.9 | 0 | ------- |
| ${\text { Min. } V_{S}}$ | 53.7 | 53.7 | 53.7 | 53.7 | 53.7 | ------- |
| S (mm) | 103 | 137 | 206 | 412 | $\infty$ | ------- |
| Max. s | 134 | 134 | 269 | 269 | 269 | 269 |


(c) $\mathrm{V}_{\mathrm{n}}$ - Diagram

(d) Spacing Curve

## EXAMPLE 6.5

Check whether the one-way ribbed slab of Example (5.) need shear reinforcement or not. The slab is simply supported on an effective span of $4.0 \mathrm{~m}(\mathrm{c} / \mathrm{c}) . f_{c}^{\prime}=20 \mathrm{MPa}$ and $f_{y}=400 \mathrm{MPa}$.

## SOLUTION

The ultimate load on the slab $=$ $w_{u}=1.2(2+2.12)+1.6 \times 5=12.94 \mathrm{kPa}$


The ultimate load on each rib $=12.94 \times 0.6=7.76 \mathrm{kN} / \mathrm{m}$
$\mathrm{d}=224 \mathrm{~mm}$.
$V_{u d}=7.76(4 / 2-0.224)=13.78 \mathrm{kN}$
$V_{n d} \succ 13.78 / 0.75=18.37 \mathrm{kN}$
The shear strength provided by concrete $\left(\mathrm{V}_{\mathrm{c}}\right)$ increased $10 \%$ due to the better load distribution in ribbed slabs.
$V_{c}=\frac{1.1 \sqrt{20}}{6} \times 0.1 \times 0.224=18.37 \mathrm{kN}$
No Min. shear reinforcement is required where $V_{n} \geq V_{c}$.

ALNOOR UNIVERSITY COLLEGE
Dept. of Building Engineering and Projects Management

REINFORCED CONCRETE (I)
C.E. 3234

FIRST SEMESTER 2023-2024

CHATER EIGHT

# STRENGTH OF MEMBERS IN COMPRESSION AND BENDING (SHORT COLUMNS) 

LECTURER<br>Dr. SA'AD A. AL-TA'AN

## CHAPTER EIGHT STRENGTH IN COMPRESSION OF MEMBERS IN COMPRESSION AND BENDING

### 8.1NTRODUCTION

Column is usually a vertical reinforced concrete member used primarily to carry axial compression loads, but can also resist moment, shear, or torsion. Compression members with height less than or equal to three times its width called pedestals, Figure (8.1a) with cross-section larger than the column and is usually used to transmit loads from columns to the foundation.
There are other structural members can be considered as compression members also as they resist compression loads and moments like some members in reinforced concrete frames, compression members in trusses, arches, and shells.

In reinforced concrete members that carry compression loads, it is preferred that concrete carries most of the load because of its low cost compared with steel. Use of reinforcement is unavoidable in columns because of many reasons:
i. Most columns are subjected to moments in addition to the compression load due to its continuity with other structural members,
ii. Some columns are constructed inclined due to mistakes in the construction process, and
iii. Use of steel with its high strength compared to concrete ( $\approx 10$ times in compression and $\approx 100$ times in tension) will reduce the required crosssection.
Columns are classified according to their heights/width ratio as:
i. Short columns with relatively small height / width ratio, and its strength is controlled by the materials strength ( $f_{c}^{\prime}$ and $f_{y}$ ) and cross-sectional dimensions, and
ii. Long (slender) columns with a relatively large height / width ratio, and its strength is controlled by the materials strength ( $f_{c}{ }^{\prime}$ and $f_{y}$ ), cross-sectional dimensions, height, and boundary conditions.
Columns are classified also according to the type of transverse reinforcement used to support the main vertical reinforcement as:
i. Tied columns with square, rectangular, or circular cross-section and the main reinforcement is wrapped by the ties, Figure (6.1b), and
ii. Spiral columns with square, circular, or polygonal cross-section with transverse reinforcement as spiral, Figure (8.1c).

There is other type of columns called composite column with steel sections as main reinforcement and with or without longitudinal bars, Figure (8.1.d).


Figure (8.1) Types of columns, (a) Pedestal, (b) tied column, (c) spiral column, (d) composite column.

### 8.2 BEHAVIOUR OF AXIALLY LOADED SHORT COLUMNS

When a reinforced concrete column is subjected to an axial compression load, the strain over the whole cross-section is constant, and the strains in concrete and steel are equal, due to the perfect bond between the two materials. When the stress in concrete is less than $\left(f_{c}^{\prime} / 2\right)$ both the concrete and steel behave elastically. Concrete and steel will share the applied external load:
$P=f_{c} \cdot A_{c}+f_{s} . A_{s t}$
Where $f_{c}=$ concrete compression stress, $A_{c}=$ net concrete area $=A_{g}-A_{s t}, A_{g}=$ gross concrete area, $A_{s t}=$ total steel area, $f_{s}=$ compression stress in steel, using the relationship $f_{s}=n f_{c}$ and substituting it in the above equation:

$$
\begin{equation*}
P=f_{c}\left(A_{c}+n A_{s t}\right)=f_{c} \cdot A_{t} \tag{8.2}
\end{equation*}
$$

The expression between brackets equal to the equivalent transformed area $\left(A_{t}\right)$ Equations ( 8.1 and 8.2) can be used to calculate the stresses in concrete and steel when the concrete stress is less than $\left(f_{d} / 2\right)$, i.e., in the working or service conditions.

Due to creep and shrinkage of concrete, the following changes may take place:
i. Stresses in concrete decreases and those in steel increase, and
ii. Heavily reinforced columns subjected to long term loads and unloaded later, tension stresses in concrete and compression stresses in steel may be created. Therefore Equations (8.1 and 8.2) will not give the true stresses and the strength design method is adopted for this reason.
When the external axial loads on a reinforced concrete column increased the steel stress reaches the yield strength $\left(f_{y}\right)$ before the concrete reaches its strength $\left(f_{c}^{\prime}\right)$. In this case the column will not reach its strength, because it will carry additional load till the concrete reaches its strength in compression, Figure (8.2).


Figure (8.2) Variation of the strains in concrete and steel for an axially loaded reinforced concrete column

Therefore the strength of an axially loaded column is the sum of the concrete strength $\left[0.85 f_{c}^{\prime}\left(A_{g}-A_{s t}\right)\right.$ and the steel strength $\left(A_{s t} \times f_{y}\right)$ :

$$
\begin{equation*}
P_{n o}=0.85 f_{c}^{\prime}\left(A_{g}-A_{s t}\right)+A_{s t} \cdot f_{y} \tag{8.3}
\end{equation*}
$$

this load is called yield strength of the column. Up to this point the behavior of both tied and spirally reinforced columns is identical, Figure (8.3), at this point also the concrete crushes and the main reinforcement buckles, Figure (8.4).

When a spirally reinforced column reaches the yield point, the concrete shell outside the spiral crushes and separates from the column and this will lead to a reduction in the column strength and the main reinforcement will not buckle, Figure (8.4) due to the lateral support provided by the closely spaced spiral ( $50-75 \mathrm{~mm}$ ). Before the column reaches the final failure, the longitudinal bars continue to carry compression stresses due to the behavior nature of steel, and this is accompanied by longitudinal compression strains leading to expansion of the concrete core (inside the spiral) creating outward radial stresses on the spiral and as a result the spiral exert radial compression stresses on the concrete core and increasing its strength more than that in Equation (8.3):

$$
\begin{equation*}
P_{n o}=0.85 f_{c}^{\prime}\left(A_{g-} A_{s t}\right)+A_{s t} \cdot f_{y}+k_{s} \cdot A_{s p} \cdot f_{y s} \tag{8.4}
\end{equation*}
$$



Figure (8.3) Comparison of tied and spiral column behaviour

Where $A_{s p}=$ volume of spiral $/$ unit length of the column, $\mathrm{k}_{\mathrm{s}}=$ factor $=1.5-2.5$ with an average of 2.0 , this means that the strength of spiral reinforcement at this stage is twice that of the longitudinal reinforcement. The increase in strength depends on the volume of the used spiral, Figure (8.3). From what is mentioned before, it looks that the spiral column exhibits ductility before final failure.


Figure (8.4) Typical failure of reinforced concrete columns, (a) tied column, (b) spiral column

### 8.3 Interaction of Bending Moment and Axial

Axially or concentrically loaded reinforced concrete columns are rarely found in practice, since moments are present due to its continuity with other structural members, the action of wind and seismic forces, or forces acting on brackets or corbels. Even if the structural analysis reveals no moments on the columns, unavoidable construction mistakes may place loads outside the column centroid resulting in moments. Therefore, most reinforced concrete columns are subjected to compression and bending moments and called eccentrically loaded columns. The interaction of compression load and moment may be presented as shown in Figure
(8.5), either a compression load on the column centroid and moment, or a compression load at a distance $\mathrm{e}=\mathrm{M} / \mathrm{P}$ from the column centroid.
The axial strength $P_{n o}$ in Figure (8.6) represent the column strength to compression load when it acts at the column centroid, i.e., the moment $=$ zero, Equation (8.3).


