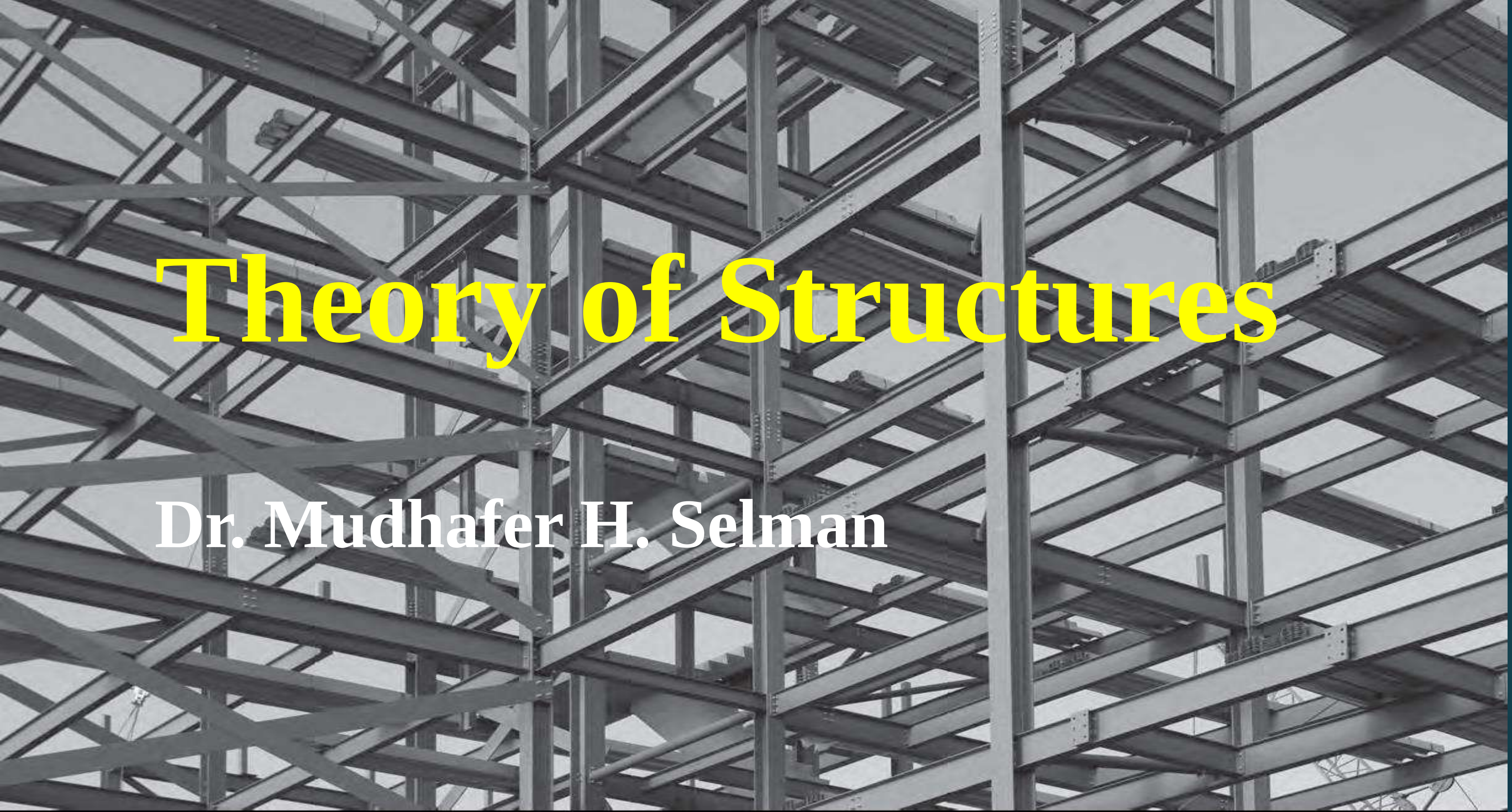

Theory of Structures

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Types of Structures and Loads

1

1.1 Introduction

A *structure* refers to a system of connected parts used to support a load. Important examples related to civil engineering include buildings, bridges, and towers; and in other branches of engineering, ship and aircraft frames, tanks, pressure vessels, mechanical systems, and electrical supporting structures are important.

1.2 Classification of Structures

It is important for a structural engineer to recognize the various types of elements composing a structure and to be able to classify structures as to their form and function. We will introduce some of these aspects now and expand on them at appropriate points throughout the text.

Structural Elements. Some of the more common elements from which structures are composed are as follows.

Tie Rods. Structural members subjected to a *tensile force* are often referred to as *tie rods* or *bracing struts.* Due to the nature of this load, these members are rather slender, and are often chosen from rods, bars, angles, or channels, Fig. 1–1.

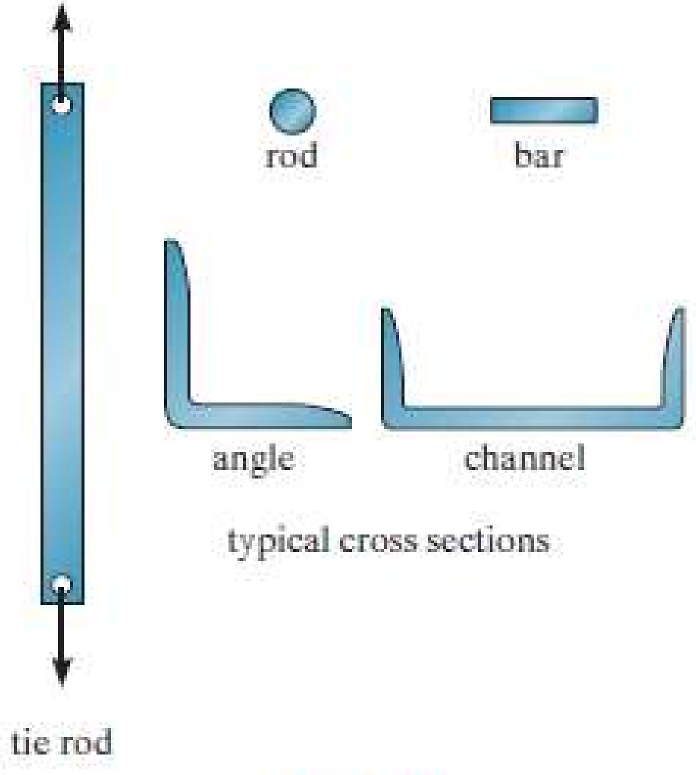


Fig. 1–1

Beams. Beams are usually straight horizontal members used primarily to carry vertical loads. Quite often they are classified according to the way they are supported, as indicated in Fig. 1–2. In particular, when the cross section varies the beam is referred to as tapered or haunched. Beam cross sections may also be "built up" by adding plates to their top and bottom.

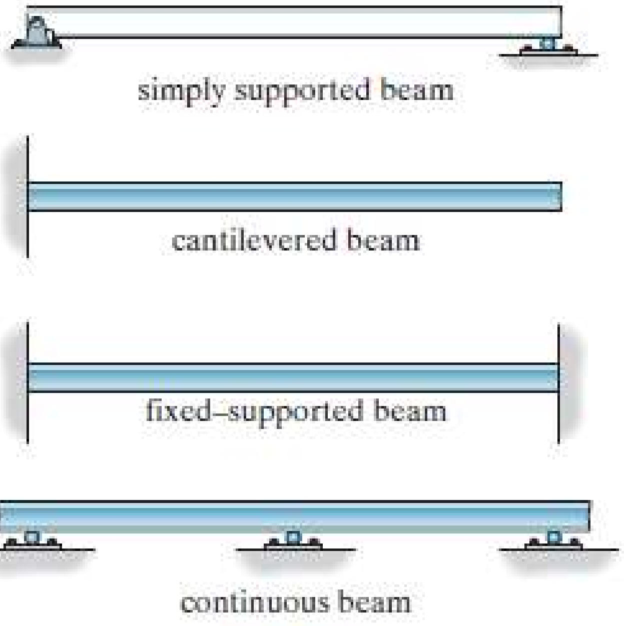
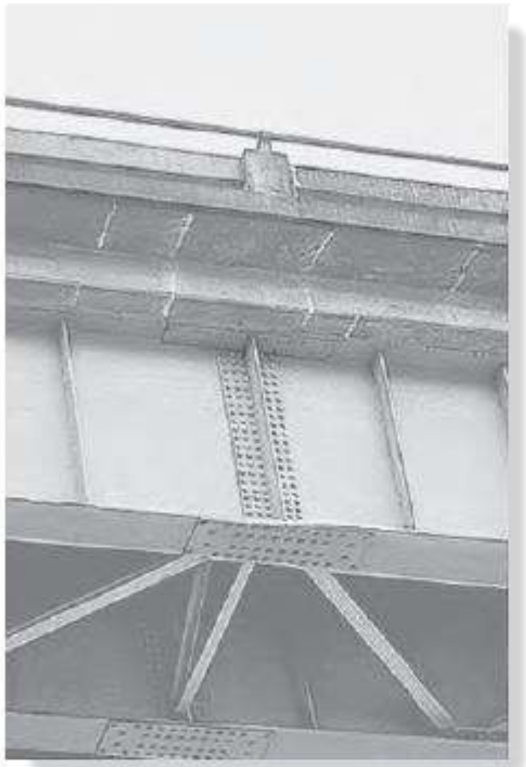


Fig. 1–2

The prestressed concrete girders are simply supported and are used for this highway bridge.



Shown are typical splice plate joints used to connect the steel girders of a highway bridge.

The steel reinforcement cage shown on the right and left is used to resist any tension that may develop in the concrete beams which will be formed around it.

Columns. Members that are generally vertical and resist axial compressive loads are referred to as *columns*, Fig. 1–4. Tubes and wide-flange cross sections are often used for metal columns, and circular and square cross sections with reinforcing rods are used for those made of concrete. Occasionally, columns are subjected to both an axial load and a bending moment as shown in the figure. These members are referred to as *beam columns*.

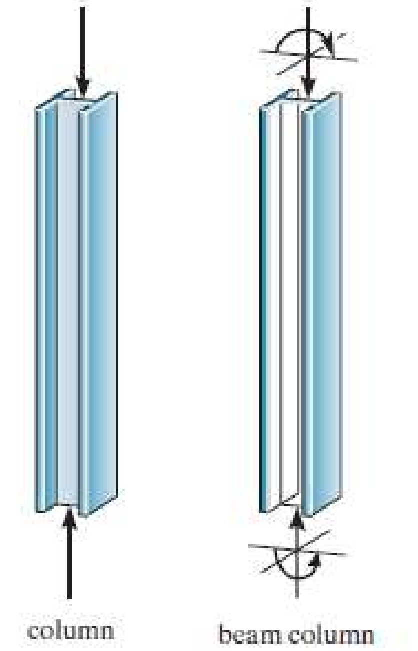


Fig. 1–4

Wide-flange members are often used for columns. Here is an example of a beam column.

Types of Structures. The combination of structural elements and the materials from which they are composed is referred to as a *structural system*. Each system is constructed of one or more of four basic types of structures. Ranked in order of complexity of their force analysis, they are as follows.

Trusses. When the span of a structure is required to be large and its depth is not an important criterion for design, a truss may be selected. *Trusses* consist of slender elements, usually arranged in triangular fashion. *Planar trusses* are composed of members that lie in the same plane and are frequently used for bridge and roof support, whereas *space trusses* have members extending in three dimensions and are suitable for derricks and towers.

Loading causes bending of truss, which develops compression in top members, tension in bottom members.

Cables and Arches. Two other forms of structures used to span long distances are the cable and the arch. *Cables* are usually flexible and carry their loads in tension. They are commonly used to support bridges, Fig. 1–6*a*, and building roofs. When used for these purposes, the cable has an advantage over the beam and the truss, especially for spans that are greater than 150 ft (46 m). Because they are always in tension, cables will not become unstable and suddenly collapse, as may happen with beams or trusses. Furthermore, the truss will require added costs for construction and increased depth as the span increases. Use of cables, on the other hand, is limited only by their sag, weight, and methods of anchorage.

The *arch* achieves its strength in compression, since it has a reverse curvature to that of the cable. The arch must be rigid, however, in order to maintain its shape, and this results in secondary loadings involving shear and moment, which must be considered in its design. Arches are frequently used in bridge structures, Fig. 1–6*b*, dome roofs, and for openings in masonry walls.

Cables support their loads in tension.
(a)

Arches support their loads in compression.
(b)

Frames. Frames are often used in buildings and are composed of beams and columns that are either pin or fixed connected, Fig. 1–7. Like trusses, frames extend in two or three dimensions. The loading on a frame causes bending of its members, and if it has rigid joint connections, this structure is generally "indeterminate" from a standpoint of analysis. The strength of such a frame is derived from the moment interactions between the beams and the columns at the rigid joints.

Here is an example of a steel frame that is used to support a crane rail. The frame can be assumed fixed connected at its top joints and pinned at the supports.

Surface Structures. A *surface structure* is made from a material having a very small thickness compared to its other dimensions. Sometimes this material is very flexible and can take the form of a tent or air-inflated structure. In both cases the material acts as a membrane that is subjected to pure tension.

The roof of the "Georgia Dome" in Atlanta, Georgia can be considered as a thin membrane.

1.3 Loads

Once the dimensional requirements for a structure have been defined, it becomes necessary to determine the loads the structure must support. Often, it is the anticipation of the various loads that will be imposed on the structure that provides the basic type of structure that will be chosen for design. For example, high-rise structures must endure large lateral loadings caused by wind, and so shear walls and tubular frame systems are selected, whereas buildings located in areas prone to earthquakes must be designed having ductile frames and connections.

TABLE 1–1 Codes
General Building Codes
Minimum Design Loads for Buildings and Other Structures, ASCE/SEI 7-10, American Society of Civil Engineers *International Building Code*
Design Codes
Building Code Requirements for Reinforced Concrete, Am. Conc. Inst. (ACI) *Manual of Steel Construction,* American Institute of Steel Construction (AISC) *Standard Specifications for Highway Bridges,* American Association of State Highway and Transportation Officials (AASHTO) *National Design Specification for Wood Construction,* American Forest and Paper Association (AFPA) *Manual for Railway Engineering,* American Railway Engineering Association (AREA)

Dead Loads. *Dead loads* consist of the weights of the various structural members and the weights of any objects that are permanently attached to the structure. Hence, for a building, the dead loads include the weights of the columns, beams, and girders, the floor slab, roofing, walls, windows, plumbing, electrical fixtures, and other miscellaneous attachments.

In some cases, a structural dead load can be estimated satisfactorily from simple formulas based on the weights and sizes of similar structures. Through experience one can also derive a "feeling" for the magnitude of these loadings. For example, the average weight for timber buildings is 40–50 lb/ft^2 (1.9–2.4 kN/m^2), for steel framed buildings it is 60–75 lb/ft^2 (2.9–3.6 kN/m^2), and for reinforced concrete buildings it is 110–130 lb/ft^2 (5.3–6.2 kN/m^2). Ordinarily, though, once the materials and sizes of the various components of the structure are determined, their weights can be found from tables that list their densities.

The densities of typical materials used in construction are listed in Table 1–2, and a portion of a table listing the weights of typical building

TABLE 1–2 Minimum Densities for Design Loads from Materials*

	lb/ft^3	kN/m^3
Aluminum	170	26.7
Concrete, plain cinder	108	17.0
Concrete, plain stone	144	22.6
Concrete, reinforced cinder	111	17.4
Concrete, reinforced stone	150	23.6
Clay, dry	63	9.9
Clay, damp	110	17.3
Sand and gravel, dry, loose	100	15.7
Sand and gravel, wet	120	18.9
Masonry, lightweight solid concrete	105	16.5
Masonry, normal weight	135	21.2
Plywood	36	5.7
Steel, cold-drawn	492	77.3
Wood, Douglas Fir	34	5.3
Wood, Southern Pine	37	5.8
Wood, spruce	29	4.5

*Reproduced with permission from American Society of Civil Engineers *Minimum Design Loads for Buildings and Other Structures*, ASCE/SEI 7-10. Copies of this standard may be purchased from ASCE at www.pubs.asce.org.

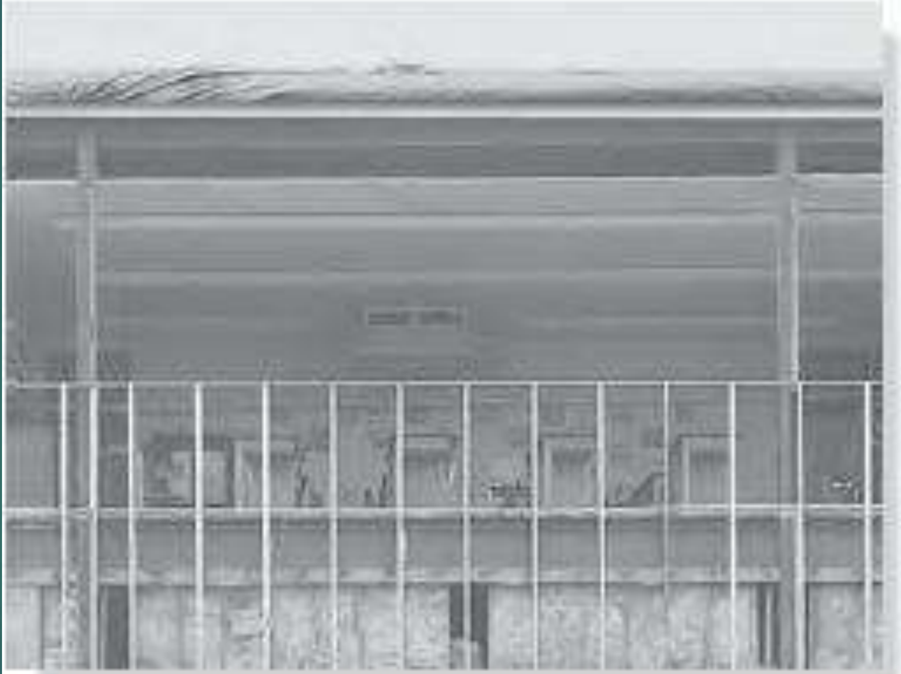
TABLE 1–3 Minimum Design Dead Loads*

Walls	psf	kN/m^2
4-in. (102 mm) clay brick	39	1.87
8-in. (203 mm) clay brick	79	3.78
12-in. (305 mm) clay brick	115	5.51
Frame Partitions and Walls		
Exterior stud walls with brick veneer	48	2.30
Windows, glass, frame and sash	8	0.38
Wood studs 2 × 4 in., (51 × 102 mm) unplastered	4	0.19
Wood studs 2 × 4 in., (51 × 102 mm) plastered one side	12	0.57
Wood studs 2 × 4 in., (51 × 102 mm) plastered two sides	20	0.96
Floor Fill		
Cinder concrete, per inch (mm)	9	0.017
Lightweight concrete, plain, per inch (mm)	8	0.015
Stone concrete, per inch (mm)	12	0.023
Ceilings		
Acoustical fiberboard	1	0.05
Plaster on tile or concrete	5	0.24
Suspended metal lath and gypsum plaster	10	0.48
Asphalt shingles	2	0.10
Fiberboard, $\frac{1}{2}$-in. (13 mm)	0.75	0.04

*Reproduced with permission from American Society of Civil Engineers *Minimum Design Loads for Buildings and Other Structures*, ASCE/SEI 7-10.

Live Loads. *Live Loads* can vary both in their magnitude and location. They may be caused by the weights of objects temporarily placed on a structure, moving vehicles, or natural forces. The minimum live loads specified in codes are determined from studying the history of their effects on existing structures. Usually, these loads include additional protection against excessive deflection or sudden overload. In Chapter 6 we will develop techniques for specifying the proper location of live loads on the structure so that they cause the greatest stress or deflection of the members. Various types of live loads will now be discussed.

Building Loads. The floors of buildings are assumed to be subjected to *uniform live loads*, which depend on the purpose for which the building is designed. These loadings are generally tabulated in local, state, or national codes. A representative sample of such *minimum live loadings*, taken from the ASCE 7-10 Standard, is shown in Table 1–4. The values are determined from a history of loading various buildings. They include some protection against the possibility of overload due to emergency situations, construction loads, and serviceability requirements due to vibration. In addition to uniform loads, some codes specify *minimum concentrated live loads*, caused by hand carts, automobiles, etc., which must also be applied anywhere to the floor system. For example, both uniform and concentrated live loads must be considered in the design of an automobile parking deck.



The live floor loading in this classroom consists of desks, chairs and laboratory equipment. For design the ASCE 7-10 Standard specifies a loading of 40 psf or 1.92 kN/m^2.

TABLE 1–4 Minimum Live Loads*

Occupancy or Use	Live Load psf	Live Load kN/m²	Occupancy or Use	Live Load psf	Live Load kN/m²
Assembly areas and theaters			Residential		
Fixed seats	60	2.87	Dwellings (one- and two-family)	40	1.92
Movable seats	100	4.79	Hotels and multifamily houses		
Garages (passenger cars only)	50	2.40	Private rooms and corridors	40	1.92
Office buildings			Public rooms and corridors	100	4.79
Lobbies	100	4.79	Schools		
Offices	50	2.40	Classrooms	40	1.92
Storage warehouse			Corridors above first floor	80	3.83
Light	125	6.00			
Heavy	250	11.97			

*Reproduced with permission from *Minimum Design Loads for Buildings and Other Structures*, ASCE/SEI 7-10.