Figure (7.5) Equivalent eccentricity
When the moment is small, the strain distribution is nearly constant, and the neutral axis lies outside the cross-section. When the moment increases, the neutral axis depth and the column axial strength decreases, Figure (8.6). The curve representing the relationship between the axial and bending strength of the column called interaction diagram. All points inside this diagram, represent the loads and moments that can be carried by the column, and those outside represent loads and moments causing failure. Any radial straight line from the origin, represent a constant ratio of moment to load (e) or a constant distance from the point of action of the load to the column centroid.
The case of balanced failure ( $M_{n b}, P_{n b}$ ) represents the load and moment at balanced strain condition. At this stage, crushing of concrete in compression and yielding of the tension steel occurs instantaneously. The balanced load $\left(P_{n b}\right)$ act at a distance ( $e_{b}=M_{n b} / P_{n b}$ ) from the column centroid. In columns that are carrying compression loads essentially, this case of failure and the compression failure can't be avoided as in beams and slabs by limiting the reinforcement ratio to ( $\rho<\rho_{\mathrm{b}}$ ) and the case of failure is controlled by the load position and not by the reinforcement ratio. To compensate for compression failure, a small strength reduction factor is used (0.650.75 ) more than that used in flexural members (beams and slabs).

When a column is subjected to load ( $P_{n}>P_{n b}$ ) and moment the compression strain reaches ( 0.003 ) before the yielding of the tension steel, i.e., compression failure, and in this case the neutral axis depth $\left(\infty>c>c_{b}\right)$ and the eccentricity $\left(e_{b}>e>0\right)$.

When a column is subjected to load ( $P_{n}<P_{n b}$ ) and moment the tension steel will reach yielding before the compression strain reaches (0.003) i.e., tension failure, and in this case the neutral axis depth $\left(c_{b}>c>0\right)$ and the eccentricity $\left(\infty>e>e_{b}\right)$.

The case when the compression stress resultant equal to the tension stress resultant is a state of pure bending, this point lies on the horizontal (M-axis) where ( $\mathrm{M}_{\mathrm{n}}=$ $\mathrm{M}_{\mathrm{n} 0}, \mathrm{P}_{\mathrm{n}}=0$, and $\mathrm{e}=\infty$ ), Figure (8.6).


Figure (8.6) Typical interaction diagram of a reinforced concrete column
There is another point that lies on the vertical (load-axis) below the zero point and represent the case of axial tensile strength (tensile strength of concrete $=0$ ) and equal to:

$$
\begin{equation*}
P_{n o}{ }^{*}=A_{s t} \times f_{y} \tag{8.5}
\end{equation*}
$$

The interaction diagram represent clearly the behaviour of a reinforced concrete column, and is used for the analysis and design of of reinforced concrete column. To draw the interaction diagram, a number of points have to be determined by selecting a suitable values of the neutral axis depth and assuming that the strain in compression $=0.003$. Usually the interaction diagrams are drawn dimensionless so that it can be used for any dimensions, shape, and reinforcement ratios.

### 8.3.1 Nominal axial Load Capacity $P_{n o}$ and ACI code Maximum Axial Load Capacity $\boldsymbol{P}_{\mathrm{n} \text { (max.) }}$

As mentioned previously, the case of axial compression is rarely found in practice, and the point is $\left(P_{n o}\right)$ is hypothetical and used to construct the interaction diagram. Equation (8.3) represents the axial strength for the tied and spiral columns. The strength reduction factor that correspond to this state is $=0.65$ for the tied columns, and 0.75 for spiral columns. For the case of axial tension, the strength reduction factor $=0.9$.
Since the case of axial compression is rarely found in practice, a minimum value of eccentricity should be used. As a result of this eccentricity, the strength of the column will be reduced as recommended by the ACI:

$$
\begin{align*}
\phi P_{n} & =0.8 \phi P_{n o}  \tag{8.6a}\\
\phi P_{n} & =0.85 \phi P_{n o} \tag{8.6b}
\end{align*}
$$

### 7.3.2 Balanced Strain Condition for Rectangular Sections

Thee balanced strain condition is the point that separates the compression and tension controlled zones, Figure (8.6). This occurs when the strain at the compression face $=0.003$ and the strain in the tension steel $=$ yield strain $\left(\varepsilon_{y}=\right.$ $\mathrm{f}_{\mathrm{y}} / \mathrm{E}_{\mathrm{s}}$ ), Figure (8.7):

$$
\begin{aligned}
& c_{b}=\frac{600}{600+f_{y}} d \\
& a_{b}=\beta 1 \cdot c_{b}
\end{aligned}
$$

The resultant of the compression stresses on concrete equal to:

$$
\begin{equation*}
C_{1}=0.85 f_{c}^{\prime} \cdot a_{b} \cdot b \tag{8.7}
\end{equation*}
$$

The resultant of the compression stresses on the compression steel equal to:
$C_{2}=A_{s}^{\prime} \cdot f_{s}^{\prime \prime}$

The effective stress in the compression steel when the compression steel reaches yielding equal to:
$f_{s}^{\prime \prime}=f_{y}-0.85 f_{c}^{\prime}$
And if it does not reach yielding, the effective stress equal to:
$f_{s}^{\prime \prime}=f_{s}^{\prime}-0.85 f_{c}^{\prime}$


Figure (8.7) Balanced strain condition in a rectangular column
Where $f_{s}^{\prime}$ equal to:
$f_{s}^{\prime}=\frac{c-d^{\prime}}{c} 600$
The resultant of stresses in the tension steel equal to:
$T=A_{s} . f_{y}$
Using the equation of equilibrium of the loads in the vertical direction, the nominal balanced strength $\left(P_{n b}\right)$ of the column can be calculated:
$C_{1}+C_{2}-T-P_{n b}=0$
$P_{n b}=C_{1}+C_{2}-T$

The third equation of equilibrium ( $\Sigma \mathrm{M}=0$ ) about the centroidal axis can be used to determine the location of $\left(P_{n b}\right)$ :

$$
\begin{equation*}
P_{n b} \cdot e_{b}=C_{1}(h / 2-a / 2)+C_{2}\left(h / 2-d^{\prime}\right)+T\left(h / 2-d_{s}\right) \tag{8.13}
\end{equation*}
$$

## EXAMPLE 87.1

A rectangular column with $\mathrm{b}=300 \mathrm{~mm}, \mathrm{~h}=500 \mathrm{~mm}$, reinforced with six \# 20 bars (three on each face) 60 mm from the each face. $f_{c}^{\prime}=25 \mathrm{MPa}$ and $f_{y}=400 \mathrm{MPa}$.
Determine the points of change on the interaction diagram, point in compression and point in the tension failure zones. Consider the load acting on the major axis (xaxis).

## SOLUTION

1- Axial strength of the column $\mathrm{P}_{\text {no }}$

$$
\begin{aligned}
& \quad P_{n o}=0.85 f_{c}^{\prime} . h . b+A_{s t} \cdot\left(f_{y}-0.85 \times f_{c}^{\prime}\right) \\
& P_{n o}=0.85 \times 25 \times 0.5 \times 0.3+6 \times 314 \times 10^{-6}(400-0.85 \times 25)=3.901 \mathrm{MN} \\
& P_{n o}=3.901 \mathrm{MN}
\end{aligned}
$$

At this point, $\mathrm{e}=0, \mathrm{M}_{\mathrm{n}}=0, \mathrm{c}=\infty$, compression is dominant, i.e., $\varepsilon_{t}=0.002$, and $\phi=$ 0.65 .

$$
\phi P_{n o}=0.65 \times 3.901=2.536 M N, \phi M_{n}=0
$$

2- Pure bending ( $P_{n}=0, M_{n}=M_{n o}$ )
The section is considered as a double reinforced section subjected to pure bending.

$$
\begin{aligned}
& C_{1}+C_{2}=T \\
& 0.85 f_{c}^{\prime} . \beta 1 . c . b+A_{s}^{\prime}\left(\frac{c-d^{\prime}}{c}\right) 600-0.85 f_{c}^{\prime}=A_{s} \cdot f_{y} \\
& 0.85 \times 25 \times 0.85(c) 0.3+3 \times 314 \times 10^{-6}\left(\frac{c-0.06}{c} 600-0.85 \times 25\right) \\
& =3 \times 314 \times 10^{-6} \times 400 \\
& c^{2}+0.031 c-0.00625=0 \\
& \mathrm{c}=65 \mathrm{~mm}, \mathrm{a}=55 \mathrm{~mm} \\
& f_{s}^{\prime}=\frac{65-60}{65} 600=46.15 \mathrm{MPa} \\
& f_{s}^{\prime \prime}=46.2-0.85 \times 25 M=24.9 \mathrm{MPa} \\
& M_{n 1}=0.85 f_{c}^{\prime} . a . b(0.44-0.055 / 2)=0.1446 M N . m=144.6 \mathrm{kN} \\
& M_{n 2}=A_{s}^{\prime} . f_{S}^{\prime \prime}\left(d-d^{\prime}\right)=3 \times 314 \times 10^{-6} \times 24.9(0.44-0.06) \\
& M_{n 2}=0.00891 M N . m=8.9 \mathrm{kN} . \mathrm{m}
\end{aligned}
$$

$$
\begin{aligned}
& M_{n o}=144.6+8.9=153.5 \mathrm{kN} . \mathrm{m} \\
& \varepsilon_{t}=\frac{d-c}{c} 0.003=\frac{440-65}{65} 0.003=0.017>0.005, \phi=0.9 \\
& \phi M_{n o}=0.9 \times 153.5 \mathrm{kN} . \mathrm{m}=138.2 \mathrm{kN} . \mathrm{m} \\
& \phi P_{n}=0
\end{aligned}
$$

3- Balanced strain condition, $c=c_{b}, P_{n}=P_{n b}, M_{n}=M_{n b}$.
$c_{b}=\frac{600}{600+f_{y}} d=0.6 d=264 \mathrm{~mm}$
$a_{b}=0.85 \times 264=224$
$N_{c 1}=0.85 f_{c}^{\prime} \cdot a_{b} \cdot b=0.85 \times 25 \times 0.224 \times 0.3$
$=1.428 M N$
$f_{s}^{\prime}=\frac{264-60}{264} 600=463.6 M P a>f_{y}$
$N_{t}=A_{s} . f_{y}=3 \times 314 \times 10^{-6} \times 400=0.377 M N$
$\sum F_{y}=0$
$N_{c 1}+N_{c 2}-N_{t}-P_{n b}=0$
$P_{n b}=1.428+0.357-0.377=1.408 M N$
$\varepsilon_{t}=\varepsilon_{y}=0.002$
$\therefore \quad \phi=0.65$
$\phi P_{n b}=0.65 \times 1.408=0.915 M N$
Taking the moment of the three internal forces and the external load about the column centroid to find $\left(\mathrm{e}_{\mathrm{b}}\right)$ :

$$
e_{b}=\frac{C_{1}(h / 2-a / 2)+C_{2}\left(h / 2-d^{\prime}\right)+T\left(h / 2-d^{\prime}\right)}{P_{n b}}
$$



Figure (8.8) Balanced strain condition for example (7.1)
$e_{b}=\frac{1.428(0.25-0.224 / 2)+0.357(0.25-0.06)+0.377(0.25-0.06)}{1.408}$
$e_{b}=0.239 m=239 \mathrm{~mm}$
$M_{n b}=P_{n b} . e_{b}=1.408 \times 0.239=0.3365 M N . m$
$\phi M_{n b}=0.65 \times 336.5=218.7 \mathrm{kN} . \mathrm{m}$

4- Point in the compression controlled region, for this region, $c>c_{b}$, try $c=400$ mm .
$a=0.85 \times 400=340 \mathrm{~mm}$
$C_{1}=0.85 \times 25 \times 0.34 \times 0.3=2.17 \mathrm{MN}$
$f_{s}^{\prime}=\frac{400-60}{400} 600=510 \mathrm{MPa}$
$\therefore f_{s}^{\prime \prime}=400-0.85 \times 25=378.75 \mathrm{MPa}$
$C_{2}=A_{s}^{\prime} \cdot f_{s}^{\prime \prime}=3 \times 314 \times 10^{-6} \times 378.75=0.357 \mathrm{MN}$
$\varepsilon_{s}=\varepsilon_{t}=\frac{d-c}{c} 0.003=\frac{440-400}{400} 0.003=0.0003<0.002$
$\therefore \phi=0.65$
$f_{S}=E_{S} \cdot \varepsilon_{S}=200000 \times 0.0003=60 \mathrm{MPa}$
$N_{t}=A_{s} \cdot f_{s}=3 \times 314 \times 10^{-6} \times 60=0.057 \mathrm{MN}$
$P_{n}=N_{c 1}+N_{c 2}-N_{t}=2.17+0.357-0.057=2.47 \mathrm{MN}$
$e=\frac{1.428(0.25-0.34 / 2)+0.357(0.25-0.06)+0.057(0.25-0.06)}{2.47}=0.102 \mathrm{~m}=102 \mathrm{~mm}$
$M_{n}=P_{n} \cdot e=2.47 \times 0.102=0.2519 \mathrm{MN} . \mathrm{m}$
$\phi P_{n}=0.65 \times 2.47=1.606 M N=1606 \mathrm{kN}$
$\phi M_{n}=0.65 \times 251.9=163.7 \mathrm{kN} . \mathrm{m}$
5- Point in the compression controlled region, in this region $\mathrm{c}<\mathrm{c}_{\mathrm{b}}$, try $\mathrm{c}=120$ mm:

$$
a=0.85 \times 120=102 \mathrm{~mm}
$$

$C_{1}=0.85 f_{c}^{\prime} \cdot a . b=0.85 \times 25 \times 0.102 \times 0.3=0.65 \mathrm{MN}$
$f_{s}^{\prime}=\frac{120-60}{120} 600=300 \mathrm{MPa}>f_{y}$
$f_{s}^{\prime \prime}=f_{s}^{\prime}-0.85 f_{c}^{\prime}=300-0.85 \times 25=278.75 \mathrm{MPa}$
$C_{2}=A_{s}^{\prime} \cdot f_{s}^{\prime \prime}=3 \times 314 \times 10^{-6} \times 278.75=0.263 \mathrm{MN}$
$T=A_{s} \cdot f_{y}=3 \times 314 \times 10^{-6} \times 400=0.377 \mathrm{MN}$
$\sum F_{y}=0$
$C_{1}+C_{2}-T-P_{n b}=0$
$P_{n}=0.65+0.263-0.377=0.536 M N$
$e=\frac{0.65(0.25-0.102 / 2)+0.263(0.25-0.06)+0.377(0.25-0.06)}{0.536}=0.468 \mathrm{~m}$
$e=468 \mathrm{~mm}$
$M_{n}=P_{n} \cdot e=0.536 \times 0.468=0.251 M N . m$
$\varepsilon_{s}=\varepsilon_{t}=\frac{d-c}{c} 0.003=\frac{440-120}{120} 0.003=0.008>0.005$
$\therefore \phi=0.9$
$\phi P_{n}=0.9 \times 0.536=0.4824 M N=482.4 \mathrm{kN}$
$\phi M_{n}=0.9 \times 251=225.9 \mathrm{kN} . \mathrm{m}$
6- The case of axial tension, in this case the steel will resist all the external load:
$P_{n o}^{*}=A_{s t} \cdot f_{y}=6 \times 314 \times 10^{-6} \times 400=0.754 M N=754 \mathrm{kN}$
The neutral axis depth $c=\infty$ and $\mathrm{e}=0$.
It is possible to take more points to draw the interaction diagram more accurately as shown in the Table below.

Table (8.1) Coordinates for the interaction diagram of Example (8.1)

| No. | c <br> mm | a <br> mm | e <br> mm | $P_{n}$ <br> kN | $M_{n}$ <br> $\mathrm{kN} . \mathrm{m}$ | $\phi$ | $\phi P_{n}$ <br> kN | $\phi M_{n}$ <br> $\mathrm{kN} . \mathrm{m}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\infty$ | h | 0 | 3901 | 0 | 0.65 | 2536 | 0 |
| 2 | 600 | 510 | 7 | 3738 | 26 | 0.65 | 2430 | 17 |
| 3 | 500 | 425 | 51 | 3113 | 158 | 0.65 | 2023 | 102 |
| 4 | 400 | 340 | 102 | 2470 | 251.9 | 0.65 | 1605.5 | 163.7 |
| 5 | 334 | 284 | 150 | 1987 | 298.1 | 0.65 | 1291.6 | 193.7 |
| 6 | 264 | 224 | 239 | 1408 | 336.5 | 0.65 | 915.2 | 218.7 |
| 7 | 190 | 162 | 311 | 1010 | 314 | 0.808 | 816 | 253.7 |
| 8 | 120 | 102 | 468 | 536 | 251 | 0.9 | 482.4 | 225.9 |
| 9 | 65 | 55 | $\infty$ | 0 | 153.7 | 0.9 | 0 | 138.4 |
| 10 | --- | --- | 0 | -754 | 0 | 0.9 | 678.6 | 0 |
| 11 | 50 | 42.5 | -512 | -219 | 112.1 | 0.9 | 197.1 | 100.9 |

### 8.3.3. Distributed Reinforcement

When the moment acting on the column is relatively large, all or most of the reinforcement should be placed near the outer faces as in the previous example. When the moment is relatively small (small or zero eccentricity), the strain distribution is almost constant or nearly so and the reinforcement should be distributed uniformly across the outer edges of the cross-section. The strain in the intermediate bars is calculated using the strain compatibility. The method of analysis is similar to that of example (8.1).