For some types of buildings having very large floor areas, many codes will allow a *reduction* in the uniform live load for a *floor*, since it is unlikely that the prescribed live load will occur simultaneously throughout the entire structure at any one time. For example, ASCE 7-10 allows a reduction of live load on a member having an *influence area* ($K_{LL}\ A_T$) of 400 ft^2 (37.2 m^2) or more. This reduced live load is calculated using the following equation:

$$L = L_o\left(0.25 + \frac{15}{\sqrt{K_{LL}A_T}}\right) \quad \text{(FPS units)}$$

$$L = L_o\left(0.25 + \frac{4.57}{\sqrt{K_{LL}A_T}}\right) \quad \text{(SI units)} \qquad (1\text{–}1)$$

where

L = reduced design live load per square foot or square meter of area supported by the member.

L_o = unreduced design live load per square foot or square meter of area supported by the member (see Table 1–4).

K_{LL} = live load element factor. For interior columns $K_{LL} = 4$.

A_T = tributary area in square feet or square meters.*

The reduced live load defined by Eq. 1–1 is limited to not less than 50% of L_o for members supporting one floor, or not less than 40% of L_o for members supporting more than one floor. No reduction is allowed for loads exceeding 100 lb/ft^2 (4.79 kN/m^2), or for structures used for public assembly, garages, or roofs. Example 1–2 illustrates Eq. 1–1's application.

EXAMPLE 1.2

A two-story office building shown in the photo has interior columns that are spaced 22 ft apart in two perpendicular directions. If the (flat) roof loading is 20 lb/ft^2, determine the reduced live load supported by a typical interior column located at ground level.

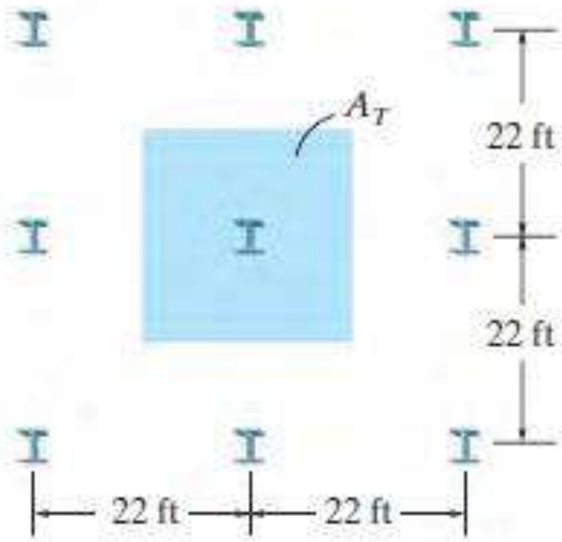


Fig. 1–9

SOLUTION

As shown in Fig. 1–9, each interior column has a tributary area or effective loaded area of $A_T = (22\text{ ft})(22\text{ ft}) = 484\text{ ft}^2$. A ground-floor column therefore supports a roof live load of

$$F_R = (20\text{ lb/ft}^2)(484\text{ ft}^2) = 9680\text{ lb} = 9.68\text{ k}$$

This load cannot be reduced, since it is not a floor load. For the second floor, the live load is taken from Table 1–4: $L_o = 50\text{ lb/ft}^2$. Since $K_{LL} = 4$, then $4A_T = 4(484\text{ ft}^2) = 1936\text{ ft}^2$ and $1936\text{ ft}^2 > 400\text{ ft}^2$, the live load can be reduced using Eq. 1–1. Thus,

$$L = 50\left(0.25 + \frac{15}{\sqrt{1936}}\right) = 29.55\text{ lb/ft}^2$$

The load reduction here is $(29.55/50)100\% = 59.1\% > 50\%$. O.K. Therefore,

$$F_F = (29.55\text{ lb/ft}^2)(484\text{ ft}^2) = 14\,300\text{ lb} = 14.3\text{ k}$$

The total live load supported by the ground-floor column is thus

$$F = F_R + F_F = 9.68\text{ k} + 14.3\text{ k} = 24.0\text{ k} \qquad \textit{Ans.}$$

Highway Bridge Loads. The primary live loads on bridge spans are those due to traffic, and the heaviest vehicle loading encountered is that caused by a series of trucks. Specifications for truck loadings on highway bridges are reported in the *LRFD Bridge Design Specifications* of the American Association of State and Highway Transportation Officials (AASHTO). For two-axle trucks, these loads are designated with an H, followed by the weight of the truck in tons and another number which gives the year of the specifications in which the load was reported. H-series truck weights vary from 10 to 20 tons. However, bridges located on major highways, which carry a great deal of traffic, are often designed for two-axle trucks plus a one-axle semitrailer as in Fig. 1–10. These are designated as HS loadings. In general, a truck loading selected for design depends upon the type of bridge, its location, and the type of traffic anticipated.

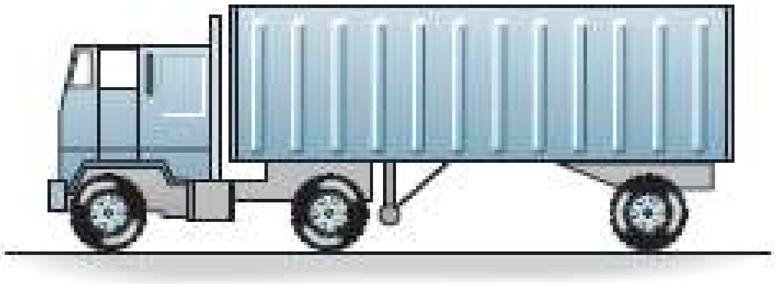


Fig. 1–10

Railroad Bridge Loads. The loadings on railroad bridges, as in Fig. 1–11, are specified in the *Specifications for Steel Railway Bridges* published by the American Railroad Engineers Association (AREA). Normally, E loads, as originally devised by Theodore Cooper in 1894, were used for design. B. Steinmann has since updated Cooper's load distribution and has devised a series of M loadings, which are currently acceptable for design. Since train loadings involve a complicated series of concentrated forces, to simplify hand calculations, tables and graphs are sometimes used in conjunction with influence lines to obtain the critical load. Also, computer programs are used for this purpose.

Fig. 1–11

Impact Loads. Moving vehicles may bounce or sidesway as they move over a bridge, and therefore they impart an *impact* to the deck. The percentage increase of the live loads due to impact is called the *impact factor*, I. This factor is generally obtained from formulas developed from experimental evidence. For example, for highway bridges the AASHTO specifications require that

$$I = \frac{50}{L + 125} \qquad \text{but not larger than } 0.3$$

Wind Loads. When structures block the flow of wind, the wind's kinetic energy is converted into potential energy of pressure, which causes a wind loading. The effect of wind on a structure depends upon the density and velocity of the air, the angle of incidence of the wind, the shape and stiffness of the structure, and the roughness of its surface. For design purposes, wind loadings can be treated using either a static or a dynamic approach.

For the static approach, the fluctuating pressure caused by a constantly blowing wind is approximated by a mean velocity pressure that acts on the structure. This pressure q is defined by its kinetic energy, $q = \frac{1}{2}\rho V^2$, where ρ is the density of the air and V is its velocity. According to the ASCE 7-10 Standard, this equation is modified to account for the importance of the structure, its height, and the terrain in which it is located. It is represented as

$$\boxed{\begin{aligned} q_z &= 0.00256 K_z K_{zt} K_d V^2 \ (\text{lb/ft}^2) \\ q_z &= 0.613 K_z K_{zt} K_d V^2 \ (\text{N/m}^2) \end{aligned}} \qquad (1\text{–}2)$$

where

V = the velocity in mi/h (m/s) of a 3-second gust of wind measured 33 ft (10 m) above the ground. Specific values depend upon the "category" of the structure obtained from a wind map. For example, the interior portion of the continental United States reports a wind speed of 105 mi/h (47 m/s) if the structure is an agricultural or storage building, since it is of low risk to human life in the event of a failure. The wind speed is 120 mi/h (54 m/s) for cases where the structure is a hospital, since its failure would cause substantial loss of human life.

Hurricane winds caused this damage to a condominium in Miami, Florida.

K_z = the velocity pressure exposure coefficient, which is a function of height and depends upon the ground terrain. Table 1–5 lists values for a structure which is located in open terrain with scattered low-lying obstructions.

K_{zt} = a factor that accounts for wind speed increases due to hills and escarpments. For flat ground $K_{zt} = 1.0$.

K_d = a factor that accounts for the direction of the wind. It is used only when the structure is subjected to combinations of loads (see Sec. 1–4). For wind acting alone, $K_d = 1.0$.

TABLE 1–5 Velocity Pressure Exposure Coefficient for Terrain with Low-Lying Obstructions

z (ft)	z (m)	K_z
0–15	0–4.6	0.85
20	6.1	0.90
25	7.6	0.94
30	9.1	0.98
40	12.2	1.04
50	15.2	1.09

Design Wind Pressure for Enclosed Buildings. Once the value for q_z is obtained, the design pressure can be determined from a list of relevant equations listed in the ASCE 7-10 Standard. The choice depends upon the flexibility and height of the structure, and whether the design is for the main wind-force resisting system, or for the building's components and cladding. For example, using a "directional procedure" the *wind-pressure* on an enclosed building of any height is determined using a two-termed equation resulting from both external and internal pressures, namely,

$$\boxed{p = qGC_p - q_h(GC_{pi})} \tag{1–3}$$

Wind blowing on a wall will tend to tip a building or cause it to sidesway. To prevent this engineers often use cross bracing to provide stability. Also, see p. 46.

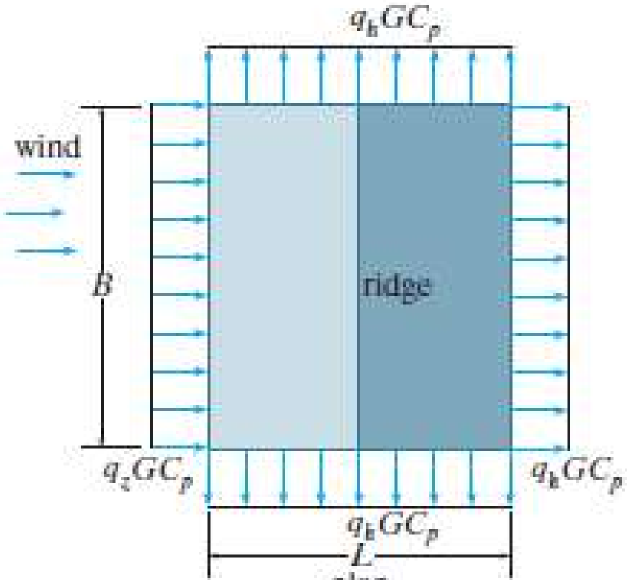
Here

$q =$ q_z for the windward wall at height z above the ground (Eq. 1–2), and $q = q_h$ for the leeward walls, side walls, and roof, where $z = h$, the mean height of the roof.

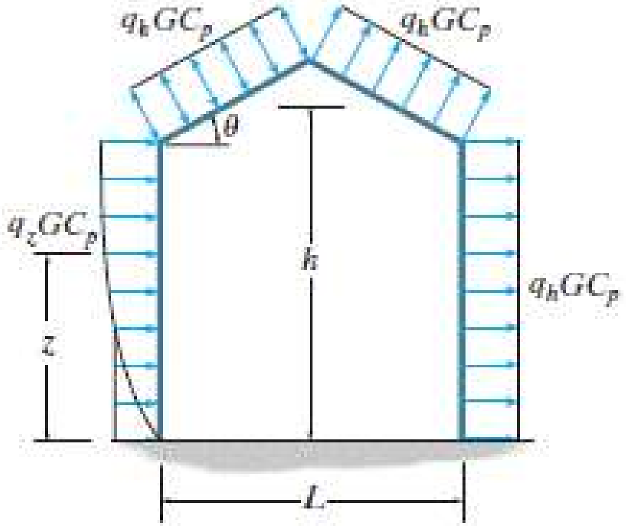
$G =$ a wind-gust effect factor, which depends upon the exposure. For example, for a rigid structure, $G = 0.85$.

$C_p =$ a wall or roof pressure coefficient determined from a table. These tabular values for the walls and a roof pitch of $\theta = 10°$ are given in Fig. 1–12. Note in the elevation view that the pressure will vary with height on the windward side of the building, whereas on the remaining sides and on the roof the pressure is assumed to be constant. Negative values indicate pressures acting away from the surface.

$(GC_{pi}) =$ the internal pressure coefficient, which depends upon the type of openings in the building. For fully enclosed buildings $(GC_{pi}) = \pm 0.18$. Here the signs indicate that either positive or negative (suction) pressure can occur within the building.



plan



elevation

Surface	L/B	C_p	Use with
Windward wall	All values	0.8	q_z
Leeward wall	0–1 2 ≥4	−0.5 −0.3 −0.2	q_h
Side walls	All values	−0.7	q_h

Wall pressure coefficients, C_p

(a)

Wind direction	Windward angle θ		Leeward angle
	h/L	10°	$\theta = 10°$
Normal to ridge	≤0.25 0.5 >1.0	−0.7 −0.9 −1.3	−0.3 −0.5 −0.7

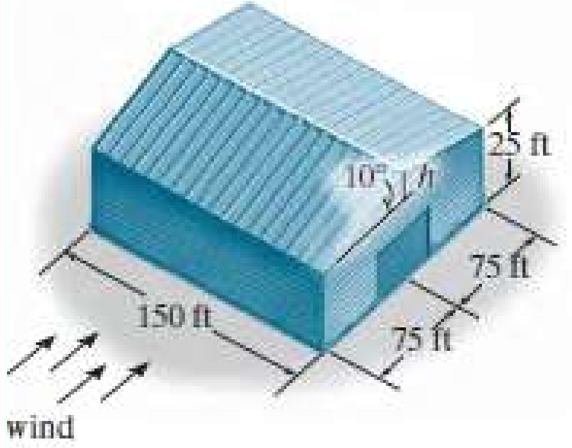
Maximum negative roof pressure coefficients, C_p, for use with q_h

(b)

Fig. 1–12

EXAMPLE 1.3

The enclosed building shown in the photo and in Fig. 1–13*a* is used for storage purposes and is located outside of Chicago, Illinois on open flat terrain. When the wind is directed as shown, determine the design wind pressure acting on the roof and sides of the building using the ASCE 7-10 Specifications.



(a)

Fig. 1–13

SOLUTION

First the wind pressure will be determined using Eq. 1–2. The basic wind speed is $V = 105$ mi/h, since the building is used for storage. Also, for flat terrain, $K_{zt} = 1.0$. Since only wind loading is being considered, $K_d = 1.0$. Therefore,

$$\begin{aligned} q_z &= 0.00256\, K_z K_{zt} K_d V^2 \\ &= 0.00256\, K_z(1.0)(1.0)(105)^2 \\ &= 28.22\, K_z \end{aligned}$$

From Fig. 1–13*a*, $h' = 75 \tan 10^\circ = 13.22$ ft so that the mean height of the roof is $h = 25 + 13.22/2 = 31.6$ ft. Using the values of K_z in Table 1–5, calculated values of the pressure profile are listed in the table in Fig. 1–13*b*. Note the value of K_z was determined by linear interpolation for $z = h$, i.e., $(1.04 - 0.98)/(40 - 30) = (1.04 - K_z)/(40 - 31.6)$, $K_z = 0.990$, and so $q_h = 28.22(0.990) = 27.9$ psf.

In order to apply Eq. 1–3 the gust factor is $G = 0.85$, and $(GC_{pi}) = \pm 0.18$. Thus,

$$
\begin{aligned}
p &= qGC_p - q_h(GC_{pi}) \\
&= q(0.85)C_p - 27.9(\pm 0.18) \\
&= 0.85qC_p \mp 5.03 \qquad (1)
\end{aligned}
$$

The pressure loadings are obtained from this equation using the calculated values for q_z listed in Fig. 1–13*b* in accordance with the wind-pressure profile in Fig. 1–12.

z (ft)	K_z	q_z (psf)
0–15	0.85	24.0
20	0.90	25.4
25	0.94	26.5
h = 31.6	0.990	27.9

(b)

Windward Wall. Here the pressure varies with height z since $q_z GC_p$ must be used. For all values of $L/B, C_p = 0.8$, so that from Eq. (1),

$$p_{0-15} = 11.3 \text{ psf} \quad \text{or} \quad 21.3 \text{ psf}$$
$$p_{20} = 12.2 \text{ psf} \quad \text{or} \quad 22.3 \text{ psf}$$
$$p_{25} = 13.0 \text{ psf} \quad \text{or} \quad 23.1 \text{ psf}$$

Leeward Wall. Here $L/B = 2(75)/150 = 1$, so that $C_p = -0.5$. Also, $q = q_h$ and so from Eq. (1),

$$p = -16.9 \text{ psf} \quad \text{or} \quad -6.84 \text{ psf}$$

Side Walls. For all values of $L/B, C_p = -0.7$, and therefore since we must use $q = q_h$ in Eq. (1), we have

$$p = -21.6 \text{ psf} \quad \text{or} \quad -11.6 \text{ psf}$$

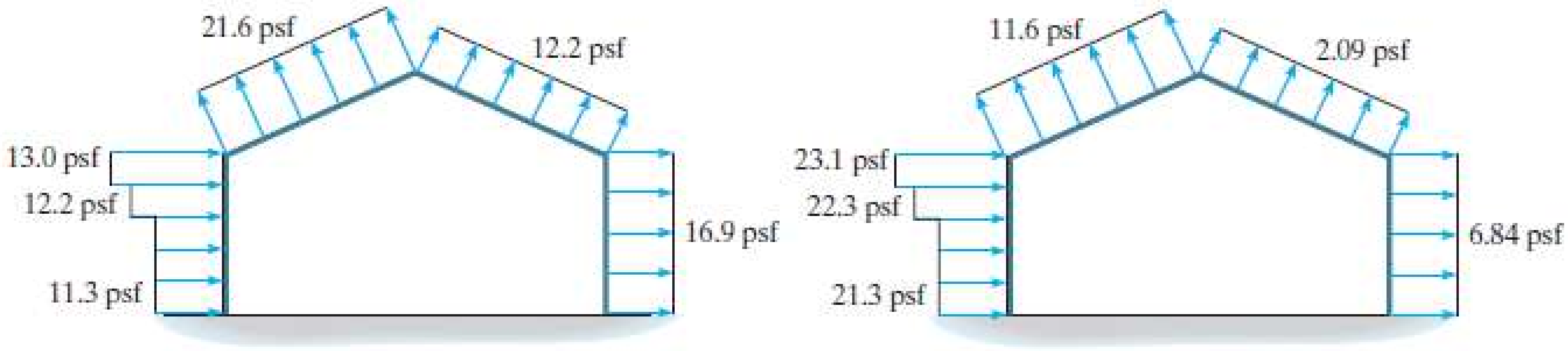
Windward Roof. Here $h/L = 31.6/2(75) = 0.211 < 0.25$, so that $C_p = -0.7$ and $q = q_h$. Thus,

$$p = -21.6 \text{ psf} \quad \text{or} \quad -11.6 \text{ psf}$$

Leeward Roof. In this case $C_p = -0.3$; therefore with $q = q_h$, we get

$$p = -12.2 \text{ psf} \quad \text{or} \quad -2.09 \text{ psf}$$

These two sets of loadings are shown on the elevation of the building, representing either positive or negative (suction) internal building pressure, Fig. 1–13*c*. The main framing structure of the building must resist these loadings as well as for separate loadings calculated from wind blowing on the front or rear of the building.



(c)

Snow Loads. In some parts of the country, roof loading due to snow can be quite severe, and therefore protection against possible failure is of primary concern. Design loadings typically depend on the building's general shape and roof geometry, wind exposure, location, its importance, and whether or not it is heated. Like wind, snow loads in the ASCE 7-10 Standard are generally determined from a zone map reporting 50-year recurrence intervals of an extreme snow depth. For example, on the relatively flat elevation throughout the mid-section of Illinois and Indiana, the ground snow loading is 20 lb/ft^2 (0.96 kN/m^2). However, for areas of Montana, specific case studies of ground snow loadings are needed due to the variable elevations throughout the state. Specifications for snow loads are covered in the ASCE 7-10 Standard, although no single code can cover all the implications of this type of loading.

If a roof is flat, defined as having a slope of less than 5%, then the pressure loading on the roof can be obtained by modifying the ground snow loading, p_g, by the following empirical formula

Excessive snow and ice loadings act on this roof.

$$\boxed{p_f = 0.7C_eC_tI_sp_g} \qquad (1\text{–}5)$$

Here

C_e = an exposure factor which depends upon the terrain. For example, for a fully exposed roof in an unobstructed area, $C_e = 0.8$, whereas if the roof is sheltered and located in the center of a large city, then $C_e = 1.2$.

C_t = a thermal factor which refers to the average temperature within the building. For unheated structures kept below freezing $C_t = 1.2$, whereas if the roof is supporting a normally heated structure, then $C_t = 1.0$.

I_s = the importance factor as it relates to occupancy. For example, $I_s = 0.80$ for agriculture and storage facilities, and $I_s = 1.20$ for schools and hospitals.

If $p_g \leq 20\ \text{lb/ft}^2$ ($0.96\ \text{kN/m}^2$), then use the *largest value* for p_f, either computed from the above equation or from $p_f = I_sp_g$. If $p_g > 20\ \text{lb/ft}^2$ ($0.96\ \text{kN/m}^2$), then use $p_f = I_s(20\ \text{lb/ft}^2)$.

EXAMPLE 1.4

The unheated storage facility shown in Fig. 1–14 is located on flat open terrain in southern Illinois, where the specified ground snow load is 15 lb/ft^2. Determine the design snow load on the roof which has a slope of 4%.