## EXAMPLE 8.2

In the previous example if two bars \# 20 are added at the middle of the long sides, Figure (8.9), calculate the balanced eccentricity.

## SOLUTION

$C_{1}=0.85 f_{c}^{\prime} \cdot a_{b} \cdot b=0.85 \times 25 \times 0.224 \times 0.3=1.428 M N$
$f_{s 2}^{\prime}=\frac{264-60}{264} 600=463.6 \mathrm{MPa}>f_{y}$
$f_{s 2}^{\prime \prime}=f_{y}-0.85 f_{c}^{\prime}=400-0.85 \times 25=378.75 \mathrm{MPa}$
$C_{2}=A_{s}^{\prime} \cdot f_{s}^{\prime \prime}=3 \times 314 \times 10^{-6} \times 378.75=0.357 \mathrm{MN}$
$f_{s 3}^{\prime}=\frac{264-250}{264} 600=31.8 M P a>f_{y}$
$f_{s 3}^{\prime \prime}=f_{s 3}-0.85 f_{c}^{\prime}=31.8-0.85 \times 25$
$=10.55 \mathrm{MPa}$
$C_{3}=A_{s 3}^{\prime} \cdot f_{s 3}^{\prime \prime}=2 \times 314 \times 10^{-6} \times 10.55$
$=0.0066 M N=6.6 \mathrm{kN}$
$T=A_{s} . f_{y}=3 \times 314 \times 10^{-6} \times 400$
$=0.377 M N$
$\sum F_{y}=0$
$C_{1}+C_{2}+C_{3}-T-P_{n b}=0$
$P_{n b}=1.428+0.357+0.0066-0.377$
$=1.414 \mathrm{MN}$
$\phi P_{n b}=0.65 \times 1414=919 \mathrm{kN}$


Figure (8.9) Balanced strain condition of Example (8.2)
$e_{b}=\frac{1.428(0.25-0.224 / 2)+0.357(0.25-0.06)+0.377(0.25-0.06)}{1.414}$
$e_{b}=0.237 \mathrm{~m}=237 \mathrm{~mm}$
$M_{n b}=P_{n b} \cdot e_{b}=1.414 \times 0.237=0.335 M N . m$
$\phi M_{n b}=0.65 \times 335=217 \mathrm{kN} . \mathrm{m}$
Figure (8.10) shows the interaction diagrams for the columns of Examples (8.1 and 8.2 ), the difference is very small near the region of the balanced strain condition, while in the tension controlled zone, the column of Example (8.2) has larger bending strength than the column of Example (8.1), the Figure shows also the interaction diagram of the column of Example (8.2) rotated $90^{\circ}$, i.e., $\mathrm{b}=500 \mathrm{~mm}$, and $\mathrm{h}=300 \mathrm{~mm}$. It can be noticed that the bending strength is reduced compared to that of Example (8.2). The Figure shows also the interaction diagram multiplied by the strength reduction factor $(\phi)$.


Figure (8.10) Interaction diagrams for columns of Examples (8.1 and 8.2)

### 8.4 Lateral Reinforcement

The transverse reinforcement in columns is either a ties with the same shape as the column spaced vertically, or continuous spiral. The transverse reinforcement has three functions:
i. Fixing the main bars and keeping it in position inside the forms,
ii. Prevent or retard the buckling of the vertical bars, and
iii. Act as shear reinforcement.
iv. The spiral reinforcement provides the column with additional strength that is lost by the separation of the concrete shell.

### 8.4.1Ties

The ACI Code recommends the use of \#10 ties when the diameter of the main reinforcement $\leq 32 \mathrm{~mm}$, and $\# 12$ when the diameter of the main reinforcement $\geq 35$ mm or when the bars are arranged in bundles. It is possible to use deformed wires or welded wire fabric as a transverse reinforcement with equivalent area as the ties. The vertical spacing between stirrups in the vertical direction should not exceed:
i. $16 \mathrm{~d}_{\mathrm{b}}$,
ii. $48 \mathrm{~d}_{\mathrm{t}}$, and
iii. Least dimension (b) of the column.


Figure (8.11) Arrangement of ties

### 8.4.2 Spiral Reinforcement

The spiral reinforcement make the column retain its strength is spite of the large deformation before failure, i.e., the failure is ductile. The volume of the spiral reinforcement should be large enough to substitute the column the strength lost by separation of the concrete shell ( $\mathrm{A}_{\mathrm{g}}-\mathrm{A}_{\mathrm{ch}}$ ). Using the third term of Equation (8.4) and assuming $\mathrm{k}_{\mathrm{s}}=2.0$ as an average, the strength contributed by the spiral reinforcement becomes:

$$
\begin{equation*}
P_{n}=2.0 f_{y t} \cdot A_{s p} \tag{8.14}
\end{equation*}
$$

Where $f_{y t}=$ yield strength of the spiral $\leq 690 \mathrm{MPa}, A_{s p}=$ area of the spiral. If $\rho_{\mathrm{s}}$ is assumed equal to volume o spiral in one turn to the volume of concrete core (out to out of spiral) or $\rho_{\mathrm{s}}=\left(A_{s p} / A_{c h}\right)$, Equation (8.14) becomes:

$$
\begin{equation*}
P_{n}=2.0 \times \rho_{s} \cdot A_{c h} \cdot f_{y} \tag{8.15}
\end{equation*}
$$

Where $A_{c h}=$ area of the concrete core (out to out of spiral). When the concrete shell separated, the column will lose strength equal to:

$$
\begin{equation*}
P_{n}=0.85 f_{c}^{\prime}\left(A_{g}-A_{c h}\right) \tag{8.16}
\end{equation*}
$$

equating Equations (7.14 1nd 7.15), the spiral reinforcement ratio becomes:

$$
\begin{align*}
& P_{n}=2.0 \times \rho_{s} \cdot A_{c h} \cdot f_{y}=0.85 f_{c}^{\prime}\left(A_{g}-A_{c h}\right) \\
& \quad \rho_{s}=0.425 \frac{f_{c}^{\prime}}{f_{y t}}\left(\frac{A_{g}}{A_{c h}}-1\right) \tag{8.17}
\end{align*}
$$

To make the spiral strength more than the concrete shell, use 04.5 instead of 0.45 in the above equation:

$$
\begin{equation*}
\rho_{s}=0.45 \frac{f_{c}^{\prime}}{f_{y t}}\left(\frac{A_{g}}{A_{c h}}-1\right) \tag{8.18}
\end{equation*}
$$

Using the definition $\rho_{\mathrm{s}}=\left(A_{s p} / A_{c h}\right)$, it is possible to use Equation (8.18) to design a spiral:
$\rho_{s}=\frac{A_{s p}}{A_{c h}}=\frac{\text { Volume of spiral in one turn }}{\text { Volume of concrete core out to out of spiral in a height } \mathbf{s}}$
Substituting the volumes of the spiral and the concrete core, the above Equation can be written as follow:

$$
\begin{equation*}
\rho_{s}=\frac{a_{s p} \cdot \pi\left(D_{c}-d_{s p}\right)}{\pi\left(D_{c}^{2} / 4\right) s}=\frac{4 a_{s p}\left(D_{c}-d_{s p}\right)}{D_{c}^{2} \cdot s} \tag{8.19}
\end{equation*}
$$

Where $\mathrm{D}_{\mathrm{c}}=$ core diameter (out to out of spiral), $\mathrm{a}_{\mathrm{sp}}=$ cross-sectional area of the spiral, $\mathrm{d}_{\mathrm{sp}}=$ spiral diameter $\geq 10 \mathrm{~mm}, \mathrm{~s}=$ spacing $\mathrm{c} / \mathrm{c}$ in the vertical direction. The clear distance between the spiral in the vertical direction should not be less than 25 mm , not more than 75 mm , or maximum coarse aggregate size.


Figure (8.12) Arrangement of spiral and longitudinal bars in reinforced concrete columns

## EXAMPLE 8.3

A reinforced concrete circular column with $\mathrm{D}=500 \mathrm{~mm}, \mathrm{D}_{\mathrm{c}}=420 \mathrm{~mm}, f_{c}^{\prime}=30$ MPa , and $f_{y}=400 \mathrm{MPa}$. Design the necessary spiral.

## SOLUTION

$A_{g}=\pi(0.5)^{2} / 4=0.1963 \mathrm{~m}^{2}$

$$
\begin{aligned}
& A_{c}=\pi(0.42)^{2} / 4=0.1385 m^{2} \\
& \rho_{s} \geq 0.45 \frac{30}{400}\left(\frac{0.1963}{0.1385}-1\right)=0.014
\end{aligned}
$$

If a 10 mm diameter spiral is used ( $\mathrm{a}_{\mathrm{s}}=79 \mathrm{~mm}^{2}$ ), using Equation (8.19) to calculate s:
$0.014=\frac{4 \times 79 \times 10^{-6}(0.42-0.01)}{(0.42)^{2} . s}$
$s=0.0524 \mathrm{~m}=52.4 \mathrm{~mm}$
Use $\mathrm{s}=50 \mathrm{~mm}$, the clear distance between spiral in the vertical direction $=50-\mathrm{d}_{\mathrm{s}}=$ $50-10=40 \mathrm{~mm}$, within the limits mentioned before, $>25 \mathrm{~mm},<75 \mathrm{~mm}$, and > max. coarse aggregate size.

### 8.5 Limits on Reinforcement Ratio

The ACI Code limited the gross reinforcement ratio $0.01 \leq \rho_{\mathrm{g}} \leq 0.08$, where $\rho_{\mathrm{g}}=$ $\left(\mathrm{A}_{\mathrm{st}} / \mathrm{A}_{\mathrm{g}}\right)$. The reasons for limiting the minimum reinforcement ratio are:
i. To prevent sudden or brittle failure,
ii. Providing minimum bending strength for the column, and
iii. Reducing the influence of creep and shrinkage of concrete.

If the area of the column is more than required for strength, the reinforcement ration should not be less than 0.005 .
The maximum limit of the reinforcement ratio is to satisfy the clear spacing requirements of bars:
i. $\geq 1.5 \mathrm{~d}_{\mathrm{b}}$,
ii. $\geq 40 \mathrm{~mm}$, and
iii. (4/3) maximum coarse aggregate size.

### 8.6 Analysis of Sections in Compression Controlled Region

When the nominal strength $\mathrm{P}_{\mathrm{n}}$ of a column exceeds the nominal balanced strength $\mathrm{P}_{\mathrm{nb}}, \mathrm{e}<\mathrm{e}_{\mathrm{b}}$, or when $\mathrm{c}>\mathrm{c}_{\mathrm{b}}$ compression will dominant, i.e., the behavior of the member is close to that of a column than that of a beam. The strain in the tension steel [which may be compression if (e) is very small] is less than the yield strain. The interaction diagrams can be used for analysis and design. Whitney's method can be used also.

### 8.6.1 Whitney Formula for Compression Failure Case

The method is suitable for symmetrically reinforced columns ( $\mathrm{A}_{\mathrm{s}}=\mathrm{A}_{\mathrm{s}}{ }^{\prime}$ ), in deriving this equation, the compression steel is assumed to reach yielding and $a=a_{b}$.

$$
\begin{equation*}
P_{n}=\frac{b . h \cdot f_{c}^{\prime}}{\frac{3 . h \cdot e}{d^{2}}+1.18}+\frac{A_{s}^{\prime} \cdot f_{y}}{\frac{e}{d-d^{\prime}}+0.5} \tag{8.20}
\end{equation*}
$$

If $\gamma . h=d-d^{\prime}, \rho_{g}=2 A_{s}^{\prime} / A_{g}, A_{g}=b h$, and $\xi=d / h$ the above Equation can be rewritten as follow:

$$
\begin{equation*}
P_{n}=A_{g}\left[\frac{f_{c}^{\prime}}{\frac{3}{\xi^{2}}\left(\frac{e}{h}\right)+1.18}+\frac{\rho_{g} \cdot f_{y}}{\left(\frac{2}{\gamma}\right)\left(\frac{e}{h}\right)+1}\right] \tag{8.21}
\end{equation*}
$$

This equation is more suitable for design.

## EXAMPLE 7.4

In Example (7.1) if the load is placed 150 mm to the right of the y -axis, find the design strength.

## SOLUTION

Since $\mathrm{e}=150 \mathrm{~mm}<\mathrm{e}_{\mathrm{b}}=239 \mathrm{~mm}$, it is compression failure and Equation (7.20) can be used.
$P_{n}=\frac{0.3 \times 0.5 \times 25}{\frac{3 \times 0.5 \times 0.15}{(0.44)^{2}+1.18}}+\frac{942 \times 10^{-6} \times 400}{\frac{0.15}{0.44-0.06}+0.5}$
$=2.022 \mathrm{MN}>P_{n b}=1.408 \mathrm{MN}$
This value is less than the exact value $\mathrm{P}_{\mathrm{n}}=1.989 \mathrm{MN}$ ( $1.63 \%$ difference). $\phi=0.65$
$\phi P_{n}=0.65 \times 2.022=1.314 \mathrm{MN}$
$\phi M_{n}=0.15 \times 1.314=0.197 M N$


Figure (8.13) Analysis of compression controlled sections

### 8.6.2 Analysis of Sections in Tension Controlled region

When the nominal strength $\mathrm{P}_{\mathrm{n}}$ of a column is less than the nominal balanced strength $P_{n b}, e>e_{b}$, or when $c<c_{b}$ tension will dominant, i.e., the behavior of the member is close to that of a beam than that of a column. The strain in the tension steel is more than the yield strain when the concrete reaches a compression strain $=0.003$. The interaction diagrams can be used for analysis and design. Whitney's method can be used also.

### 8.7.2 Approximate Formula for Tension Failure Cases

For symmetrically reinforced sections $\left(\left(\mathrm{A}_{\mathrm{s}}=\mathrm{A}_{\mathrm{s}}{ }^{\prime}\right)\right.$, in deriving this equation, the compression steel is assumed to reach yielding:

$$
\begin{equation*}
P_{n}=0.85 f_{c}^{\prime} b \cdot d\left\{-\rho+1-e^{\prime} / d+\sqrt{\left(1-e^{\prime} / d\right)^{2}+2 \rho\left[(m-1)\left(1-d^{\prime} / d\right)+e^{\prime} / d\right]}\right\} \tag{8.22}
\end{equation*}
$$

Where $\rho^{\prime}=A_{s}^{\prime} /(b d), \rho=A_{s} /(b d), m=f_{y}\left(0.85 f_{c}^{\prime}\right)$, and $e^{\prime}=e+\left(d-d^{\prime}\right) / 2$.

## EXAMPLE 8.5

In Example (8.1) if the load is placed 350 mm to the right of the y -axis, find the design strength.

## SOLUTION

Since $\mathrm{e}=350 \mathrm{~mm}<\mathrm{e}_{\mathrm{b}}=239 \mathrm{~mm}$, it is tension failure and
Equation (7.22) can be used.

$$
\begin{aligned}
& e^{\prime}=350+\frac{440-60}{2}=540 \mathrm{~mm} \\
& \rho=\rho^{\prime}=\frac{942}{300 \times 440}=0.00714
\end{aligned}
$$

$$
m=\frac{400}{0.85 \times 25}=18.82
$$



$$
P_{n}=0.85 \times 25 \times 0.3 \times 0.44\{-0.00714+1-540 / 440
$$

$$
\begin{aligned}
& \left\{+\sqrt{(1-540 / 440)^{2}+2 \times 0.00714[(18.82-1)(1-60 / 440+540 / 440]}\right\}=0.851 \mathrm{MN} \\
& =851 \mathrm{kN} \\
& M_{n}=0.851 \times 0.35=0.298 \mathrm{MN} . \mathrm{m}=298 \mathrm{kN} . \mathrm{m}
\end{aligned}
$$

This value is $1 \%$ more than the exact value.

### 8.7.3 Using the Interaction Diagrams or the Analysis

In the analysis, the cross-sectional dimensions, and the area of steel are known or given, and the required unknowns are either:
i. The design strength of the column $\phi P_{n}$ if e is given, or
ii. The load position if $P_{n}$ is given.