Fig. 1–14

SOLUTION

Since the roof slope is < 5%, we will use Eq. 1–5. Here, $C_e = 0.8$ due to the open area, $C_t = 1.2$ and $I_s = 0.8$. Thus,

$$p_f = 0.7C_eC_tI_sp_g$$
$$= 0.7(0.8)(1.2)(0.8)(15\text{ lb/ft}^2) = 8.06\text{ lb/ft}^2$$

Since $p_g = 15\text{ lb/ft}^2 < 20\text{ lb/ft}^2$, then also

$$p_f = Ip_g = 1.2(15\text{ lb/ft}^2) = 18\text{ lb/ft}^2$$

By comparison, choose

$$p_f = 18\text{ lb/ft}^2 \qquad \textit{Ans.}$$

Earthquake Loads. Earthquakes produce loadings on a structure through its interaction with the ground and its response characteristics. These loadings result from the structure's distortion caused by the ground's motion and the lateral resistance of the structure. Their magnitude depends on the amount and type of ground accelerations and the mass and stiffness of the structure. In order to provide some insight as to the nature of earthquake loads, consider the simple structural model shown in Fig. 1–15. This model may represent a single-story building, where the top block is the "lumped" mass of the roof, and the middle block is the lumped stiffness of all the building's columns. During an earthquake the ground vibrates both horizontally and vertically. The horizontal accelerations create shear forces in the column that put the block in sequential motion with the ground. If the column is *stiff* and the block has a *small* mass, the period of vibration of the block will be *short* and the block will accelerate with the same motion as the ground and undergo only slight relative displacements. For an actual structure which is designed to have large amounts of bracing and stiff connections this can be beneficial, since less stress is developed in the members. On the other hand, if the column in Fig 1–15 is very flexible and the block has a large mass, then earthquake-induced motion will cause small accelerations of the block and large relative displacements.

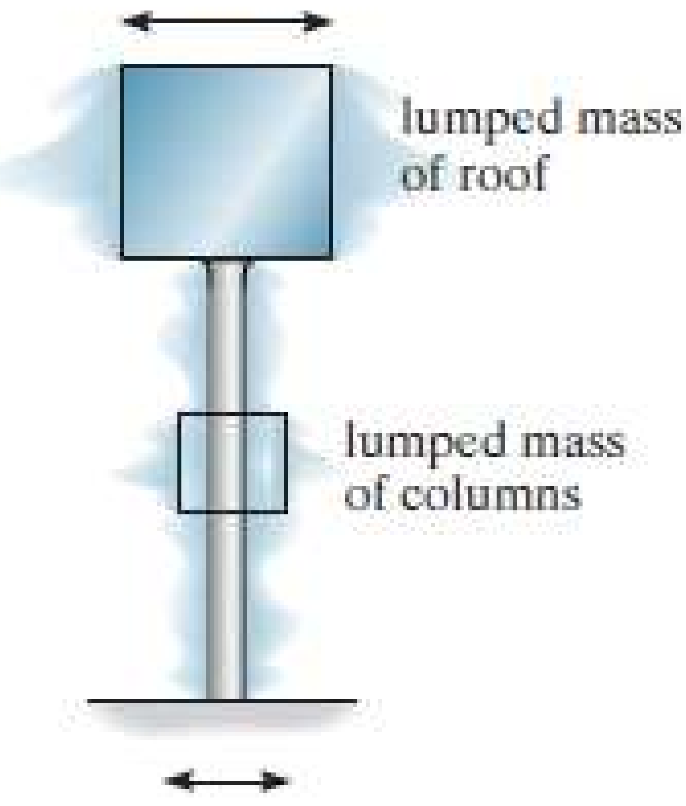


Fig. 1–15

For small structures, a *static analysis* for earthquake design may be satisfactory. This case approximates the dynamic loads by a set of externally applied *static forces* that are applied laterally to the structure. One such method for doing this is reported in the ASCE 7-10 Standard. It is based upon finding a seismic response coefficient, C_s, determined from the soil properties, the ground accelerations, and the vibrational response of the structure. For most structures, this coefficient is then multiplied by the structure's total dead load W to obtain the "base shear" in the structure. The value of C_s is actually determined from

$$C_s = \frac{S_{DS}}{R/I_e}$$

where

S_{DS} = the spectral response acceleration for short periods of vibration.

R = a response modification factor that depends upon the ductility of the structure. Steel frame members which are highly ductile can have a value as high as 8, whereas reinforced concrete frames can have a value as low as 3.

I_e = the importance factor that depends upon the use of the building. For example, $I_e = 1$ for agriculture and storage facilities, and $I_e = 1.5$ for hospitals and other essential facilities.

Hydrostatic and Soil Pressure. When structures are used to retain water, soil, or granular materials, the pressure developed by these loadings becomes an important criterion for their design. Examples of such types of structures include tanks, dams, ships, bulkheads, and retaining walls. Here the laws of hydrostatics and soil mechanics are applied to define the intensity of the loadings on the structure.

The design of this retaining wall requires estimating the soil pressure acting on it. Also, the gate of the lock will be subjected to hydrostatic pressure that must be considered for its design.

Other Natural Loads. Several other types of live loads may also have to be considered in the design of a structure, depending on its location or use. These include the effect of blast, temperature changes, and differential settlement of the foundation.

1.4 Structural Design

Whenever a structure is designed, it is important to give consideration to both material and load uncertainties. These uncertainties include a possible variability in material properties, residual stress in materials, intended measurements being different from fabricated sizes, loadings due to vibration or impact, and material corrosion or decay.

ASD. Allowable-stress design (ASD) methods include *both* the material and load uncertainties into a single factor of safety. The many types of loads discussed previously can occur simultaneously on a structure, but it is very unlikely that the maximum of all these loads will occur at the same time. For example, both maximum wind and earthquake loads normally do not act simultaneously on a structure. For *allowable-stress design* the computed elastic stress in the material must not exceed the allowable stress for each of various load combinations. Typical load combinations as specified by the ASCE 7-10 Standard include

- dead load
- 0.6 (dead load) + 0.6 (wind load)
- 0.6 (dead load) + 0.7 (earthquake load)

LRFD. Since uncertainty can be considered using probability theory, there has been an increasing trend to separate material uncertainty from load uncertainty. This method is called *strength design* or LRFD (load and resistance factor design). For example, to account for the uncertainty of loads, this method uses load factors applied to the loads or combinations of loads. According to the ASCE 7-10 Standard, some of the load factors and combinations are

- 1.4 (dead load)
- 1.2 (dead load) + 1.6 (live load) + 0.5 (snow load)
- 0.9 (dead load) + 1.0 (wind load)
- 0.9 (dead load) + 1.0 (earthquake load)

In all these cases, the combination of loads is thought to provide a maximum, yet realistic loading on the structure.

Analysis of Statically Determinate Structures

2

2.1 Idealized Structure

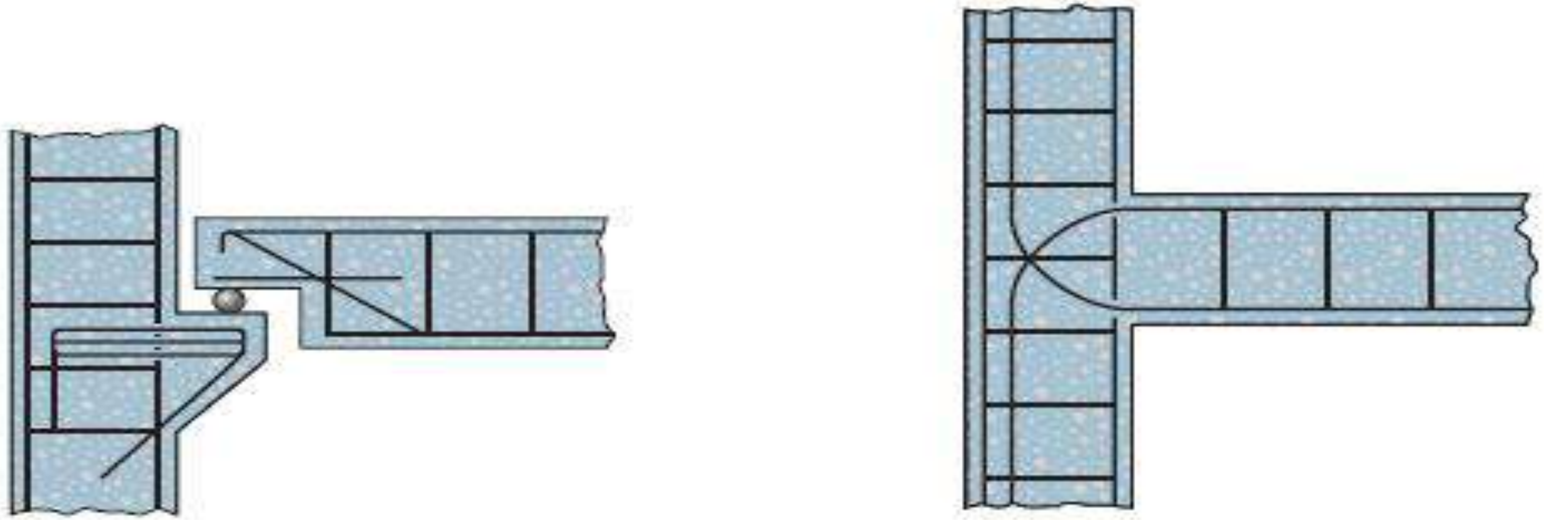
An exact analysis of a structure can never be carried out, since estimates always have to be made of the loadings and the strength of the materials composing the structure. Furthermore, points of application for the loadings must also be estimated. It is important, therefore, that the structural engineer develop the ability to model or idealize a structure so that he or she can perform a practical force analysis of the members. In this section we will develop the basic techniques necessary to do this.

Support Connections. Structural members are joined together in various ways depending on the intent of the designer. The three types of joints most often specified are the pin connection, the roller support, and the fixed joint. A pin-connected joint and a roller support allow some freedom for slight rotation, whereas a fixed joint allows no relative rotation between the connected members and is consequently more expensive to fabricate.

typical "pin-supported" connection (metal)
(a)

typical "fixed-supported" connection (metal)
(b)

Fig. 2–1



typical "roller-supported" connection (concrete)
(a)

typical "fixed-supported" connection (concrete)
(b)

Fig. 2–2

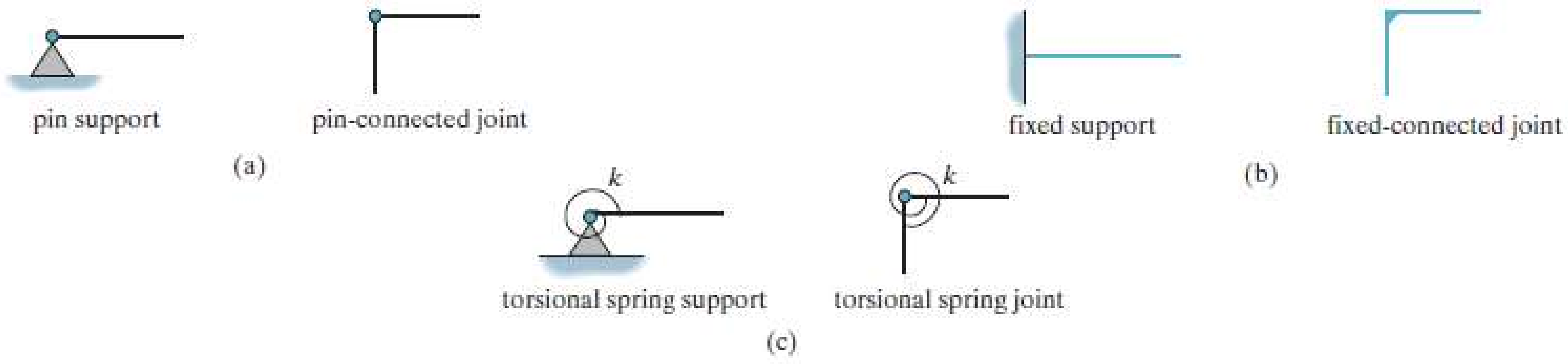

pin support
pin-connected joint
(a)
fixed support
fixed-connected joint
(b)
k
k
torsional spring support
torsional spring joint
(c)

Fig. 2–3

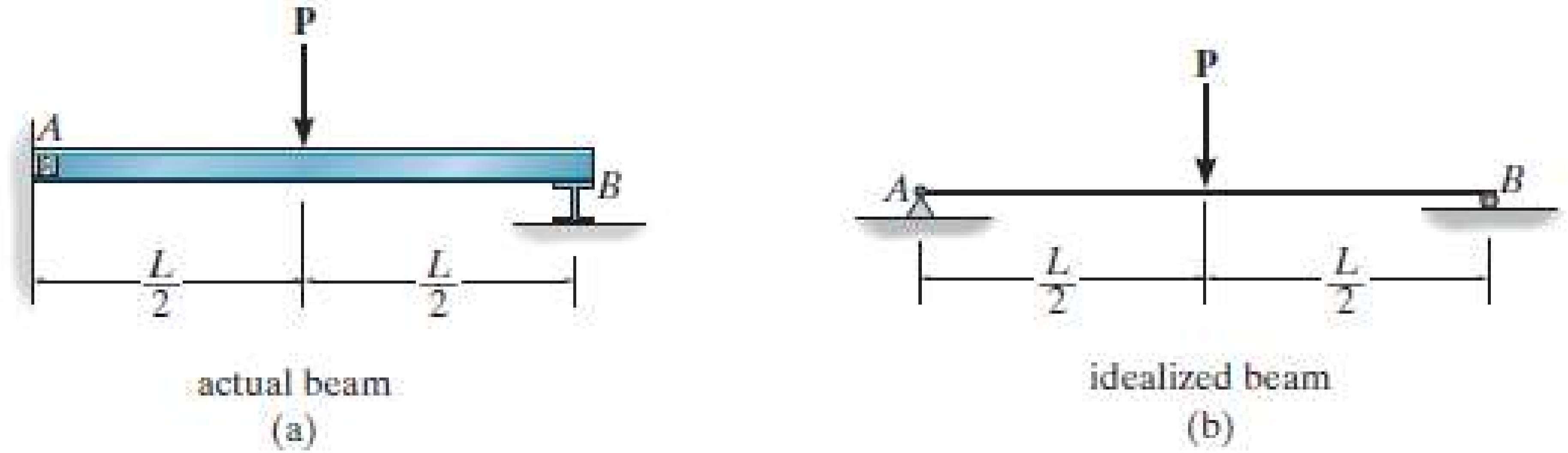
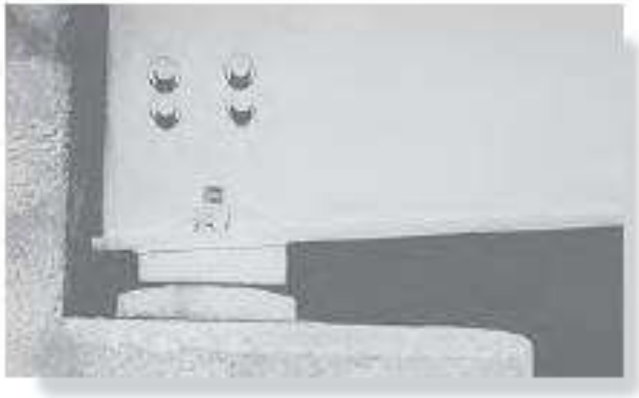

P
A
B
L/2
L/2
actual beam
(a)
P
A
B
L/2
L/2
idealized beam
(b)

Fig. 2–4



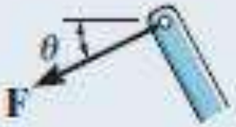
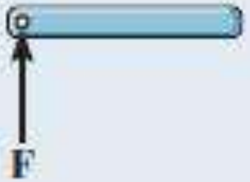
A typical rocker support used for a bridge girder.

Rollers and associated bearing pads are used to support the prestressed concrete girders of a highway bridge.

The short link is used to connect the two girders of the highway bridge and allow for thermal expansion of the deck.

Typical pin used to support the steel girder of a railroad bridge.

TABLE 2–1 Supports for Coplanar Structures

Type of Connection	Idealized Symbol	Reaction	Number of Unknowns
(1)  light cable, weightless link			One unknown. The reaction is a force that acts in the direction of the cable or link.
(2)  rollers  rocker			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(3)  smooth contacting surface			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.

(4)

smooth pin-connected collar

One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.

(5)

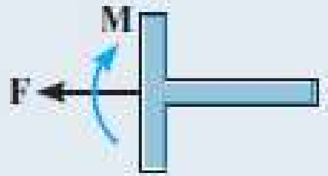
smooth pin or hinge

Two unknowns. The reactions are two force components.

(6)

slider

fixed-connected collar



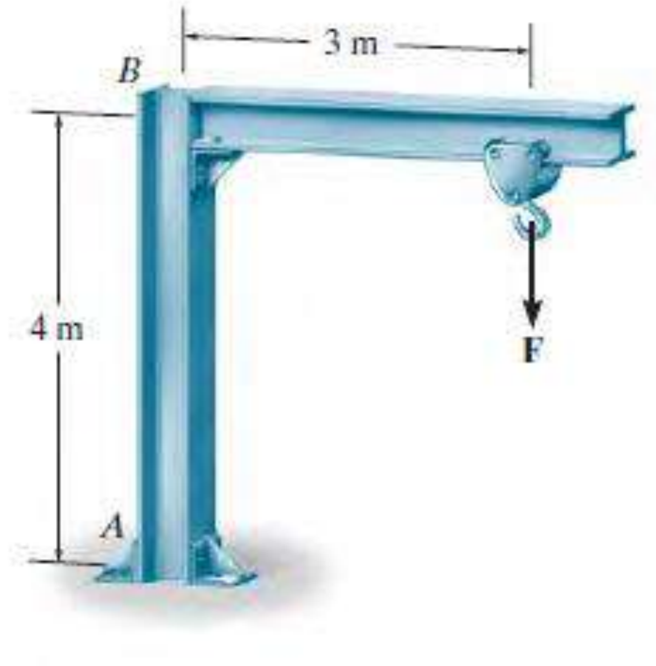
Two unknowns. The reactions are a force and a moment.

(7)

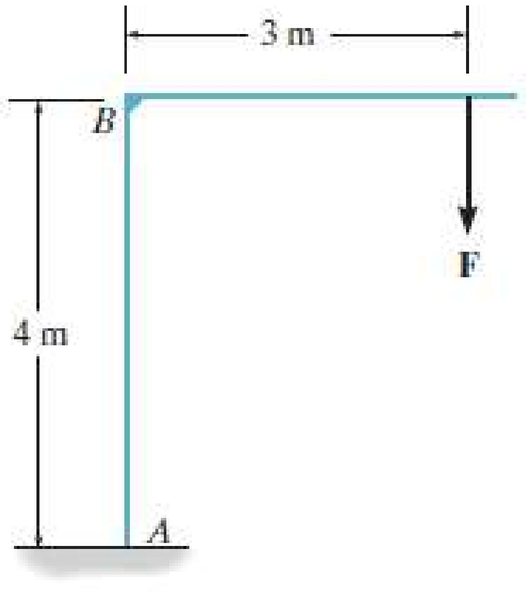
fixed support

Three unknowns. The reactions are the moment and the two force components.

Idealized Structure. Having stated the various ways in which the connections on a structure can be idealized, we are now ready to discuss some of the techniques used to represent various structural systems by idealized models.

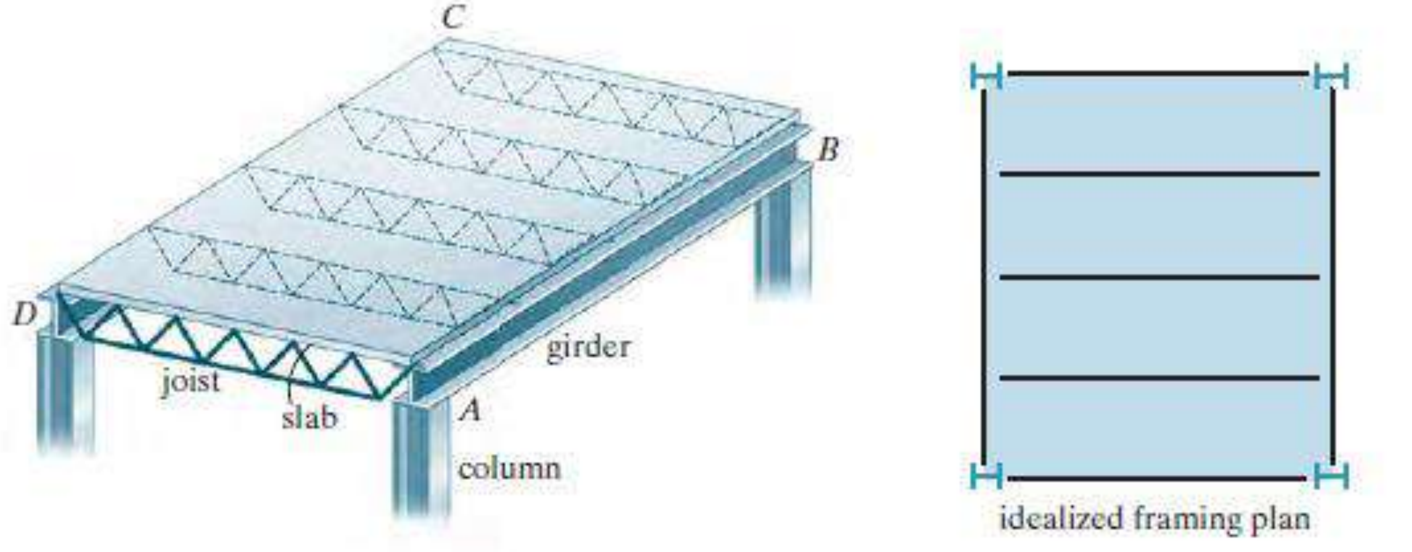


actual structure

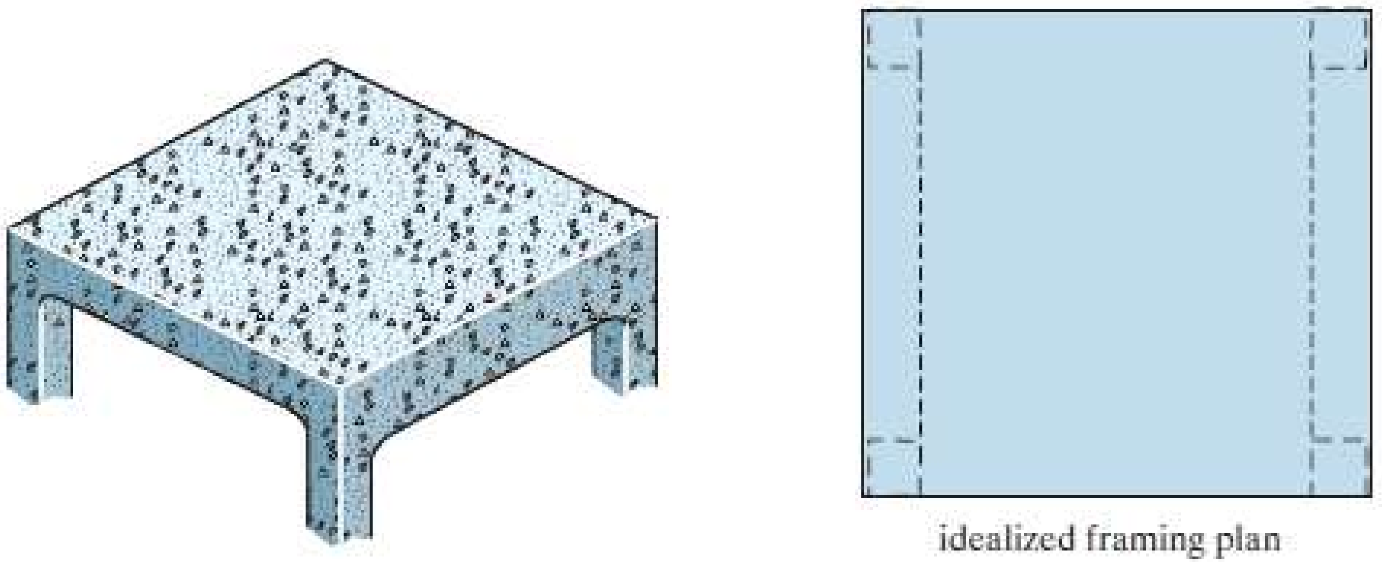


idealized structure

(b)


C
B
D
girder
joist
slab
A
column
idealized framing plan

(a) (b)

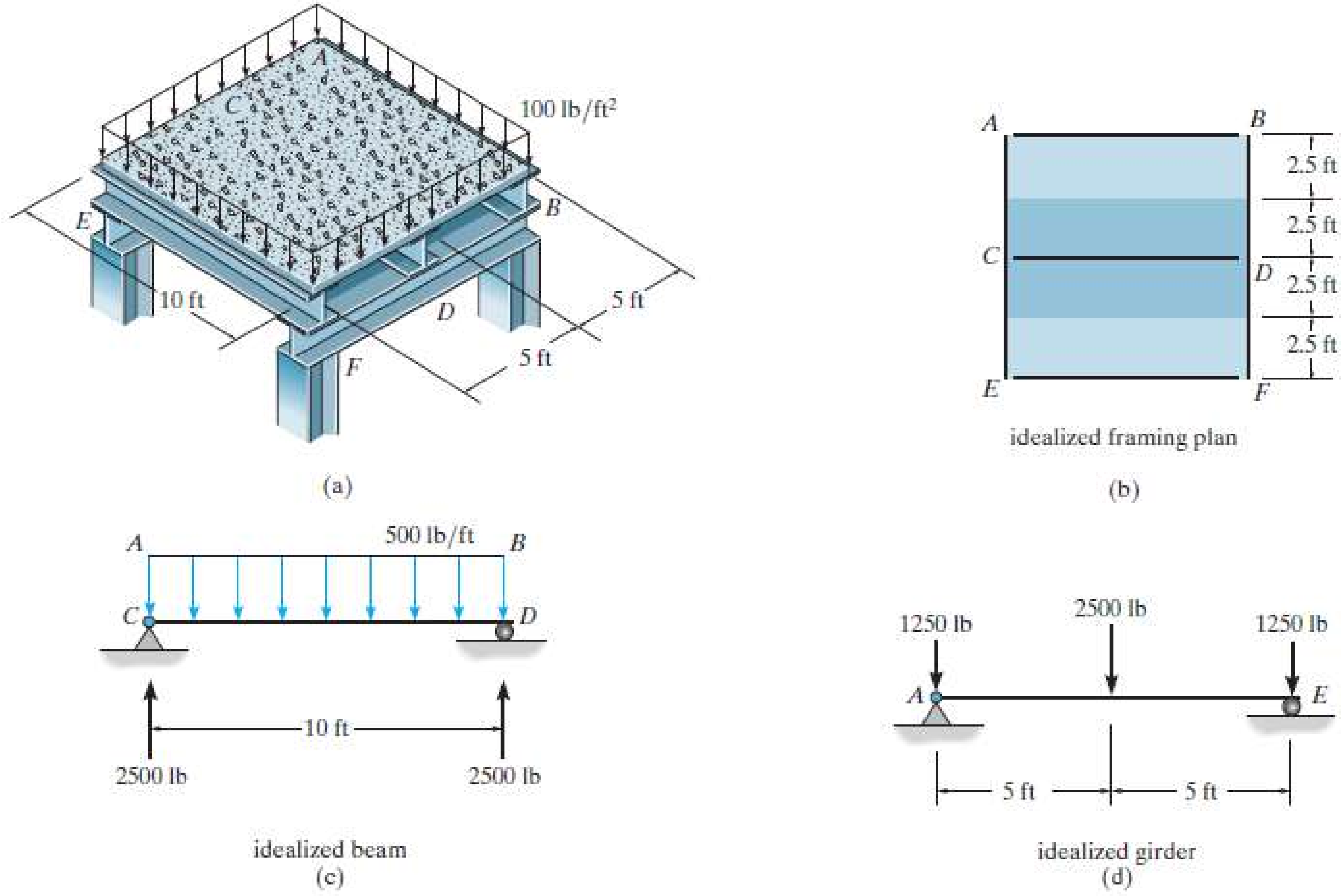

idealized framing plan

(a) (b)

Tributary Loadings. When flat surfaces such as walls, floors, or roofs are supported by a structural frame, it is necessary to determine how the load on these surfaces is transmitted to the various structural elements used for their support. There are generally two ways in which this can be done. The choice depends on the geometry of the structural system, the material from which it is made, and the method of its construction.

One-Way System. A slab or deck that is supported such that it delivers its load to the supporting members by one-way action, is often referred to as a *one-way slab*. To illustrate the method of load transmission, consider the framing system shown in Fig. 2–11a where the beams AB, CD, and EF rest on the girders AE and BF. If a uniform load of 100 lb/ft^2 is placed on the slab, then the center beam CD is assumed to support the load acting on the *tributary area* shown dark shaded on the structural framing plan in Fig. 2–11b. Member CD is therefore subjected to a *linear* distribution of load of $(100 \text{ lb/ft}^2)(5 \text{ ft}) = 500 \text{ lb/ft}$, shown on the idealized beam in Fig. 2–11c. The reactions on this beam (2500 lb) would then be applied to the center of the girders AE (and BF), shown idealized in Fig. 2–11d. Using this same concept, do you see how the remaining portion of the slab loading is transmitted to the ends of the girder as 1250 lb?

The structural framework of this building consists of concrete floor joists, which were formed on site using metal pans. These joists are simply supported on the girders, which in turn are simply supported on the columns.



A
C
100 lb/ft²
B
E
10 ft
D
5 ft
5 ft
F
(a)
A
B
2.5 ft
2.5 ft
C
D
2.5 ft
2.5 ft
E
F
idealized framing plan
(b)
A
500 lb/ft
B
C
D
10 ft
2500 lb
2500 lb
idealized beam
(c)
1250 lb
2500 lb
1250 lb
A
E
5 ft
5 ft
idealized girder
(d)

Fig. 2–11

According to the American Concrete Institute, ACI 318 code, if $L_2 > L_1$ *and if the span ratio* $(L_2/L_1) > 2$, *the slab will behave as a one-way slab,* since as L_1 becomes smaller, the beams *AB*, *CD*, and *EF* provide the greater stiffness to carry the load.

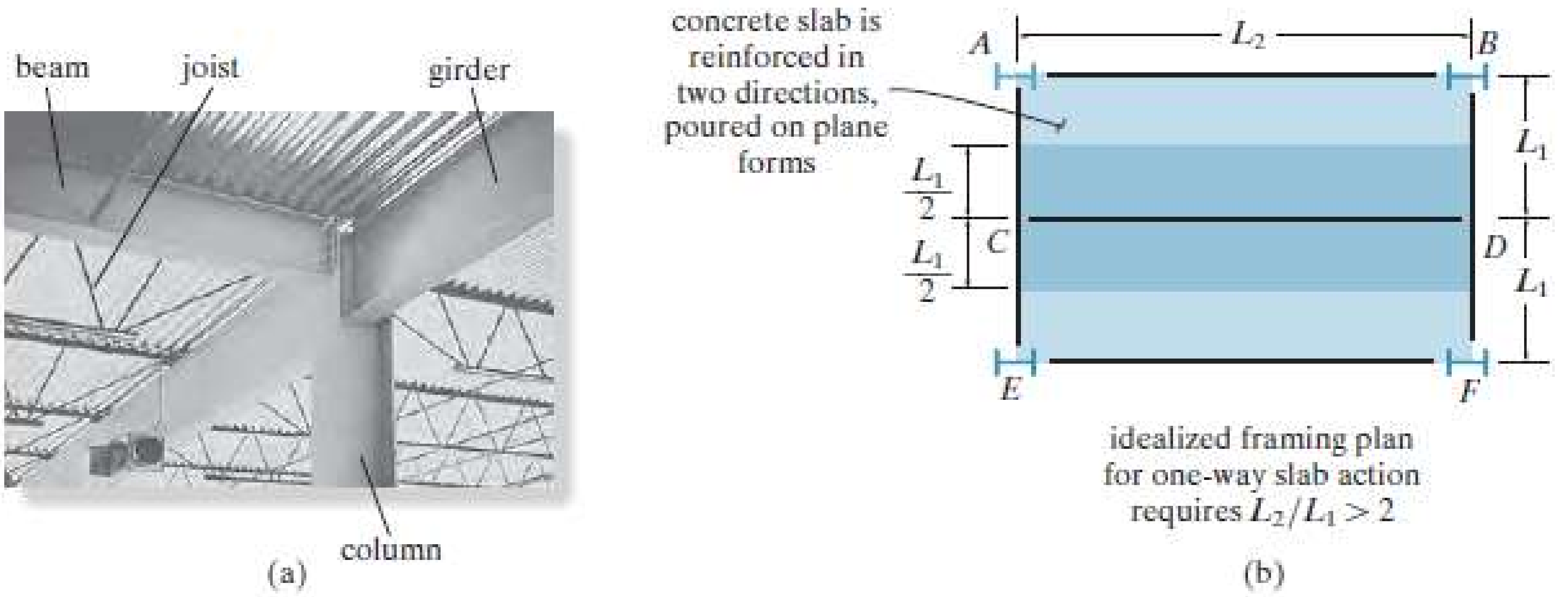


Fig. 2–12

Two-Way System. If, according to the ACI 318 concrete code the support ratio in Fig. 2–12*b* is $(L_2/L_1) \leq 2$, the load is assumed to be delivered to the supporting beams and girders in two directions. When this is the case the slab is referred to as a *two-way slab*. To show one method of treating this case, consider the square reinforced concrete slab in Fig. 2–13*a*, which is supported by four 10-ft-long edge beams, AB, BD, DC, and CA. Here $L_2/L_1 = 1$. Due to two-way slab action, the assumed *tributary area* for beam AB is shown dark shaded in Fig. 2–13*b*. This area is determined by constructing diagonal 45° lines as shown. Hence if a uniform load of 100 lb/ft^2 is applied to the slab, a peak intensity of $(100\ \text{lb/ft}^2)(5\ \text{ft}) = 500\ \text{lb/ft}$ will be applied to the center of beam AB, resulting in a *triangular* load distribution shown in Fig. 2–13*c*. For other geometries that cause two-way action, a similar procedure can be used.

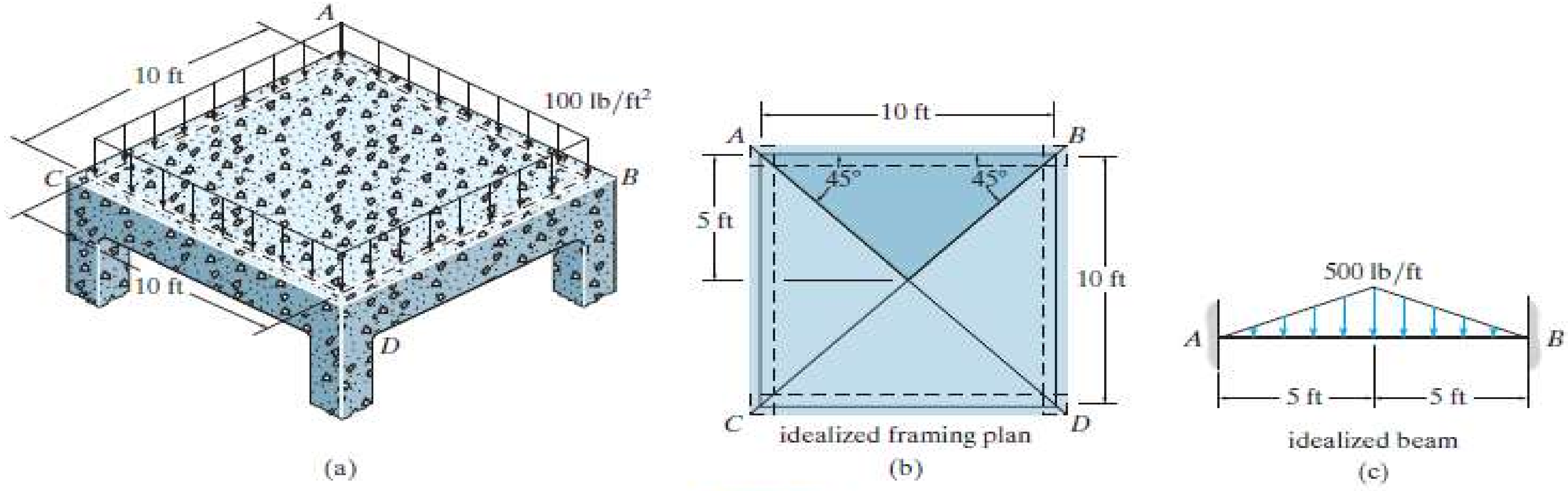


Fig. 2–13

For example, if $L_2/L_1 = 1.5$ it is then necessary to construct 45° lines that intersect as shown in Fig. 2–14*a*. A 100-lb/ft^2 loading placed on the slab will then produce *trapezoidal* and *triangular* distributed loads on members *AB* and *AC*, Fig. 2–14*b* and 2–14*c*, respectively.

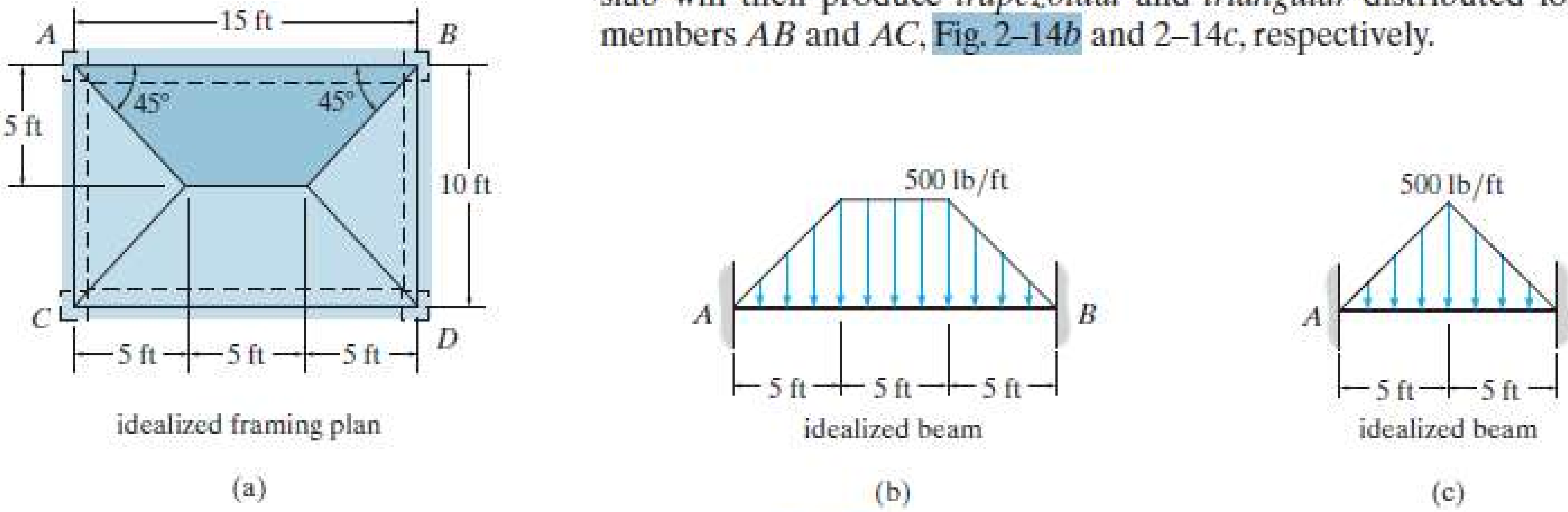


Fig. 2–14

EXAMPLE 2.1

The floor of a classroom is to be supported by the bar joists shown in Fig. 2–15*a*. Each joist is 15 ft long and they are spaced 2.5 ft on centers. The floor itself is to be made from lightweight concrete that is 4 in. thick. Neglect the weight of the joists and the corrugated metal deck, and determine the load that acts along each joist.

(a)

SOLUTION

The dead load on the floor is due to the weight of the concrete slab. From Table 1–3 for 4 in. of lightweight concrete it is $(4)(8\ \text{lb/ft}^2) = 32\ \text{lb/ft}^2$. From Table 1–4, the live load for a classroom is $40\ \text{lb/ft}^2$. Thus the total floor load is $32\ \text{lb/ft}^2 + 40\ \text{lb/ft}^2 = 72\ \text{lb/ft}^2$. For the floor system, $L_1 = 2.5$ ft and $L_2 = 15$ ft. Since $L_2/L_1 > 2$ the concrete slab is treated as a one-way slab. The tributary area for each joist is shown in Fig. 2–15*b*. Therefore the uniform load along its length is

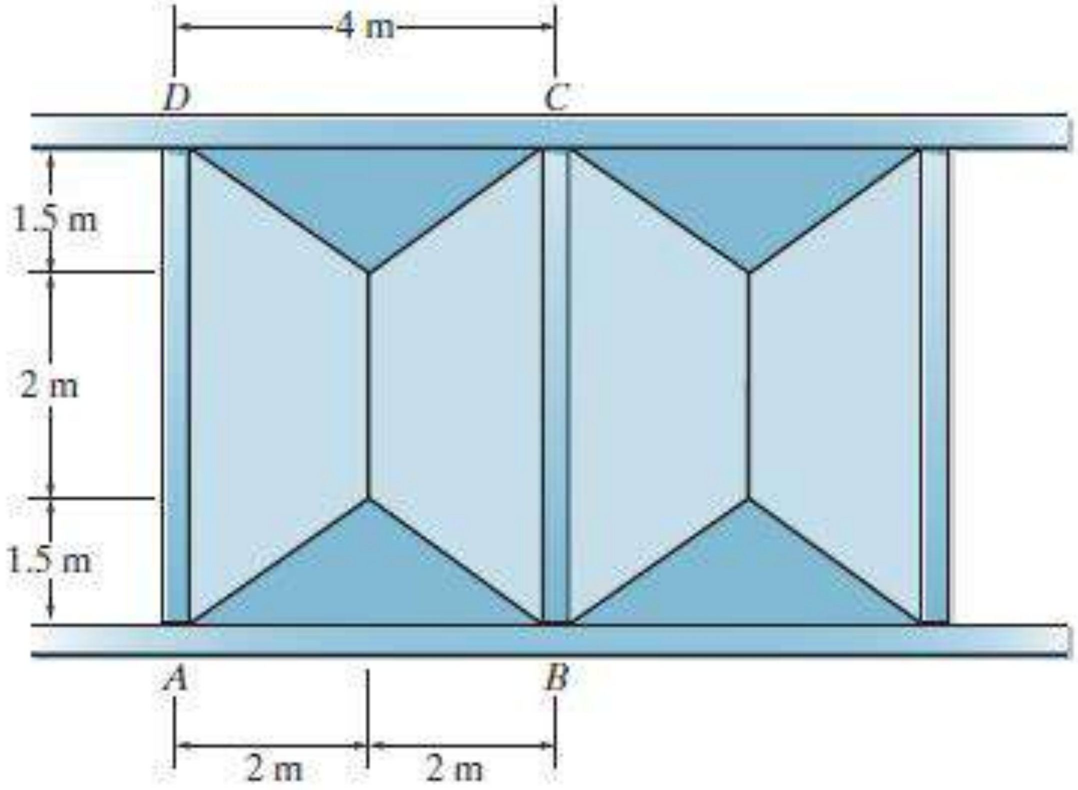
$$w = 72\ \text{lb/ft}^2(2.5\ \text{ft}) = 180\ \text{lb/ft}$$

This loading and the end reactions on each joist are shown in Fig. 2–15*c*.