## EXAMPLE 8.6

Solve Example (8.4) using the interaction diagram.
$\gamma=\left(d-d^{\prime}\right) / h=(440-60) / 500=0.76$
e / $\mathrm{h}=150 / 500=0.3$
$\rho_{g}=6 \times 314 /(300 \times 500)=0.01256$
Enter the diagram for $\gamma=0.7$ with $\mathrm{e} / \mathrm{h}=0.3$ and $\rho_{g}=0.01256$, and read $\mathrm{K}_{\mathrm{n}}=0.52$
Enter the diagram for $\gamma=0.8$ with e/h $=0.3$ and $\rho_{g}=0.01256$, and read $\mathrm{K}_{\mathrm{n}}=0.54$
Therefore $\mathrm{K}_{\mathrm{n}}=0.532$
$K_{n}=\frac{P_{n}}{f_{c}^{\prime} A_{g}}, P_{n}=K_{n} f_{c}^{\prime} A_{g}=0.532 \times 25 \times 0.15=1.995 \mathrm{MN}$

## EXAMPLE 8.7

Solve Example (8.5) using the interaction diagram.
$\gamma=\left(d-d^{\prime}\right) / h=(440-60) / 500=0.76$
e / h $=350 / 500=0.7$
Enter the diagram for $\gamma=0.7$ with $\mathrm{e} / \mathrm{h}=0.7$ and $\rho_{g}=0.01256$, and read $\mathrm{K}_{\mathrm{n}}=0.20$
Enter the diagram for $\gamma=0.8$ with e/h $=0.7$ and $\rho_{g}=0.01256$, and read $\mathrm{K}_{\mathrm{n}}=0.24$
Therefore $\mathrm{K}_{\mathrm{n}}=0.224$
$K_{n}=\frac{P_{n}}{f_{c}^{\prime} A_{g}}, P_{n}=K_{n} f_{c}^{\prime} A_{g}=0.224 \times 25 \times 0.15=0.84 M N=850 \mathrm{kN}, \phi P_{n}=0.65 \times 840=546 \mathrm{kN}$

## EXAMPLE 8.8

If a load of $\mathrm{P}_{\mathrm{u}}=650 \mathrm{kN}$ is applied on the column of example (8.1), what is the eccentricity.

## SOLUTION

$\gamma=\left(d-d^{\prime}\right) / h=(440-60) / 500=0.76$
$K_{n}=\frac{P_{u}}{\phi f_{c} A_{g}}=\frac{0.65}{0.65 \times 25 \times 0.15}=0.267$
Enter the diagram for $\gamma=0.7$ with $K_{n}=0.267$ and $\rho_{g}=0.01256$, and read e/h $=0.6$
Enter the diagram for $\gamma=0.8$ with $K_{n}=0.267$ and $\rho_{g}=0.01256$, and read e/h=0.65 $e / h=0.63, e=0.63 \times 500=315 \mathrm{~mm}$.

### 8.7 Design of Rectangular Reinforced Concrete Columns

The strength design for columns can divided into three categories, Figure (8.15):
i. Sections subjected to axial compression, or axial compression and small bending moments, (e $\left.<\mathrm{e}_{\mathrm{min}}\right),\left(\mathbf{0 . 8} \mathbf{P}_{\mathbf{n o}}<\mathbf{P}_{\mathbf{n}}<\mathbf{P n o}\right)$ in this case the design is according to Equations (8.6a and 8.6b),
ii. Compression controlled region ( $\mathrm{e}_{\min }<\mathrm{e}<\mathrm{e}_{\mathrm{b}}$ ), ( $\mathrm{P}_{\mathrm{nb}}<\mathrm{P}_{\mathrm{n}}<0.8 \mathrm{P}_{\mathrm{no}}$ ) and the section in this case is less than that required for the balanced strain condition, and
iii. Tension controlled region $\left(\mathrm{e}_{\mathrm{b}}<\mathrm{e}<\infty\right),\left(0<\mathrm{P}_{\mathrm{n}}<\mathrm{P}_{\mathrm{nb}}\right)$ and the section in this case is more than that required for the balanced strain condition.
The following equations can be used as a first estimate of the required area of the cross-section for tied and spiral columns respectively:

$$
\begin{align*}
A_{g} & =\frac{P_{u}}{0.4\left(f_{c}^{\prime}+\rho f_{y}\right)}  \tag{8.23}\\
A_{g} & =\frac{P_{u}}{0.5\left(f_{c}^{\prime}+\rho f_{y}\right)} \tag{8.24}
\end{align*}
$$

If the section is subjected to large bending moment, a higher cross-section is required.


Figure (8.15) Cases for Design of Reinforced Concrete Columns

### 8.7.1 Design for the First Region

## EXAMPLE 8.9

A reinforced concrete tied column is subjected to $P_{d}=1200 \mathrm{kN}$ and $\mathrm{P}_{\mathrm{l}}=1400 \mathrm{kN}$.
Choose a suitable dimension for the column (square) $f_{c}^{\prime}=30 \mathrm{MPa}, f_{y}=400 \mathrm{MPa}$, and $\rho_{g} \approx 0.03$

## SOLUTION

$P_{u}=1.2 \times 1200+1.6 \times 1400=3680 \mathrm{kN}$
$A_{g}=\frac{P_{u}}{0.4\left(f_{c}^{\prime}+\rho_{g} f_{y}\right)}=\frac{3.68}{0.4(30+0.03 \times 400)}=0.2190476$
$b=h=\sqrt{A_{g}}=0.468 \mathrm{~m}=468 \mathrm{~mm}$, try $\mathrm{b}=\mathrm{h}=450 \mathrm{~mm}$,
$P_{n} \geq 3680 / 0.65=5661.5 \mathrm{kN}$
$P_{n \text { (max.) }}=0.8 P_{n o}=0.8\left[0.85 f_{c}^{\prime}\left(A_{g}-A_{s t}\right)+A_{s t} \cdot f_{y}\right]=$
$0.8\left[0.85 f_{c}^{\prime}\left(A_{g}-\rho_{g} \cdot A_{g}\right)+\rho_{g} \cdot A_{g} \cdot f_{y}\right]=0.8 A_{g}\left[0.85 f_{c}^{\prime}\left(1-\rho_{g}\right)+\rho_{g} \cdot f_{y}\right]$
$5.6615=0.8(0.45)^{2}\left[0.85 \times 30\left(1-\rho_{g}\right)+\rho_{g} \times 400\right]$
$\rho_{g}=0.0252 \approx 0.03$
$A_{s t}=0.0252(450)^{2}=5111 \mathrm{~mm}^{2}$
Since the column is axially loaded, the strain and compression stress is constant across the cross-section, and it is preferred to distribute the bars along the four faces. Use $12 \# 25$ bars (four bars on each face) $=5892 \mathrm{~mm}^{2}$, Figure (8.16). Ties diameter $=\# 10 \mathrm{~mm}$ (Bar dia. $<32 \mathrm{~mm}$ ), with spacing's:
i. $16 \mathrm{~d}_{\mathrm{b}}=16 \times 25=400 \mathrm{~mm}$,
ii. $48 \mathrm{~d}_{\mathrm{t}}=48 \times 10=480 \mathrm{~mm}$, or
iii. $\mathrm{b}=450 \mathrm{~mm}$.

Therefore use 3\#10 mm @ 400 mm c/c, Figure (8.16).
Clear spacing between bars $=(450-2 \times 40-2 \times 30-3 \times 25) / 3=78 \mathrm{~mm}$
This value is:
i. $>40 \mathrm{~mm}$,
ii. $>1.5 \mathrm{~d}_{\mathrm{b}}=1.5 \times 25=37.5 \mathrm{~mm}$, or
iii. (4/3) Max. coarse aggregate size, (4/3) $20=27 \mathrm{~mm}$.


Figure (8.16) Arrangement of steel for Example (7.9)

### 8.8.2 Design for Region two (Compression Controlled)

EXAMPLE (8.10)
A reinforced concrete square column is subjected to $\mathrm{P}_{\mathrm{d}}=1250 \mathrm{kN}, \mathrm{P}_{\mathrm{l}}=1000 \mathrm{kN}$, $\mathrm{DLM}=150 \mathrm{kN} . \mathrm{m}, \mathrm{LLm}=100 \mathrm{kN} . \mathrm{m}, f_{c}^{\prime}=30 \mathrm{MPa}, f_{y}=400 \mathrm{MPa}$, choose a suitable dimensions and reinforcement to make $\rho_{g} \approx 0.025$.

SOLUTION

$$
\begin{aligned}
& \hline P_{u}=1.2 \times 1250+1.6 \times 1000=3100 \mathrm{kN} \\
& M_{u}=1.2 \times 150+1.6 \times 100=340 \mathrm{kN} . \mathrm{m} \\
& e=M_{u} / P_{u}=340 / 3100=0.11 \mathrm{~m}=110 \mathrm{~mm} \\
& A_{g}=\frac{P_{u}}{0.4\left(f_{c}^{\prime}+\rho_{g} f_{y}\right)}=\frac{3.10}{0.4(30+0.025 \times 400)}=0.19375 \\
& b=h=\sqrt{A_{g}}=0.440 \mathrm{~m}=440 \mathrm{~mm}
\end{aligned}
$$

Try $\mathrm{b}=\mathrm{h}=450 \mathrm{~mm}$, since $\mathrm{e}=110 \mathrm{~mm}<\left(\mathrm{e}_{\mathrm{b}} \approx \mathrm{h} / 225 \mathrm{~mm}\right)$ it is a compression failure case.

$$
\begin{aligned}
& P_{n} \geq P_{u} / \phi=3100 / 0.65=4769.2 \mathrm{kN} \\
& M_{n} \geq M_{u} / \phi=340 / 0.65=523.1 \mathrm{kN} . \mathrm{m}
\end{aligned}
$$

Equation (7.20) can be used to find the necessary area of steel:

$$
\begin{aligned}
& P_{n}=\frac{b \cdot h \cdot f_{c}^{\prime}}{\frac{3 . h \cdot e}{d^{2}}+1.18}+\frac{A_{s}^{\prime} \cdot f_{y}}{\frac{e}{d-d^{\prime}}+0.5} \\
& =\frac{0.45^{2} \times 30}{\frac{3 \times 0.45 \times 0.11}{(0.387)^{2}}+1.18}+\frac{A_{s}^{\prime} \times 400}{\frac{0.11}{0.387-0.063}+0.5}=4.769
\end{aligned}
$$

$$
A_{s}^{\prime}=4137 \mathrm{~mm}^{2}
$$

$$
A_{s t}=2 \times 4137=8274 \mathrm{~mm}^{2}
$$

$$
\rho_{g}=8274 /(450 \times 450)=0.0409>0.025
$$

Try $\mathrm{b}=\mathrm{h}=475 \mathrm{~mm}$, since $\mathrm{e}=110 \mathrm{~mm}<\left(\mathrm{e}_{\mathrm{b}} \approx \mathrm{h} / 237.5 \mathrm{~mm}\right)$ it is still a compression failure case.

$$
=\frac{0.475^{2} \times 30}{\frac{3 \times 0.475 \times 0.11}{(0.412)^{2}}+1.18}+\frac{A_{s}^{\prime} \times 400}{\frac{0.11}{0.412-0.063}+0.5}=4.769
$$

$$
\begin{aligned}
& A_{s}^{\prime}=3079 \mathrm{~mm}^{2} \\
& A_{s t}=2 \times 3079=6158 \mathrm{~mm}^{2} \\
& \rho_{g}=6158 /(475 \times 475)=0.0273 \approx 0.025
\end{aligned}
$$

Use $10 \# 28=6160 \mathrm{~mm}^{2}$, use five bars on each face, Figure (8.17),
Clear spacing between bars $=(475-2 \times 40-2 \times 30-4 \times 28) / 4=56 \mathrm{~mm}$, this is within the required limits of clear spacing:
i. $>40 \mathrm{~mm}$,
ii. $>1.5 \mathrm{~d}_{\mathrm{b}}=1.5 \times 25=37.5 \mathrm{~mm}$, or
iii. (4/3) Max. coarse aggregate size, (4/3) $20=27 \mathrm{~mm}$.

Ties diameter $=\# 10 \mathrm{~mm}($ Bar dia. $<32 \mathrm{~mm})$, with spacing's:
i. $16 \mathrm{~d}_{\mathrm{b}}=16 \times 28=448 \mathrm{~mm}$,
ii. $48 \mathrm{~d}_{\mathrm{t}}=48 \times 10=480 \mathrm{~mm}$, or
iii. $\mathrm{b}=475 \mathrm{~mm}$.

Therefore use 3 \# 10 mm @ $400 \mathrm{~mm} \mathrm{c} / \mathrm{c}$, Figure (8.17).


Figure (8.17) Arrangement of bars for Example (8.10)

### 8.8.2 Design for Region Three (Tension Controlled)

In this case there is a change of behavior from a column failing in compression to a beam failing in tension, and therefore an increase of the strength reduction factor from 0.65 (when $\varepsilon_{\mathrm{t}}=0.002$ ) for tied columns and 0.75 for spiral columns to 0.9 ( when $\varepsilon_{t}=0.005$ ).

## EXAMPLE (8.11)

A reinforced concrete rectangular column is subjected to $\mathrm{P}_{\mathrm{d}}=500 \mathrm{kN}, \mathrm{P}_{\mathrm{l}}=400 \mathrm{kN}$, DLM $=200 \mathrm{kN} . \mathrm{m}, \mathrm{LLm}=150 \mathrm{kN} . \mathrm{m}, f_{c}^{\prime}=25 \mathrm{MPa}, f_{y}=400 \mathrm{MPa}$, choose a suitable dimensions and reinforcement to make $\rho_{g} \approx 0.03$.

## SOLUTION

$$
\begin{aligned}
& P_{u}=1.2 \times 500+1.6 \times 400=1240 \mathrm{kN} \\
& M_{u}=1.2 \times 200+1.6 \times 150=480 \mathrm{kN} . \mathrm{m} \\
& e=M_{u} / P_{u}=480 / 1240=0.387 \mathrm{~m}=378 \mathrm{~mm}
\end{aligned}
$$

$$
A_{g}=\frac{P_{u}}{0.4\left(f_{c}^{\prime}+\rho_{g} f_{y}\right)}=\frac{1.24}{0.4(25+0.03 \times 400)}=0.083783 \mathrm{~m}^{2}
$$

$$
h=1.25 b,
$$

$$
1.25 b^{2}=0.083783, \mathrm{~b}=259 \mathrm{~mm}, \mathrm{~h}=324 \mathrm{~mm} .
$$

Assume $\mathrm{b}=400 \mathrm{~mm}, \mathrm{~h}=500 \mathrm{~mm}$.
Since $\mathrm{e}=378 \mathrm{~mm}>\left(\mathrm{e}_{\mathrm{b}} \approx \mathrm{h} / 250 \mathrm{~mm}\right)$ it is a tension failure case. Try a strength reduction factor $\varphi=0.7$,

$$
\begin{aligned}
& P n \geq 1240 / 0.7=1771 \mathrm{kN} \\
& M_{n} \geq 480 / 0.7=686 \mathrm{kN} . \mathrm{m}
\end{aligned}
$$

Since it is tension failure, the tension steel will reach yielding, if the compression assumed to reach yielding also, the tensile force T will approximately equal to $\mathrm{C}_{2}$, and in this case $\mathrm{P}_{\mathrm{n}}=\mathrm{C}_{1}$

$$
P_{n}=1.771=0.85 f_{c}^{\prime} \cdot a . b=0.85 \times 25 \times a \times 0.4
$$

$\mathrm{a}=208 \mathrm{~mm}$, the couple created by $\left(\mathrm{T}\right.$ and $\left.\mathrm{C}_{2}\right)$ and $\left(\mathrm{P}_{\mathrm{n}}\right.$ and $\left.\mathrm{C}_{1}\right)$ will equalize each other and the area of the tension steel can be found:

$$
\left.P_{n}[e-h / 2+a / 2)\right]=A_{s} \cdot f_{y}\left(d-d^{\prime}\right)
$$

$$
1.771[0.378-0.25+0.208 / 2]=A_{s} \times 400(0 . .437-0.063)
$$

$A_{s}=2746 \mathrm{~mm}^{2}, 4 \# 32$ on each face $=3216 \mathrm{~mm}^{2} . \rho_{\mathrm{g}}=0.0322$. To check the strength use Equation (8.22)

$$
\rho^{\prime}=\rho=A_{s} /(b d)=3216 /(400 \times 434)=0.0185, e^{\prime}=378+(434-66) / 2=562 \mathrm{~mm}
$$

$$
e^{\prime} / d=562 / 434=1.295, d^{\prime} / d=66 / 434=0.152, m=400 /(0.85 \times 25)=18.82
$$

$$
\begin{aligned}
& P_{n}=0.85 f_{c}^{\prime} b . d\left\{-\rho+1-e^{\prime} / d+\sqrt{\left.\left(1-e^{\prime} / d\right)^{2}+2 \rho\left[(m-1)\left(1-d^{\prime} / d\right)+e^{\prime} / d\right]\right\}}\right. \\
& P_{n}=0.85 \times 25 \times 0.4 \times 0.434\left\{-0.0185+1-1.295+\sqrt{(1-1.295)^{2}+2 \times 0.0185[(18.82-1)(1-0.152)+1.295]}\right\} \\
& P_{n}=1.917 M N=1917 k N
\end{aligned}
$$

If the equilibrium method is used, $\mathrm{c}=275 \mathrm{~mm}, \mathrm{P}_{\mathrm{n}}=1.919 \mathrm{MN}, \epsilon_{\mathrm{t}}=0.00173$, and $\phi=$ $0.65, \phi \mathrm{P}_{\mathrm{n}}=1.247 \mathrm{MN}>1.24 \mathrm{MN}$
Clear spacing between bars $=(400-2 \times 40-2 \times 30-3 \times 32) / 3=55 \mathrm{~mm}$ (O.K.)