Fig. 2–15

EXAMPLE 2.2

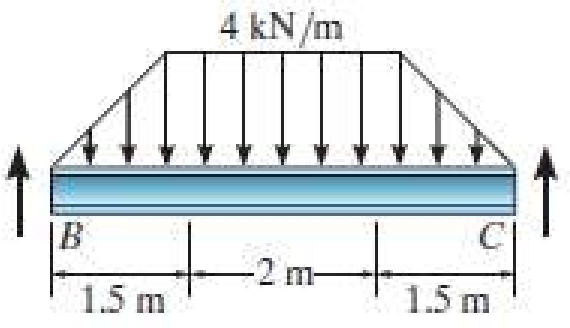
The flat roof of the steel-frame building shown in the photo is intended to support a total load of $2\ \mathrm{kN/m^2}$ over its surface. Determine the roof load within region $ABCD$ that is transmitted to beam BC. The dimensions are shown in Fig. 2–16a.



(a)

SOLUTION

In this case $L_2 = 5$ m and $L_1 = 4$ m. Since $L_2/L_1 = 1.25 < 2$, we have two-way slab action. The tributary loading along each edge beam is shown in Fig. 2–16*a*, where the lighter shaded trapezoidal area of loading is transmitted to member *BC*. The peak intensity of this loading is $(2\text{ kN/m}^2)(2\text{ m}) = 4\text{ kN/m}$. As a result, the distribution of load along *BC* is shown in Fig. 2–16*b*.



(b)

Fig. 2–16

This process of tributary load transmission should *also* be calculated for the region to the right of *BC* shown in the photo, and this load should *also* be placed on *BC*. See the next example.

EXAMPLE 2.3

The concrete girders shown in the photo of the passenger car parking garage span 30 ft and are 15 ft on center. If the floor slab is 5 in. thick and made of reinforced stone concrete, and the specified live load is 50 lb/ft^2 (see Table 1–4), determine the distributed load the floor system transmits to each interior girder.

SOLUTION

Here, $L_2 = 30$ ft and $L_1 = 15$ ft, so that $L_2/L_1 = 2$. We have a two-way slab. From Table 1–2, for reinforced stone concrete, the specific weight of the concrete is 150 lb/ft^3. Thus the design floor loading is

$$p = 150\ \text{lb/ft}^3\left(\frac{5}{12}\ \text{ft}\right) + 50\ \text{lb/ft}^2 = 112.5\ \text{lb/ft}^2$$

$$p = 150\ \text{lb/ft}^3\left(\frac{5}{12}\ \text{ft}\right) + 50\ \text{lb/ft}^2 = 112.5\ \text{lb/ft}^2$$

A trapezoidal distributed loading is transmitted to each interior girder AB from each of its sides. The maximum intensity of each of these distributed loadings is $(112.5\ \text{lb/ft}^2)(7.5\ \text{ft}) = 843.75\ \text{lb/ft}$, so that on the girder this intensity becomes $2(843.75\ \text{lb/ft}) = 1687.5\ \text{lb/ft}$, Fig. 2–17*b*. *Note:* For design, consideration should also be given to the weight of the girder.

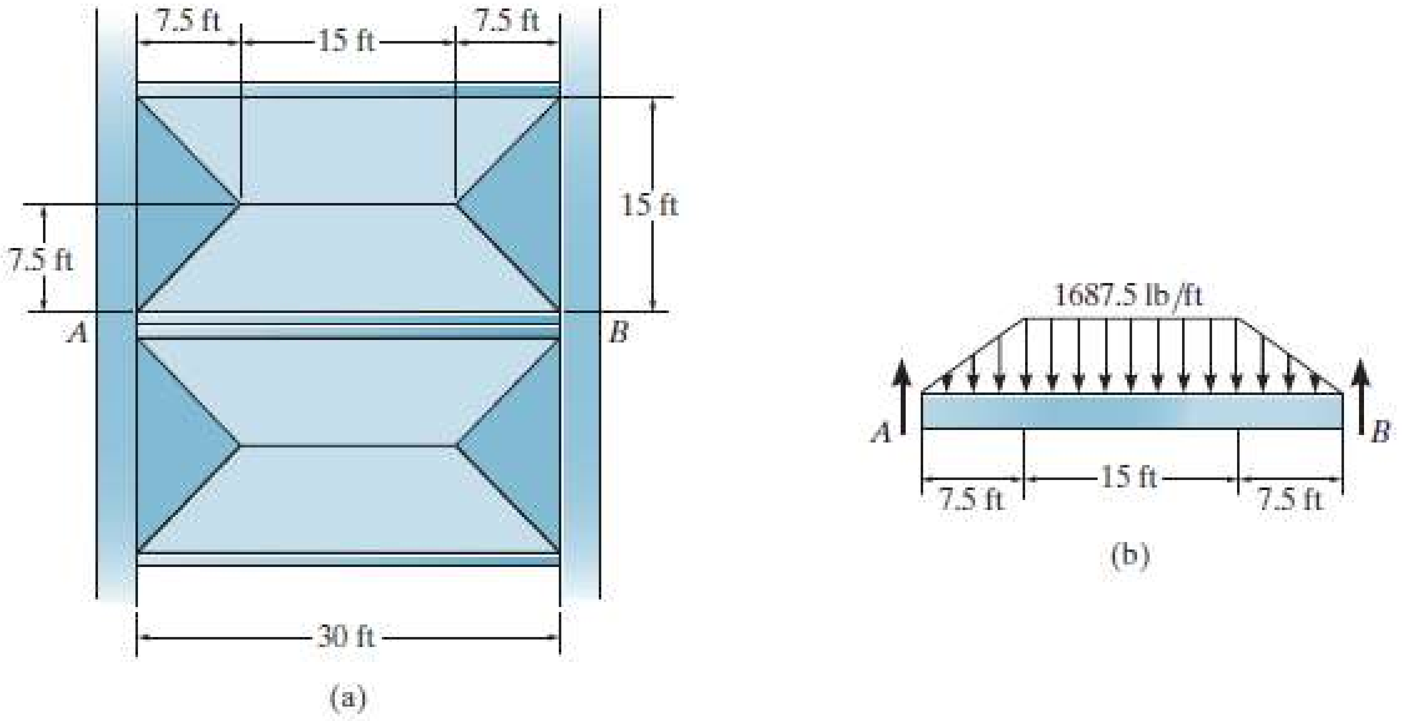


Fig. 2–17

2.2 Principle of Superposition

The principle of superposition forms the basis for much of the theory of structural analysis. It may be stated as follows: *The total displacement or internal loadings (stress) at a point in a structure subjected to several external loadings can be determined by adding together the displacements or internal loadings (stress) caused by each of the external loads acting separately*. For this statement to be valid it is necessary that a *linear relationship* exist among the loads, stresses, and displacements.

Two requirements must be imposed for the principle of superposition to apply:

1. The material must behave in a linear-elastic manner, so that Hooke's law is valid, and therefore the load will be proportional to displacement.
2. The geometry of the structure must not undergo significant change when the loads are applied, i.e., small displacement theory applies. Large displacements will significantly change the position and orientation of the loads. An example would be a cantilevered thin rod subjected to a force at its end.

Throughout this text, these two requirements will be satisfied. Here only linear-elastic material behavior occurs; and the displacements produced by the loads will not significantly change the directions of applied loadings nor the dimensions used to compute the moments of forces.

The walls on the sides of this building are used to strengthen its structure when the building is subjected to large hurricane wind loadings applied to its front or back. These walls are called "shear walls."

2.3 Equations of Equilibrium

It may be recalled from statics that a structure or one of its members is in equilibrium when it maintains a balance of force and moment. In general this requires that the force and moment equations of equilibrium be satisfied along three independent axes, namely,

$$\begin{aligned} \Sigma F_x &= 0 & \Sigma F_y &= 0 & \Sigma F_z &= 0 \\ \Sigma M_x &= 0 & \Sigma M_y &= 0 & \Sigma M_z &= 0 \end{aligned} \tag{2–1}$$

The principal load-carrying portions of most structures, however, lie in a single plane, and since the loads are also coplanar, the above requirements for equilibrium reduce to

$$\boxed{\begin{aligned} \Sigma F_x &= 0 \\ \Sigma F_y &= 0 \\ \Sigma M_O &= 0 \end{aligned}} \tag{2–2}$$

If the *internal loadings* at a specified point in a member are to be determined, the *method of sections* must be used. This requires that a "cut" or section be made perpendicular to the axis of the member at the point where the internal loading is to be determined. A free-body diagram of either segment of the "cut" member is isolated and the internal loads are then determined from the equations of equilibrium applied to the segment. In general, the internal loadings acting at the section will consist of a normal force **N**, shear force **V**, and bending moment **M**, as shown in Fig. 2–18.

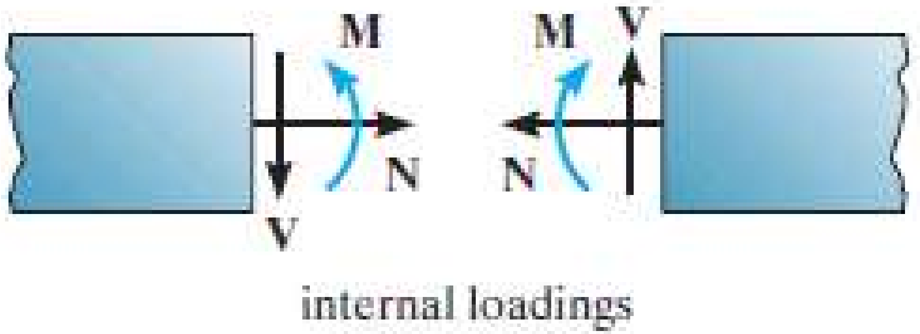


Fig. 2–18

2.4 Determinacy and Stability

Before starting the force analysis of a structure, it is necessary to establish the determinacy and stability of the structure.

Determinacy. The equilibrium equations provide both the *necessary and sufficient* conditions for equilibrium. When all the forces in a structure can be determined strictly from these equations, the structure is referred to as *statically determinate*. Structures having more unknown forces than available equilibrium equations are called *statically indeterminate*. As a general rule, a structure can be identified as being either statically determinate or statically indeterminate by drawing free-body diagrams of all its members, or selective parts of its members, and then comparing the total number of unknown reactive force and moment components with the total number of available equilibrium equations.* For a coplanar structure there are at most *three* equilibrium equations for each part, so that if there is a total of n parts and r force and moment reaction components, we have

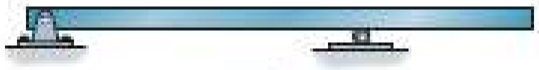
$$\begin{aligned} r &= 3n, \text{ statically determinate} \\ r &> 3n, \text{ statically indeterminate} \end{aligned} \qquad (2\text{–}3)$$

EXAMPLE 2.4

Classify each of the beams shown in Fig. 2–19*a* through 2–19*d* as statically determinate or statically indeterminate. If statically indeterminate, report the number of degrees of indeterminacy. The beams are subjected to external loadings that are assumed to be known and can act anywhere on the beams.

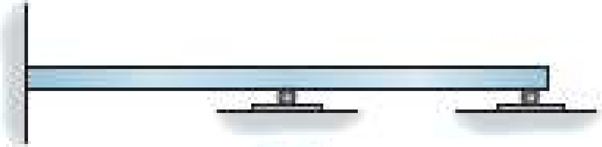
SOLUTION

Compound beams, i.e., those in Fig. 2–19*c* and 2–19*d*, which are composed of pin-connected members must be disassembled. Note that in these cases, the unknown reactive forces acting between each member must be shown in equal but opposite pairs. The free-body diagrams of each member are shown. Applying $r = 3n$ or $r > 3n$, the resulting classifications are indicated.

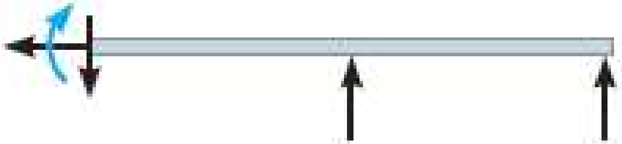


(a)

$r = 3, n = 1, 3 = 3(1)$ Statically determinate *Ans.*

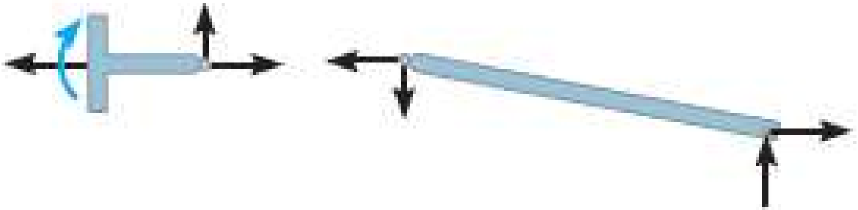


(b)



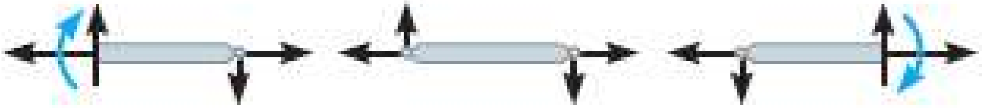
$r = 5, n = 1, 5 > 3(1)$ Statically indeterminate to the second degree *Ans.*

(c)



$r = 6, n = 2, 6 = 3(2)$ Statically determinate *Ans.*

(d)



$r = 10, n = 3, 10 > 3(3)$ Statically indeterminate to the first degree *Ans.*

Fig. 2–19

EXAMPLE 2.5

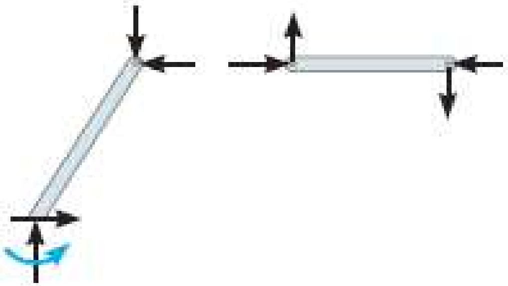
Classify each of the pin-connected structures shown in Fig. 2–20*a* through 2–20*d* as statically determinate or statically indeterminate. If statically indeterminate, report the number of degrees of indeterminacy. The structures are subjected to arbitrary external loadings that are assumed to be known and can act anywhere on the structures.

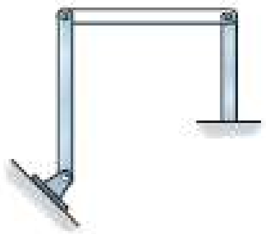
SOLUTION

Classification of pin-connected structures is similar to that of beams. The free-body diagrams of the members are shown. Applying $r = 3n$ or $r > 3n$, the resulting classifications are indicated.

(a)

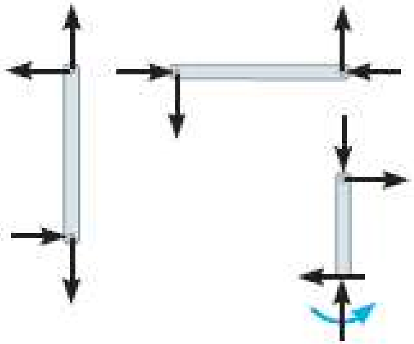
$r = 7$, $n = 2$, $7 > 6$
Statically indeterminate to the first degree *Ans.*

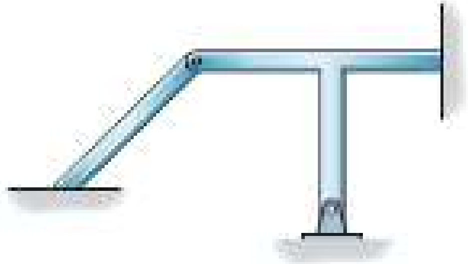




(b)

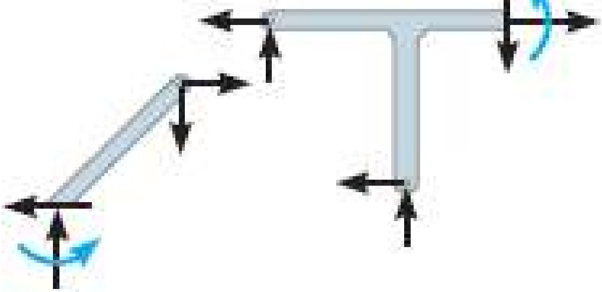
$r = 9$, $n = 3$, $9 = 9$,
Statically determinate *Ans.*

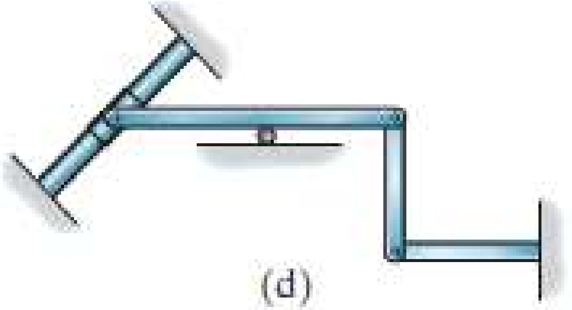




(c)

$r = 10$, $n = 2$, $10 > 6$,
Statically indeterminate to the fourth degree *Ans.*





(d)

$r = 9$, $n = 3$, $9 = 9$,
Statically determinate *Ans.*

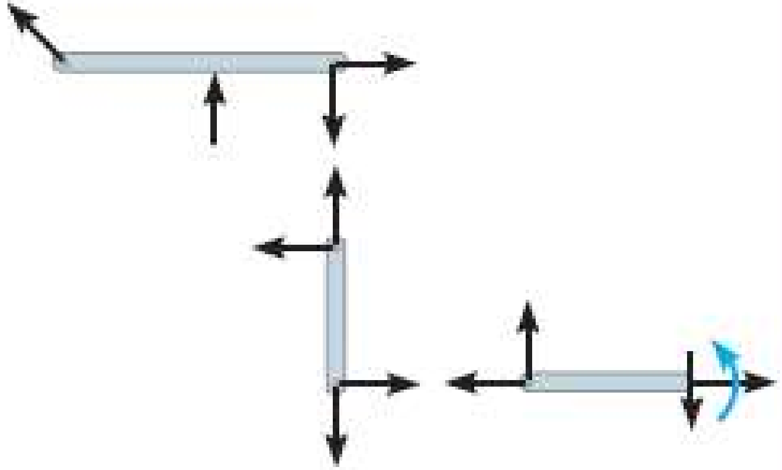
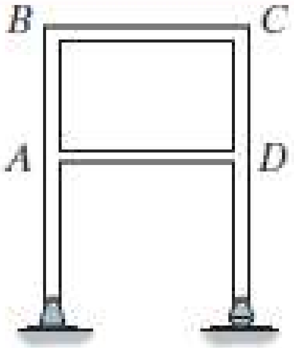


Fig. 2–20

EXAMPLE 2.6

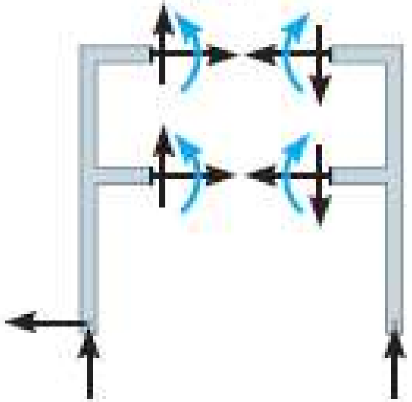
Classify each of the frames shown in Fig. 2–21*a* and 2–21*b* as statically determinate or statically indeterminate. If statically indeterminate, report the number of degrees of indeterminacy. The frames are subjected to external loadings that are assumed to be known and can act anywhere on the frames.



(a)

SOLUTION

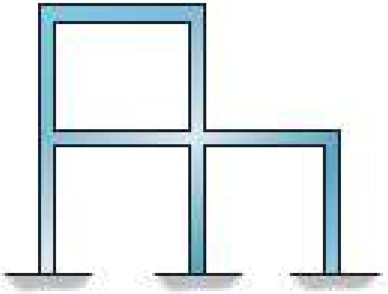
Unlike the beams and pin-connected structures of the previous examples, frame structures consist of members that are connected together by rigid joints. Sometimes the members form internal loops as in Fig. 2–21*a*. Here *ABCD* forms a closed loop. In order to classify these structures, it is necessary to use the method of sections and "cut" the loop apart. The free-body diagrams of the sectioned parts are drawn and the frame can then be classified. Notice that only *one section* through the loop is required, since once the unknowns at the section are determined, the internal forces at any point in the members can then be found using the method of sections and the equations of equilibrium. A second example of this is shown in Fig. 2–21*b*. Although the frame in Fig. 2–21*c* has no closed loops

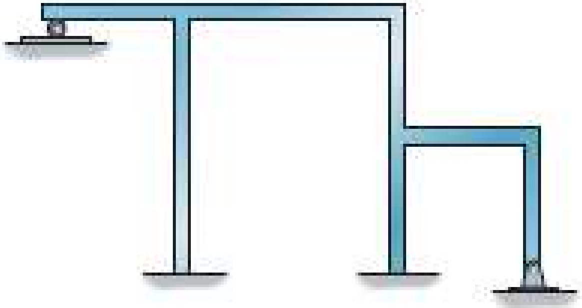


$r = 9$, $n = 2$, $9 > 6$,
Statically indeterminate to the third degree *Ans.*

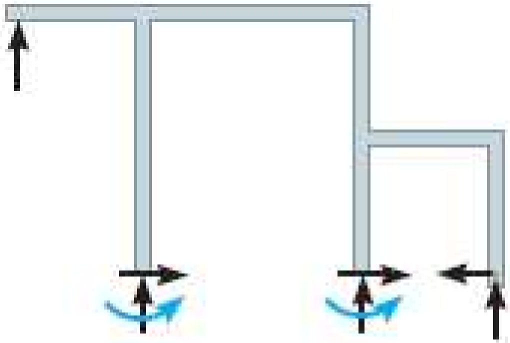
(a)

we can use this same method, using vertical sections, to classify it. For this case we can *also* just draw its complete free-body diagram. The resulting classifications are indicated in each figure.

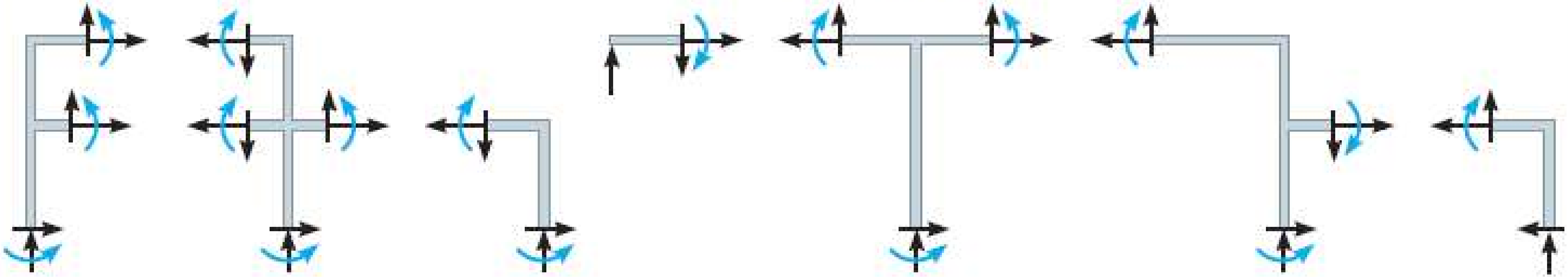




(This frame has no closed loops.)



$r = 9$, $n = 1$, $9 > 3$,
Statically indeterminate to the sixth degree *Ans.*



$r = 18$, $n = 3$, $18 > 9$,
Statically indeterminate to the ninth degree *Ans.*

(b)

(c)

$r = 18$, $n = 4$, $18 > 12$,
Statically indeterminate to the sixth degree *Ans.*

Fig. 2–21

Stability. To ensure the equilibrium of a structure or its members, it is not only necessary to satisfy the equations of equilibrium, but the members must also be properly held or constrained by their supports. Two situations may occur where the conditions for proper constraint have not been met.

Partial Constraints. In some cases a structure or one of its members may have *fewer* reactive forces than equations of equilibrium that must be satisfied. The structure then becomes only *partially constrained*. For example, consider the member shown in Fig. 2–22 with its corresponding free-body diagram. Here the equation $\Sigma F_x = 0$ will not be satisfied for the loading conditions and therefore the member will be unstable.

Improper Constraints. In some cases there may be as many unknown forces as there are equations of equilibrium; however, *instability* or movement of a structure or its members can develop because of *improper constraining* by the supports. This can occur if all the *support reactions are concurrent* at a point. An example of this is shown in Fig. 2–23. From the free-body diagram of the beam it is seen that the summation of moments about point O will *not* be equal to zero ($Pd \neq 0$); thus rotation about point O will take place.

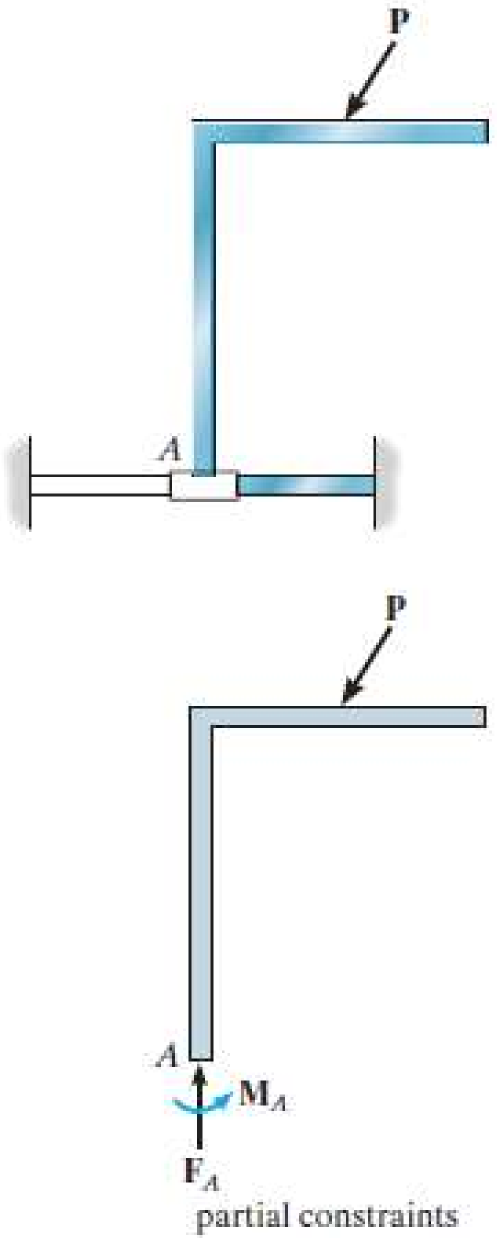


Fig. 2–22

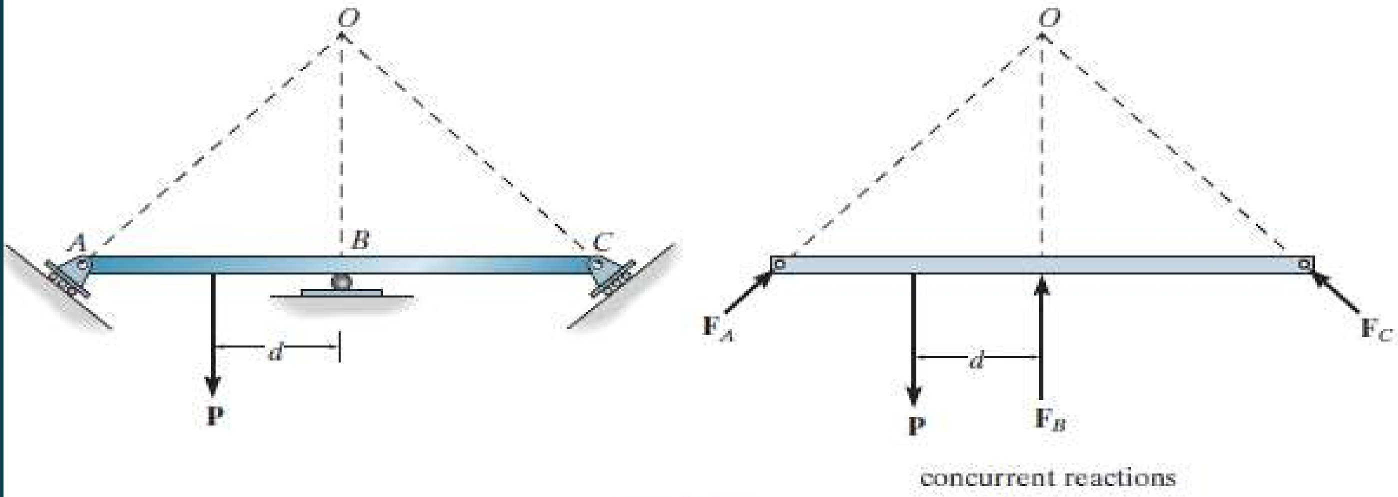


Fig. 2–23

Another way in which improper constraining leads to instability occurs when the *reactive forces* are all *parallel*. An example of this case is shown in Fig. 2–24. Here when an inclined force **P** is applied, the summation of forces in the horizontal direction will not equal zero.

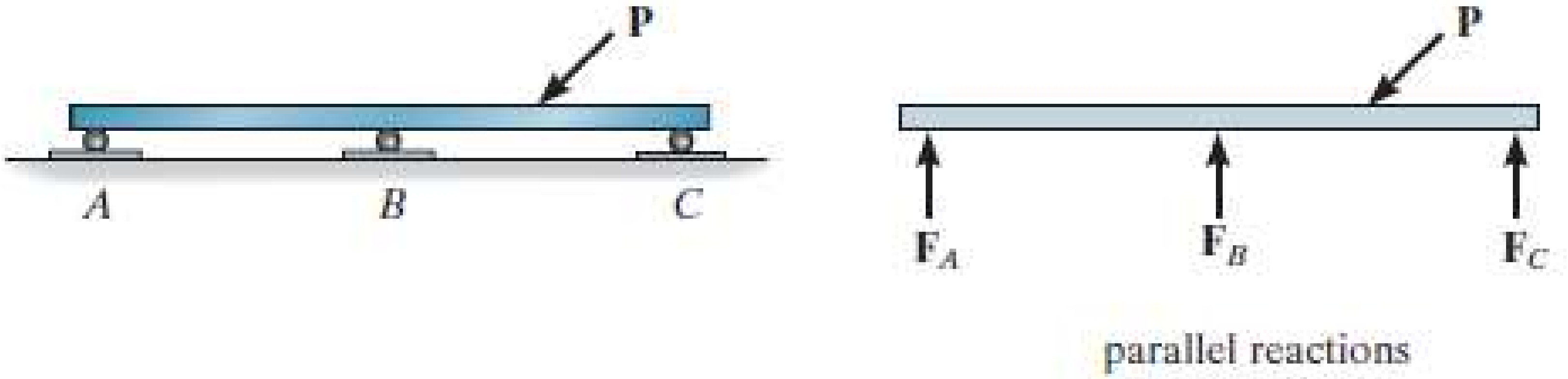


Fig. 2–24

In general, then, a structure will be geometrically unstable—that is, it will move slightly or collapse—if there are fewer reactive forces than equations of equilibrium; or if there are enough reactions, instability will occur if the lines of action of the reactive forces intersect at a common point or are parallel to one another. If the structure consists of several members or components, local instability of one or several of these members can generally be determined *by inspection*. If the members form a collapsible mechanism, the structure will be unstable. We will now formalize these statements for a *coplanar structure* having n members or components with r unknown reactions. Since three equilibrium equations are available for each member or component, we have

$r < 3n$	unstable	
$r \geq 3n$	unstable if member reactions are concurrent or parallel or some of the components form a collapsible mechanism	(2–4)

If the structure is unstable, *it does not matter* if it is statically determinate or indeterminate. In all cases such types of structures must be avoided in practice.

EXAMPLE 2.7

Classify each of the structures in Fig. 2–25*a* through 2–25*d* as stable or unstable. The structures are subjected to arbitrary external loads that are assumed to be known.

SOLUTION

The structures are classified as indicated.

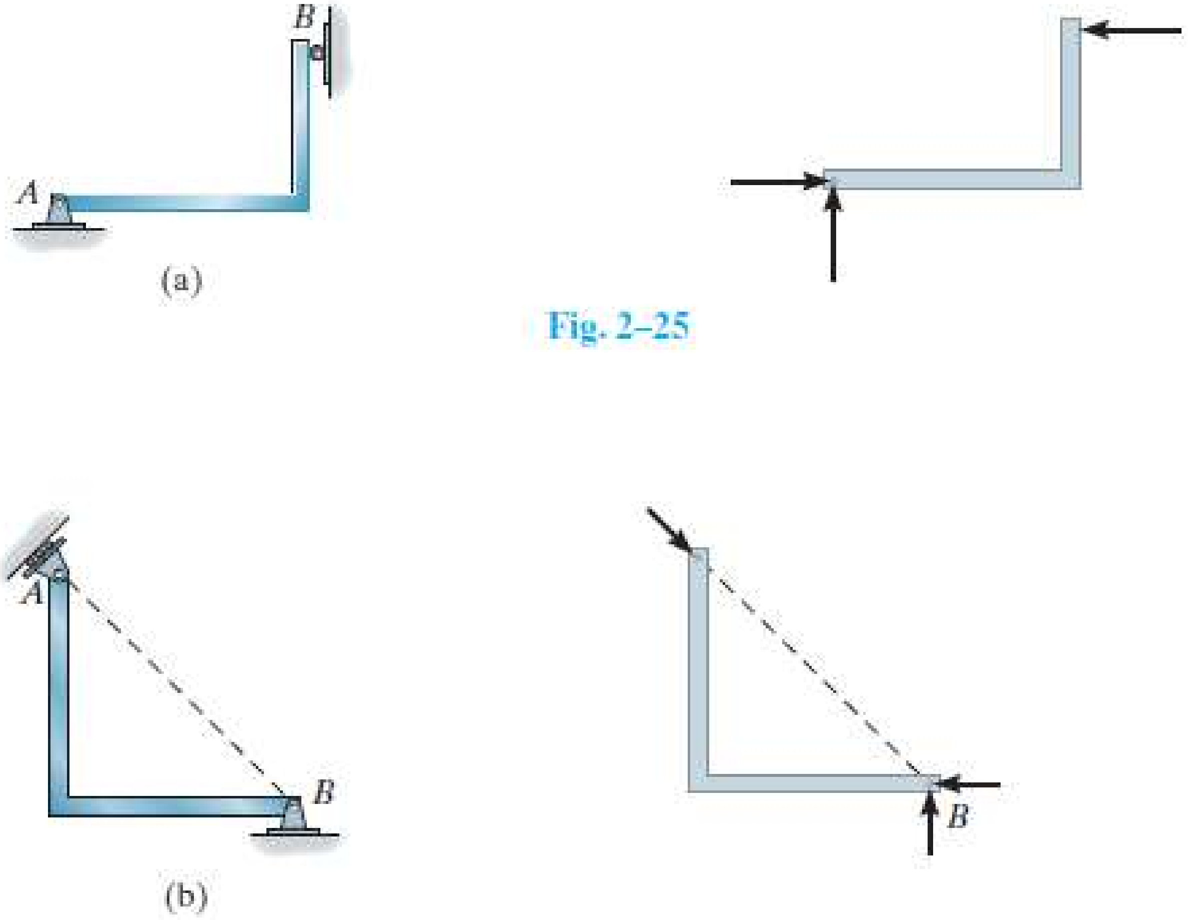


Fig. 2–25

2.5 Application of the Equations of Equilibrium

Occasionally, the members of a structure are connected together in such a way that the joints can be assumed as pins. Building frames and trusses are typical examples that are often constructed in this manner. Provided a pin-connected coplanar structure is properly constrained and contains no more supports or members than are necessary to prevent collapse, the forces acting at the joints and supports can be determined by applying the three equations of equilibrium ($\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M_O = 0$) to each member. Understandably, once the forces at the joints are obtained, the size of the members, connections, and supports can then be determined on the basis of design code specifications.

Procedure for Analysis

The following procedure provides a method for determining the *joint reactions* for structures composed of pin-connected members.

Free-Body Diagrams

- Disassemble the structure and draw a free-body diagram of each member. Also, it may be convenient to supplement a member free-body diagram with a free-body diagram of the *entire structure*. Some or all of the support reactions can then be determined using this diagram.
- Recall that reactive forces common to two members act with equal magnitudes but opposite directions on the respective free-body diagrams of the members.
- All two-force members should be identified. These members, regardless of their shape, have no external loads on them, and therefore their free-body diagrams are represented with equal but opposite collinear forces acting on their ends.
- In many cases it is possible to tell by inspection the proper arrowhead sense of direction of an unknown force or couple moment; however, if this seems difficult, the directional sense can be assumed.

Equations of Equilibrium

- Count the total number of unknowns to make sure that an equivalent number of equilibrium equations can be written for solution. Except for two-force members, recall that in general three equilibrium equations can be written for each member.
- Many times, the solution for the unknowns will be straightforward if the moment equation $\Sigma M_O = 0$ is applied about a point (O) that lies at the intersection of the lines of action of as many unknown forces as possible.
- When applying the force equations $\Sigma F_x = 0$ and $\Sigma F_y = 0$, orient the x and y axes along lines that will provide the simplest reduction of the forces into their x and y components.
- If the solution of the equilibrium equations yields a *negative* magnitude for an unknown force or couple moment, it indicates that its arrowhead sense of direction is *opposite* to that which was assumed on the free-body diagram.

EXAMPLE 2.8

Determine the reactions on the beam shown in Fig. 2–28*a*.

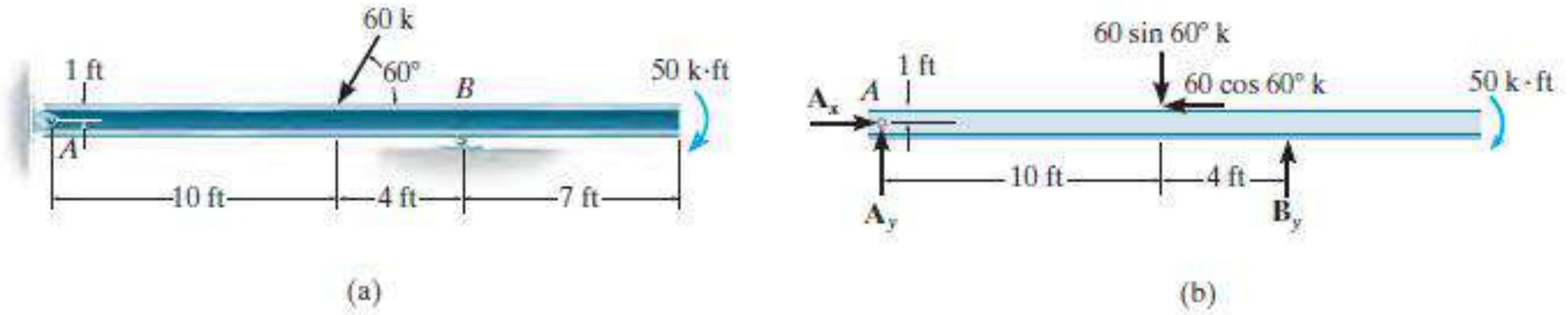


Fig. 2–28

SOLUTION

Free-Body Diagram. As shown in Fig. 2–28*b*, the 60-k force is resolved into *x* and *y* components. Furthermore, the 7-ft dimension line is not needed since a couple moment is a *free vector* and can therefore act anywhere on the beam for the purpose of computing the external reactions.

Equations of Equilibrium. Applying Eqs. 2–2 in a sequence, using previously calculated results, we have

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad A_x - 60\cos 60^\circ = 0 \quad A_x = 30.0\text{ k}$ *Ans.*

$\curvearrowleft + \Sigma M_A = 0;$ $\quad -60\sin 60^\circ(10) + 60\cos 60^\circ(1) + B_y(14) - 50 = 0 \quad B_y = 38.5\text{ k}$ *Ans.*

$+\uparrow \Sigma F_y = 0;$ $\quad -60\sin 60^\circ + 38.5 + A_y = 0 \quad A_y = 13.4\text{ k}$ *Ans.*

EXAMPLE 2.9

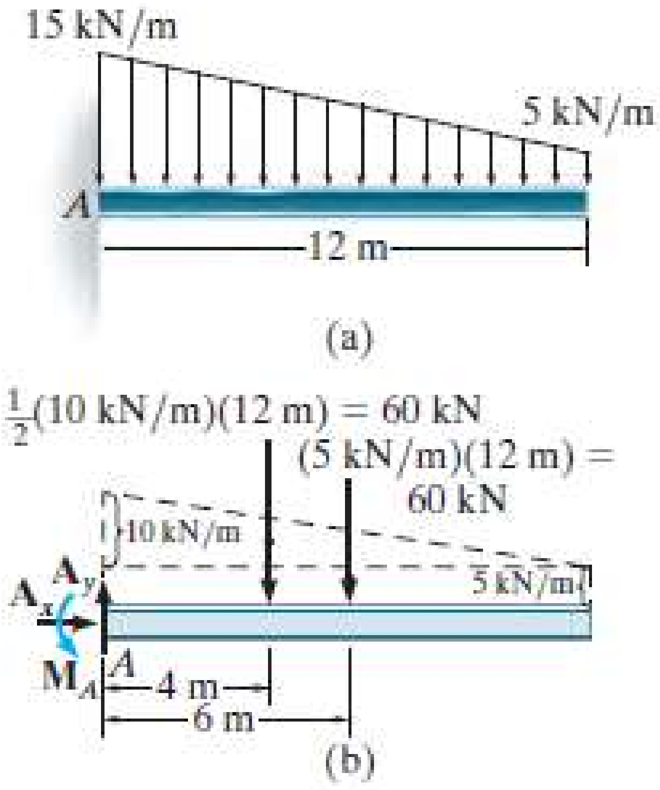


Fig. 2–29

Determine the reactions on the beam in Fig. 2–29*a*.

SOLUTION

Free-Body Diagram. As shown in Fig. 2–29*b*, the trapezoidal distributed loading is segmented into a triangular and a uniform load. The *areas* under the triangle and rectangle represent the *resultant* forces. These forces act through the centroid of their corresponding areas.