When the bending moment is high, it is preferred to use rectangular section, with its width parallel to the axis of rotation.

### 8.10 Circular Spirally Reinforced Columns

المقاطع الدائرية تستعمل عادة في الاماكن التي يكون فيها العمود منفردا، أي غير محاط بجدران أو قو اطع مثل بعض الأبنية العامة وأماكن وقوف الليبارات. كما أنها تستعمل أيضـا عندما تكون اللامركزية فليالا حيث أنها تظهر صلابة ومطيلية أكثر من الأعمدة المطوقة، ولهذا السبب يكون عامل تخفيض المقاومة = 0.75 ولا مقابل 0.65 للاعمدة المطوقة. ولكن الفرق بين سلوك النو عين من الأعمدة يقل عند زيادة اللامركزية وذلك لكون سلوك الاثثين يقترب من سلوك العتبة. مقاو مة العمود للحمل المحوري أو عندما تكون اللامركزيـة فلبلة تكون أكثر من العمود المطوق وبنفس المقطع
 خاصـة عندما تكون اللامركزيـة قليلة.

لتحليل أو لتصميم الأعمدة الدائرية يمكن الاستعانة بمخططات التداخل الموجودة في بعض المصـادر . كما
 الانضغاط يساوي أيضا (0.003) وأن معدل الاجهاد في الخرسانة في منطقة الانضغاط ثابت ويساوي أيضـا (0.85 f $f_{c}^{\prime}$ ). عند ذلك يتطلب معرفة خو اص قطعة الدائرة مثل المساحة وبعد المحور الوسطي للقطعة عن مركز الدائرة، الثنكل (22.9).

كما يمكن استعمال بعض المعادلات النقريبية الني اشتقها Whitney. فعندما يكون الانضغاط سائدا، تكون $P_{n}=\frac{A_{g} \cdot f_{c}^{\prime}}{\frac{9.6 h \cdot e}{\left(0.8 h+0.67 d_{s}\right)^{2}}+1.18}+\frac{A_{s t} \cdot f_{y}}{\frac{3 e}{d_{s}}+1}$

مقاومة الاضغاط الاسمية كما يأتي:
 مر اكز القضبان = قطر المقطع (أو عرضه) - 2 (الغطاء الخرساني الصـافي) - قطر القضبان المستعملة. عند اشنتقاق هذه المعادلة اعنبر أن تسليح الانضغاط سيصل الى الخضو ع عند وصول المقطع الى مقاو منه، كما أن المعادلة تعطي نتائجا تقريبية عندما تكون مقاومة الخضو ع للتنليح و كذلك نسبة التسليح كبيرة. أما عندما يكون الثد سائدا، فيمكن حساب مقاو مة الانضغاط الاسمية كما يلي:

$$
\begin{equation*}
P_{n}=0.85 f_{c}^{\prime} \cdot h^{2}\left[\sqrt{\left(\frac{0.85 e}{h}-0.38\right)^{2}+\frac{\rho_{g} \cdot m \cdot d_{s}}{2.5 h}}-\left(\frac{0.85 e}{h}-0.38\right)\right] \tag{8.30}
\end{equation*}
$$



## Curve A arm to centroid of the circle

Curve B Area

## x/h



بعض المقاطع المربعة توضع فيها القضبان على محيط دائرة أيضا، الثكل (8.23). لتحليل أو لتصميم مثل هذه المقاطع يمكن استعمال توافق الانفعالات ومعادلات توازن القوى، كما يمكن اسنعمال المعادلات التقريبية التي اشتقها Whitney. فعندما يكون الانضغاط سائدا تكون مقاومة الانضغاط الاسمية كما يلي:

$$
\begin{equation*}
P_{n}=\frac{A_{g} \cdot f_{c}^{\prime}}{\frac{12 h \cdot e}{\left(h+0.67 d_{s}\right)^{2}}+1.18}+\frac{A_{s t} \cdot f_{y}}{\frac{3 e}{d_{s}}+1} \tag{8.31}
\end{equation*}
$$

أما عندما يكون الشد سائدا، فيمكن حساب مقاومة الانضغاط الاسمية كما يلي:

$$
\begin{equation*}
P_{n}=0.85 f_{c}^{\prime} \cdot h^{2}\left[\sqrt{\left(\frac{e}{h}-0.5\right)^{2}+\frac{0.67 \rho_{g} \cdot m \cdot d_{s}}{h}}-\left(\frac{e}{h}-0.5\right)\right] \tag{8.32}
\end{equation*}
$$

مثال (8.12)
عمود خرساني مربع المقطع (500×500 mm) محمل بحمل معامل مقاره $1500 k N=P_{u}$ () ييعد (250mm) عن الدحور الوسطي للمقطع. '20MPa مساحة التسليح اللازمة لو رتبت القضبان على محيط دائرة.


مقطع مربع


مقطع دائري

الثكل (23.8) الأعمدة الخرسانية المسلحة حلزونياً

الحل

$$
D_{c}=500-2 \times 40=420 \mathrm{~mm}
$$

$$
A_{g}=0.5^{2}=0.25 m^{2}
$$

$$
A_{c}=\pi(0.42)^{2} / 4=0.1385 m^{2}
$$

$\rho_{s} \geq 0.45 \frac{20}{276}\left(\frac{0.25}{0.1385}-1\right)=0.0263$
لو افترض قطر الحلزون $\left.79 \mathrm{~mm}^{2}=a_{s}\right) 10 \mathrm{~mm}=d_{s}$ : وباستعمال المعادلة (19.9) لصساب $0.0263=\frac{4 \times 79 \times 10^{-6}(0.42-0.01)}{(0.42)^{2} . s}$
$s=0.028 m=28.0 \mathrm{~mm}$
و وهن القيمة فليلة لان المسافة الصافية بين الحلزون في الاتجاه العمودي ستكون (28-10 mm= 18 18 ) و هذه أقل من (25 mm). يجرب قطر (12 mm) للحلزون:
$0.0263=\frac{4 \times 113 \times 10^{-6}(0.42-0.012)}{(0.42)^{2} . s}$
$s=0.04 \mathrm{~m}=40 \mathrm{~mm}$
وتكون المسافة الصافية بين الحلزون في الاتجاه العمودي = 28 و وذا الرقم يقع ضمن الحدود المذكورة
المقاومة الاسمية اللازمة
$P_{n} \geq 1.5 / 0.75=2.0 \mathrm{MN}$
$d_{s}=500-2 \times 40-2 \times 12-25=371 \mathrm{~mm}$
بما أن قيمة (e ) غير معلومة فلا يمكن التتبؤ بحالة الفشل هل هي انضغاط أم شد، للّك تستعمل المعادلتين للانضغاط والثد ويعتمد ايهما أكبر من مساحة التنليح:

$$
2.0=\frac{0.5^{2} \times 20}{\frac{12 \times 0.5 \times 0.25}{(0.5+0.67 \times 0.371)^{2}}+1.18}+\frac{A_{s t} \times 276}{\frac{3 \times 0.25}{0.371}+1}
$$

$$
7703 \mathrm{~mm}^{2}=A_{s t}
$$

أما اذا كان الثند سائدا، فيمكن حساب مقاومة الانضغاط الاسمية كما يلي:

$$
2.0=0.85 \times 20 \times 0.5^{2}\left[\sqrt{\left(\frac{0.25}{0.5}-0.5\right)^{2}+\frac{0.67 \rho_{g} \times 16.235 \times 0.371}{0.5}}-\left(\frac{0.25}{0.5}-0.5\right)\right]
$$

$$
\text { 6850mm² = } A_{s t} 0.0274=\rho_{g} \text { ، اذن الانضغاط هو السائد وتعتمد قيمة } 7703 \mathrm{~mm}^{2}=A_{s t} \text { ، }
$$ تستعمل 7856mm² $=25 \phi 16$. المسافة الصافية بين القضبان تساوي:

$$
\begin{aligned}
& (\pi \times 371-16 \times 25) / 16=48 \mathrm{~mm}
\end{aligned}
$$

$$
\begin{aligned}
& \text { 据 } \frac{4}{3} \text { < } 48 \mathrm{~mm}
\end{aligned}
$$