Equations of Equilibrium

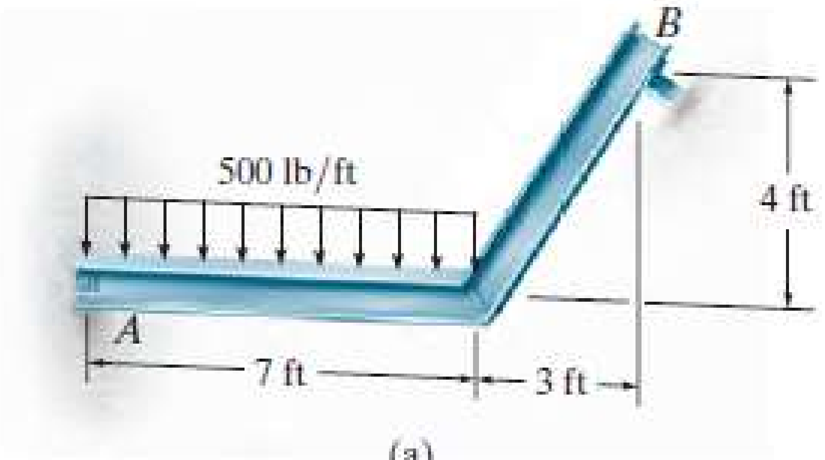
$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x = 0$ *Ans.*

$+\uparrow \Sigma F_y = 0; \quad A_y - 60 - 60 = 0 \qquad A_y = 120\text{ kN}$ *Ans.*

$\circlearrowleft + \Sigma M_A = 0; \quad -60(4) - 60(6) + M_A = 0 \quad M_A = 600\text{ kN}\cdot\text{m}$ *Ans.*

EXAMPLE 2.10

Determine the reactions on the beam in Fig. 2–30*a*. Assume *A* is a pin and the support at *B* is a roller (smooth surface).

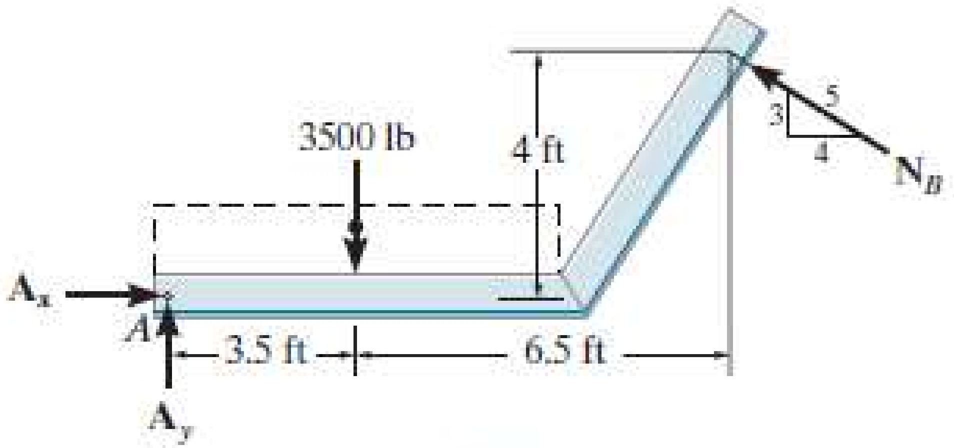


(a)

Fig. 2–30

SOLUTION

Free-Body Diagram. As shown in Fig. 2–30*b*, the support ("roller") at *B* exerts a *normal force* on the beam at its point of contact. The line of action of this force is defined by the 3–4–5 triangle.



(b)

Equations of Equilibrium. Resolving $\mathbf{N}_B$ into x and y components and summing moments about A yields a direct solution for N_B. Why? Using this result, we can then obtain A_x and A_y.

$\circlearrowleft + \Sigma M_A = 0;$ $\quad -3500(3.5) + \left(\frac{4}{5}\right)N_B(4) + \left(\frac{3}{5}\right)N_B(10) = 0$ *Ans.*

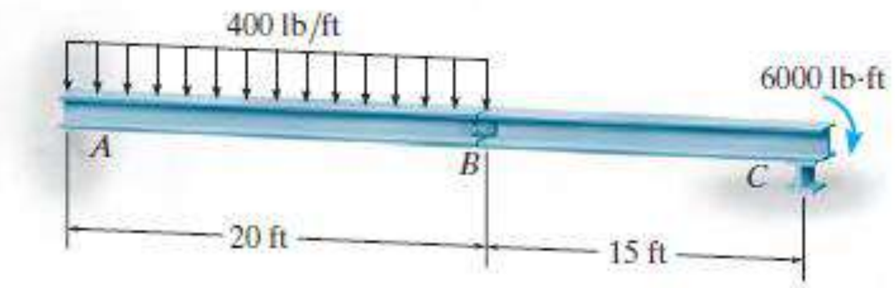
$$N_B = 1331.5 \text{ lb} = 1.33 \text{ k}$$

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad A_x - \frac{4}{5}(1331.5) = 0 \qquad A_x = 1.07 \text{ k}$ *Ans.*

$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 3500 + \frac{3}{5}(1331.5) = 0 \qquad A_y = 2.70 \text{ k}$ *Ans.*

EXAMPLE 2.11

The compound beam in Fig. 2–31*a* is fixed at *A*. Determine the reactions at *A*, *B*, and *C*. Assume that the connection at *B* is a pin and *C* is a roller.

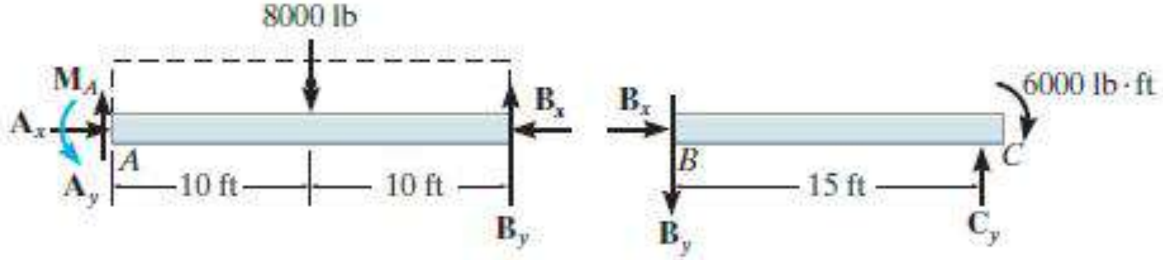


(a)

Fig. 2–31

SOLUTION

Free-Body Diagrams. The free-body diagram of each segment is shown in Fig. 2–31*b*. Why is this problem statically determinate?



(b)

Equations of Equilibrium. There are six unknowns. Applying the six equations of equilibrium, using previously calculated results, we have

Segment *BC*:

$\curvearrowright\!+\Sigma M_C = 0;\qquad -6000 + B_y(15) = 0 \qquad B_y = 400\text{ lb}$ *Ans.*

$+\uparrow\Sigma F_y = 0;\qquad -400 + C_y = 0 \qquad C_y = 400\text{ lb}$ *Ans.*

$\overset{+}{\rightarrow}\Sigma F_x = 0;\qquad B_x = 0$ *Ans.*

Segment *AB*:

$\curvearrowright\!+\Sigma M_A = 0;\qquad M_A - 8000(10) + 400(20) = 0$

$M_A = 72.0\text{ k}\cdot\text{ft}$ *Ans.*

$+\uparrow\Sigma F_y = 0;\qquad A_y - 8000 + 400 = 0 \qquad A_y = 7.60\text{ k}$ *Ans.*

$\overset{+}{\rightarrow}\Sigma F_x = 0;\qquad A_x - 0 = 0 \qquad A_x = 0$ *Ans.*

EXAMPLE 2.12

Determine the horizontal and vertical components of reaction at the pins A, B, and C of the two-member frame shown in Fig. 2–32a.

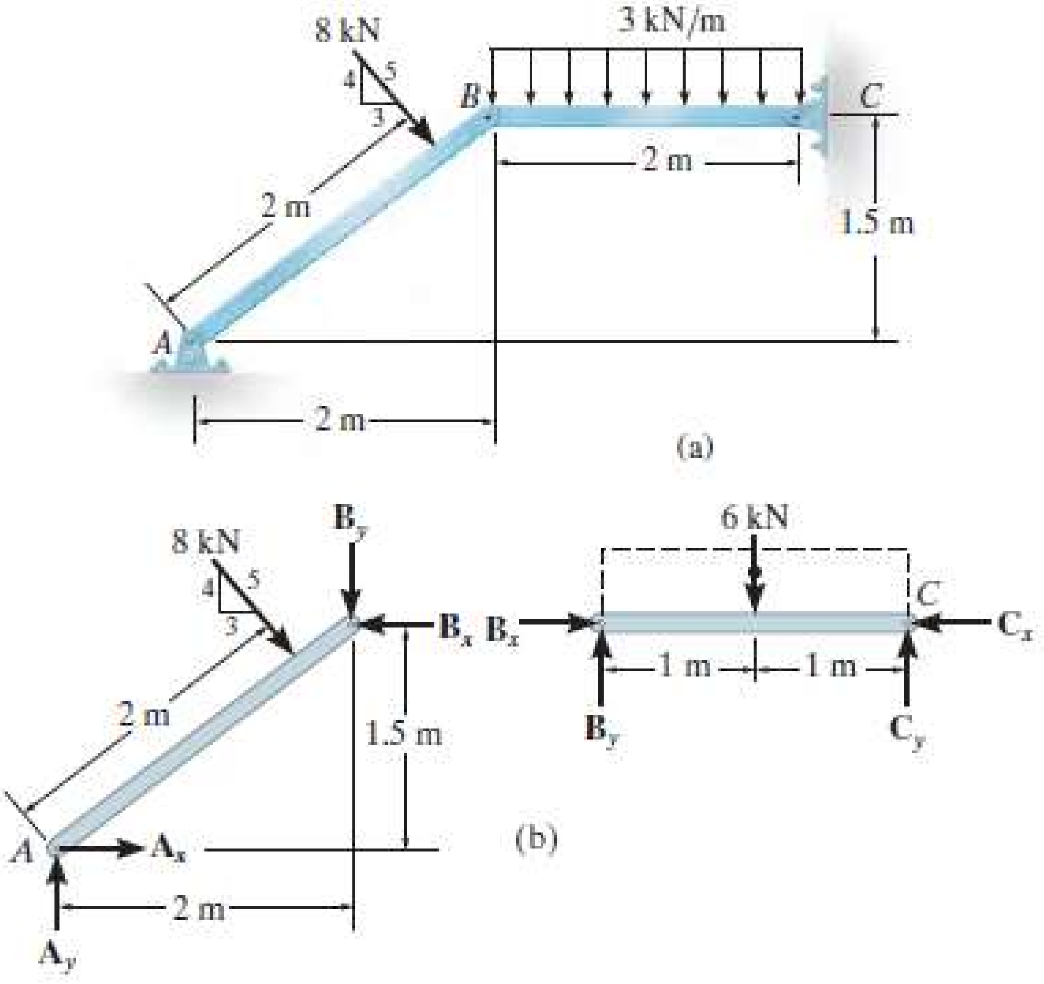


Fig. 2–32

SOLUTION

Free-Body Diagrams. The free-body diagram of each member is shown in Fig. 2–32*b*.

Equations of Equilibrium. Applying the six equations of equilibrium in the following sequence allows a direct solution for each of the six unknowns.

Member *BC*:

$\curvearrowleft + \Sigma M_C = 0; \quad -B_y(2) + 6(1) = 0 \qquad B_y = 3 \text{ kN}$ *Ans.*

Member *AB*:

$\curvearrowleft + \Sigma M_A = 0; \quad -8(2) - 3(2) + B_x(1.5) = 0 \qquad B_x = 14.7 \text{ kN}$ *Ans.*

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x + \frac{3}{5}(8) - 14.7 = 0 \qquad A_x = 9.87 \text{ kN}$ *Ans.*

$+\uparrow \Sigma F_y = 0; \quad A_y - \frac{4}{5}(8) - 3 = 0 \qquad A_y = 9.40 \text{ kN}$ *Ans.*

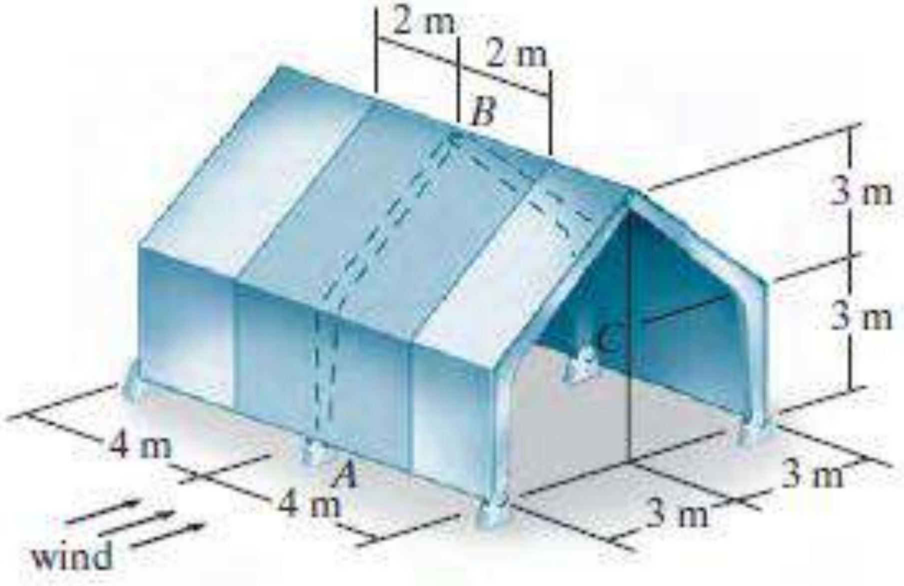
Member *BC*:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 14.7 - C_x = 0 \qquad C_x = 14.7 \text{ kN}$ *Ans.*

$+\uparrow \Sigma F_y = 0; \quad 3 - 6 + C_y = 0 \qquad C_y = 3 \text{ kN}$ *Ans.*

EXAMPLE 2.13

The side of the building in Fig. 2–33*a* is subjected to a wind loading that creates a uniform *normal* pressure of 15 kPa on the windward side and a suction pressure of 5 kPa on the leeward side. Determine the horizontal and vertical components of reaction at the pin connections A, B, and C of the supporting gable arch.

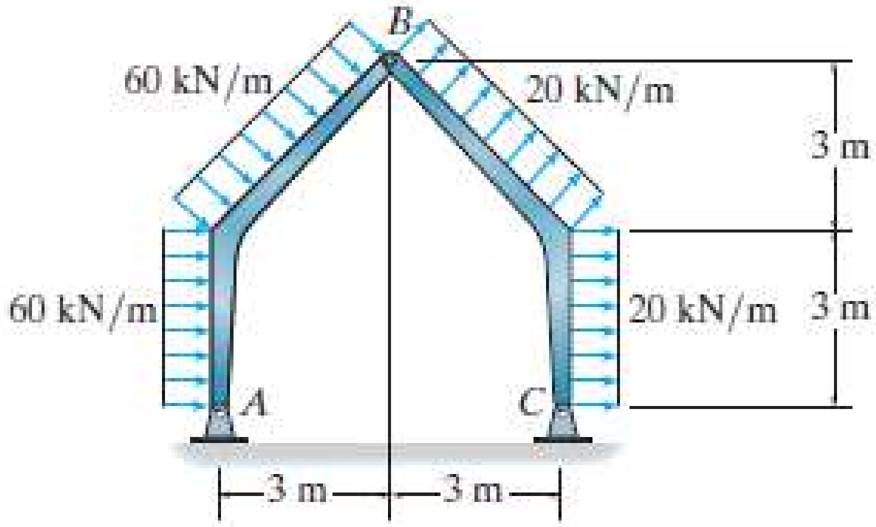


(a)

Fig. 2–33

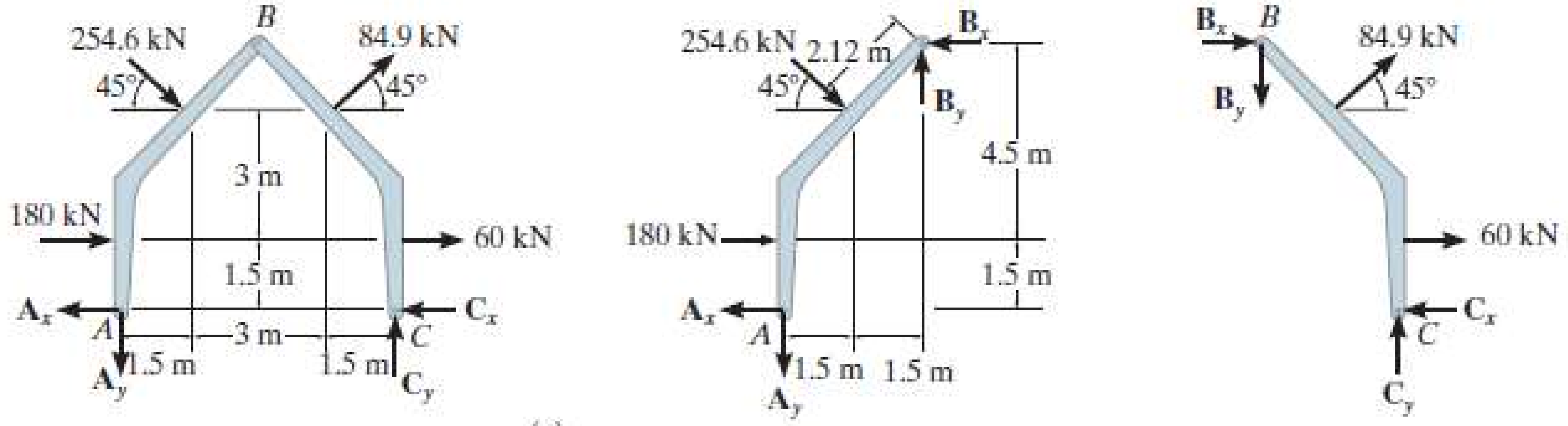
SOLUTION

Since the loading is evenly distributed, the central gable arch supports a loading acting on the walls and roof of the dark-shaded tributary area. This represents a uniform distributed load of $(15\ \text{kN/m}^2)(4\ \text{m}) = 60\ \text{kN/m}$ on the windward side and $(5\ \text{kN/m}^2)(4\ \text{m}) = 20\ \text{kN/m}$ on the leeward side, Fig. 2–33*b*.



(b)

Free-Body Diagrams. Simplifying the distributed loadings, the free-body diagrams of the entire frame and each of its parts are shown in Fig. 2–33*c*.



(c)

Equations of Equilibrium. Simultaneous solution of equations is avoided by applying the equilibrium equations in the following sequence using previously computed results.*

Entire Frame:

$$\circlearrowleft + \Sigma M_A = 0; \quad -(180 + 60)(1.5) - (254.6 + 84.9)\cos 45°(4.5)$$
$$- (254.6 \sin 45°)(1.5) + (84.9 \sin 45°)(4.5) + C_y(6) = 0$$
$$C_y = 240.0 \text{ kN} \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad -A_y - 254.6 \sin 45° + 84.9 \sin 45° + 240.0 = 0$$
$$A_y = 120.0 \text{ kN} \qquad \textit{Ans.}$$

Member AB:

$$\circlearrowright + \Sigma M_B = 0; \quad -A_x(6) + 120.0(3) + 180(4.5) + 254.6(2.12) = 0$$

$$A_x = 285.0 \text{ kN} \qquad \textit{Ans.}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad -285.0 + 180 + 254.6 \cos 45^\circ - B_x = 0$$

$$B_x = 75.0 \text{ kN} \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad -120.0 - 254.6 \sin 45^\circ + B_y = 0$$

$$B_y = 300.0 \text{ kN} \qquad \textit{Ans.}$$

Member CB:

$$\xrightarrow{+} \Sigma F_x = 0; \quad -C_x + 60 + 84.9 \cos 45^\circ + 75.0 = 0$$

$$C_x = 195.0 \text{ kN} \qquad \textit{Ans.}$$

*The problem can also be solved by applying the six equations of equilibrium only to the two members. If this is done, it is best to first sum moments about point A on member AB, then point C on member CB. By doing this, one obtains two equations to be solved simultaneously for B_x and B_y.

3

Analysis of Statically Determinate Trusses

3.1 Common Types of Trusses

A *truss* is a structure composed of slender members joined together at their end points. The members commonly used in construction consist of wooden struts, metal bars, angles, or channels. The joint connections are usually formed by bolting or welding the ends of the members to a common plate, called a *gusset plate*, as shown in Fig. 3–1, or by simply passing a large bolt or pin through each of the members. Planar trusses lie in a single plane and are often used to support roofs and bridges.

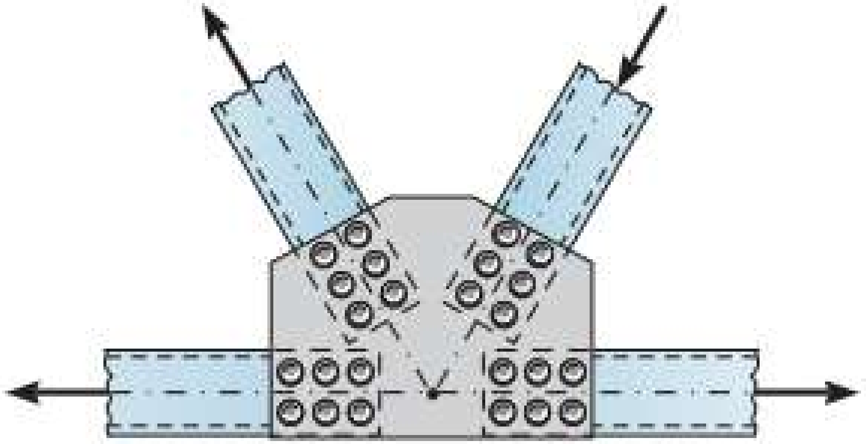


Fig. 3–1

The gusset plate is used to connect eight members of the truss supporting structure for a water tank.

Roof Trusses. Roof trusses are often used as part of an industrial building frame, such as the one shown in Fig. 3–2. Here, the roof load is transmitted to the truss at the joints by means of a series of purlins. The roof truss along with its supporting columns is termed a bent.

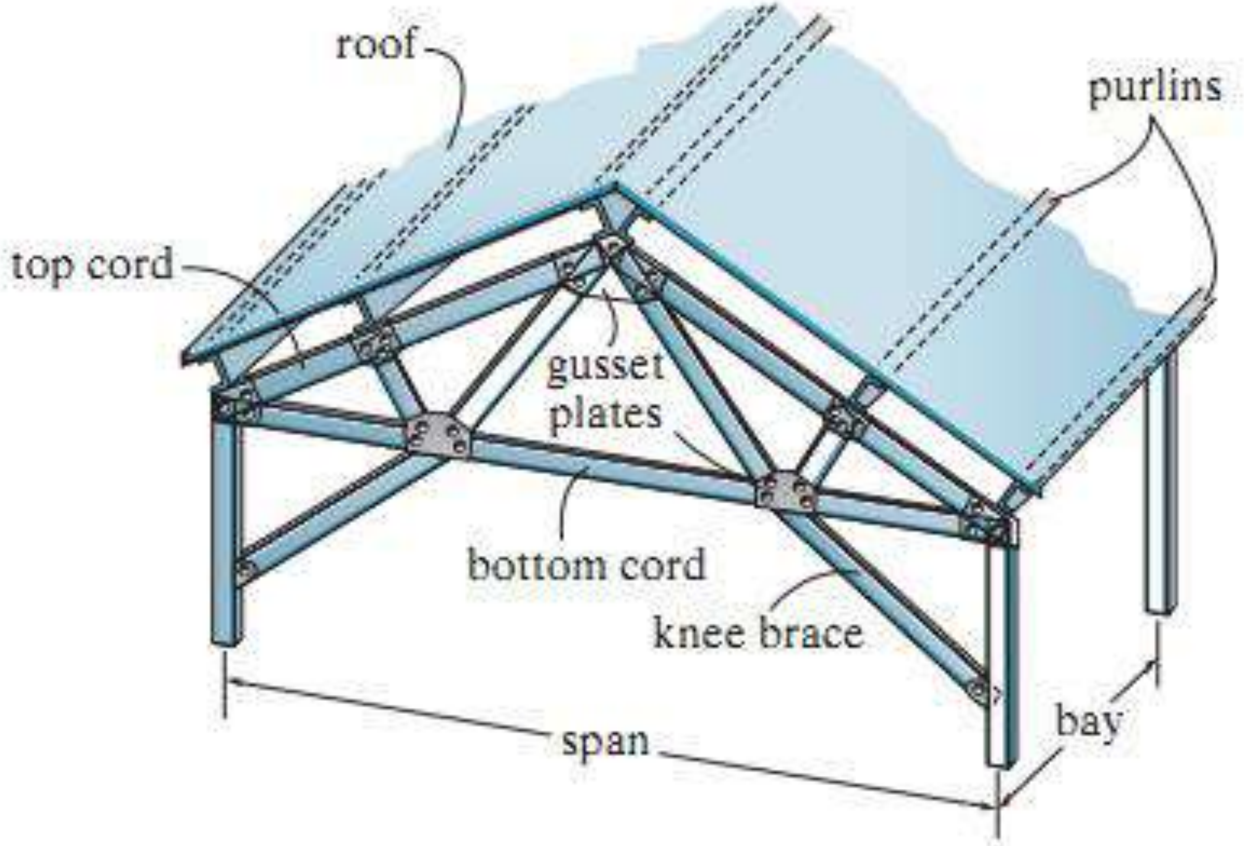


Fig. 3–2

Although more decorative than structural, these simple Pratt trusses are used for the entrance of a building.

Bridge Trusses. The main structural elements of a typical bridge truss are shown in Fig. 3–4. Here it is seen that a load on the *deck* is first transmitted to *stringers*, then to *floor beams*, and finally to the joints of the two supporting side trusses. The top and bottom cords of these side trusses are connected by top and bottom *lateral bracing*, which serves to resist the lateral forces caused by wind and the sidesway caused by moving vehicles on the bridge. Additional stability is provided by the *portal* and *sway bracing*. As in the case of many long-span trusses, a roller is provided at one end of a bridge truss to allow for thermal expansion.

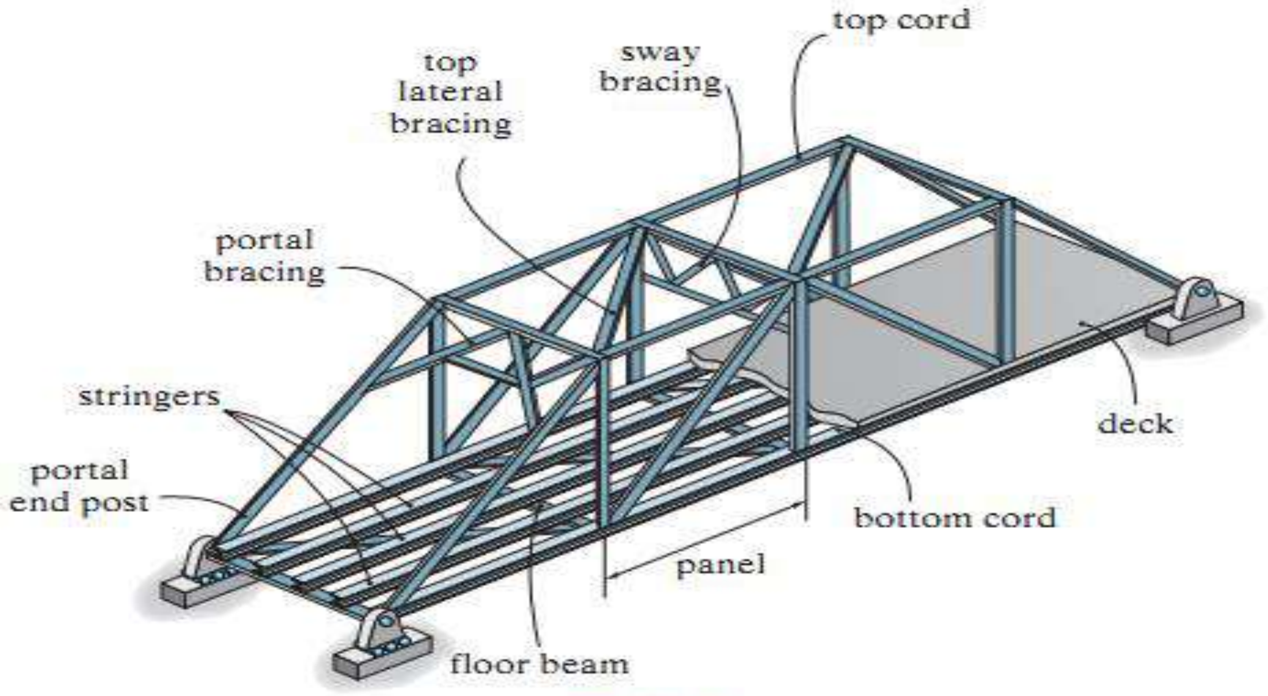


Fig. 3–4

3.2 Classification of Coplanar Trusses

Before beginning the force analysis of a truss, it is important to classify the truss as simple, compound, or complex, and then to be able to specify its determinacy and stability.

Simple Truss. To prevent collapse, the framework of a truss must be rigid. Obviously, the four-bar frame $ABCD$ in Fig. 3–7 will collapse unless a diagonal, such as AC, is added for support. The simplest framework that is rigid or stable is a *triangle*. Consequently, a *simple truss* is constructed by starting with a basic triangular element, such as ABC in Fig. 3–8, and connecting two members (AD and BD) to form an additional element. Thus it is seen that as each additional element of two members is placed on the truss, the number of joints is increased by one.

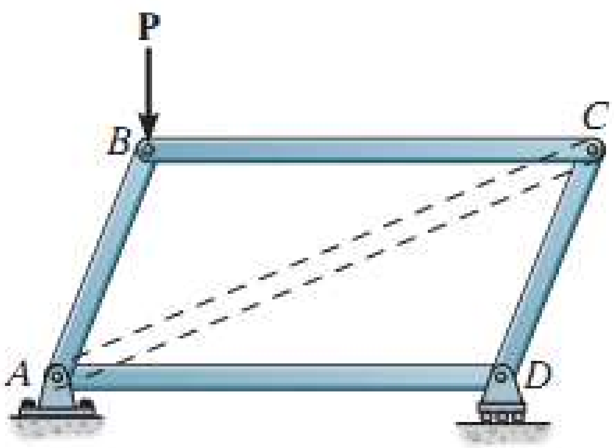


Fig. 3–7

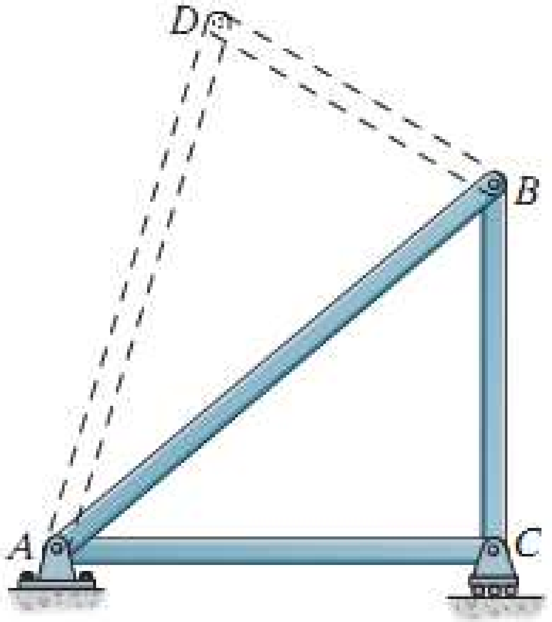
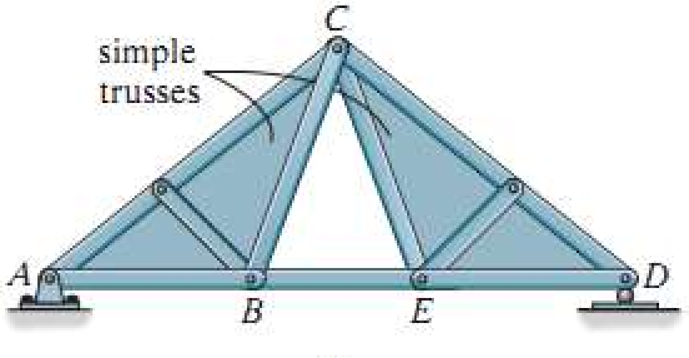


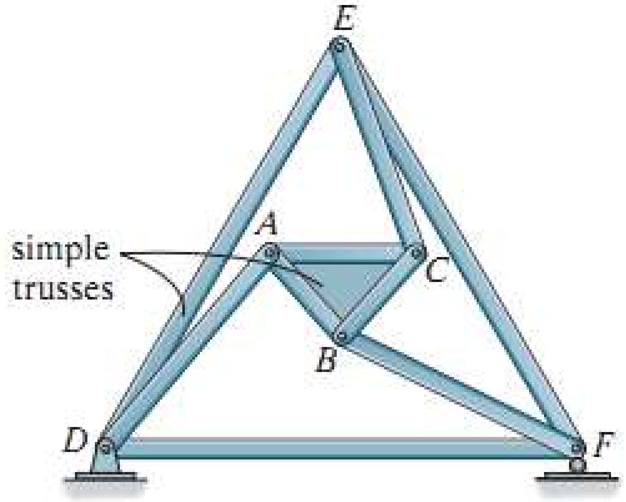
Fig. 3–8

Compound Truss. A *compound truss* is formed by connecting two or more simple trusses together. Quite often this type of truss is used to support loads acting over a *large span*, since it is cheaper to construct a somewhat lighter compound truss than to use a heavier single simple truss.

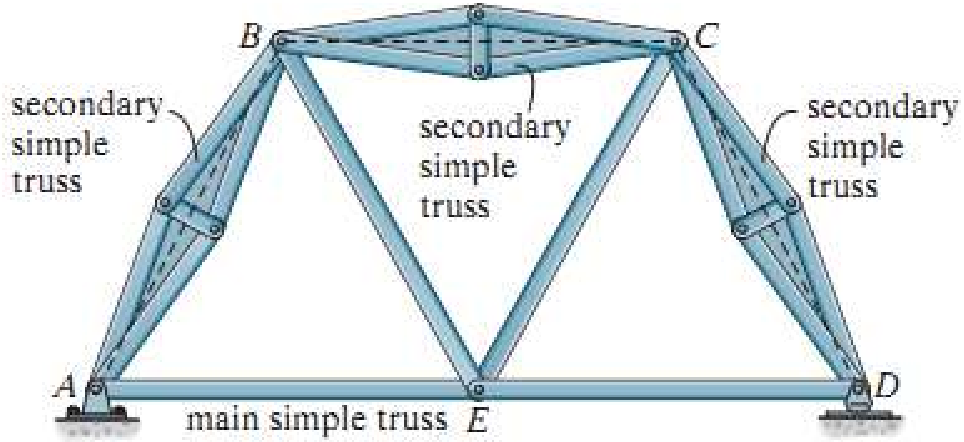
Complex Truss. A *complex truss* is one that cannot be classified as being either simple or compound. The truss in Fig. 3–12 is an example.



(a)



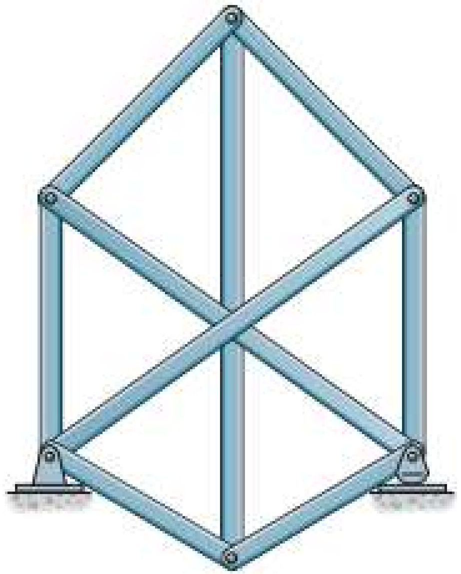
(b)



(c)

Various types of compound trusses

Fig. 3–11



Complex truss

Fig. 3–12

Determinacy. For any problem in truss analysis, it should be realized that the total number of *unknowns* includes the forces in b number of bars of the truss and the total number of external support reactions r. Since the truss members are all straight axial force members lying in the same plane, the force system acting at each joint is *coplanar and concurrent*. Consequently, rotational or moment equilibrium is automatically satisfied at the joint (or pin), and it is only necessary to satisfy $\Sigma F_x = 0$ and $\Sigma F_y = 0$ to ensure translational or force equilibrium. Therefore, only two equations of equilibrium can be written for each joint, and if there are j number of joints, the total number of equations available for solution is $2j$. By simply comparing the total number of unknowns $(b + r)$ with the total number of available equilibrium equations, it is therefore possible to specify the determinacy for either a simple, compound, or complex truss. We have

$$\boxed{\begin{array}{ll} b + r = 2j & \text{statically determinate} \\ b + r > 2j & \text{statically indeterminate} \end{array}} \qquad (3\text{–}1)$$

In particular, the *degree of indeterminacy* is specified by the difference in the numbers $(b + r) - 2j$.

Stability. If $b + r < 2j$, a truss will be *unstable*, that is, it will collapse, since there will be an insufficient number of bars or reactions to constrain all the joints. Also, a truss can be unstable if it is statically determinate or statically indeterminate. In this case the stability will have to be determined either by inspection or by a force analysis.

External Stability. As stated in Sec. 2–4, a *structure (or truss) is externally unstable if all of its reactions are concurrent or parallel*. For example, the two trusses in Fig. 3–13 are externally unstable since the support reactions have lines of action that are either concurrent or parallel.

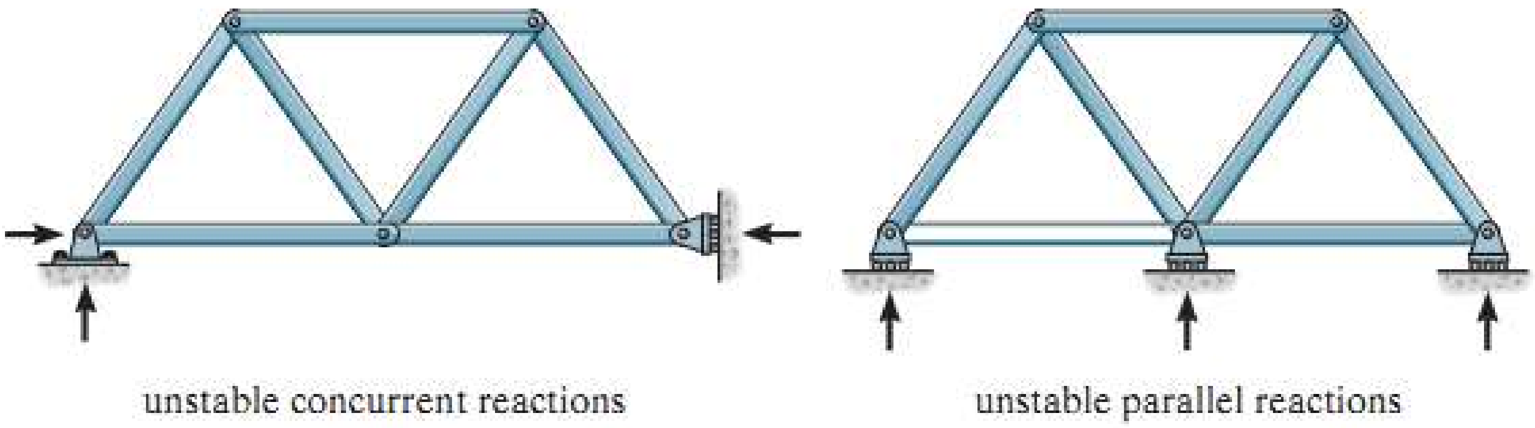


Fig. 3–13

Internal Stability. The internal stability of a truss can often be checked by careful inspection of the arrangement of its members. If it can be determined that each joint is held fixed so that it cannot move in a "rigid body" sense with respect to the other joints, then the truss will be stable. Notice that *a simple truss will always be internally stable*, since by the nature of its construction it requires starting from a basic triangular element and adding successive "rigid elements," each containing two additional members and a joint. The truss in Fig. 3–14 exemplifies this construction, where, starting with the shaded triangle element ABC, the successive joints D, E, F, G, H have been added.

If a truss is constructed so that it does not hold its joints in a fixed position, it will be unstable or have a "critical form." An obvious example of this is shown in Fig. 3–15, where it can be seen that no restraint or fixity is provided between joints C and F or B and E, and so the truss will collapse under load.

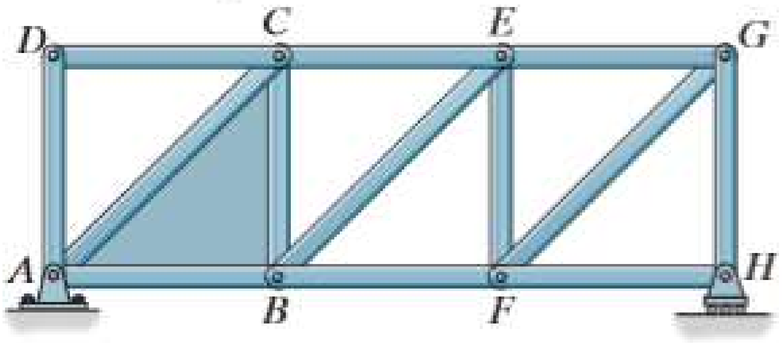


Fig. 3–14

If the truss has b bars, r external reactions, and j joints, then if:

$b + r < 2j$	unstable	
$b + r \geq 2j$	unstable if truss support reactions are concurrent or parallel or if some of the components of the truss form a collapsible mechanism	(3–2)

Bear in mind, however, that if a truss is unstable, it does not matter whether it is statically determinate or indeterminate. Obviously, the use of an unstable truss is to be avoided in practice.

EXAMPLE 3.1

Classify each of the trusses in Fig. 3–18 as stable, unstable, statically determinate, or statically indeterminate. The trusses are subjected to arbitrary external loadings that are assumed to be known and can act anywhere on the trusses.

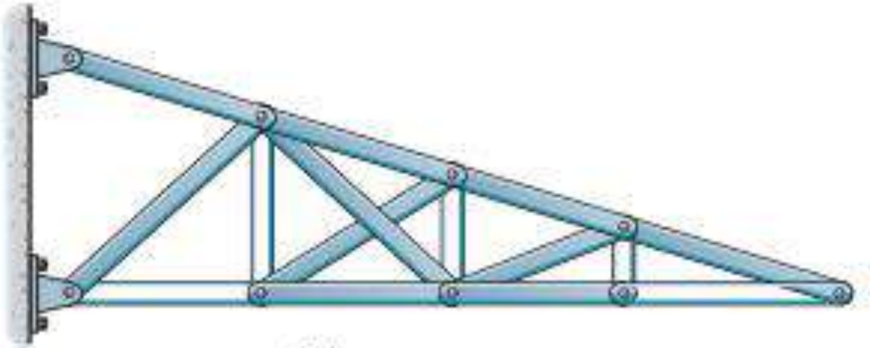
SOLUTION

Fig. 3–18*a*. *Externally stable*, since the reactions are not concurrent or parallel. Since $b = 19$, $r = 3$, $j = 11$, then $b + r = 2j$ or $22 = 22$. Therefore, the truss is *statically determinate*. By inspection the truss is *internally stable*.

(a)

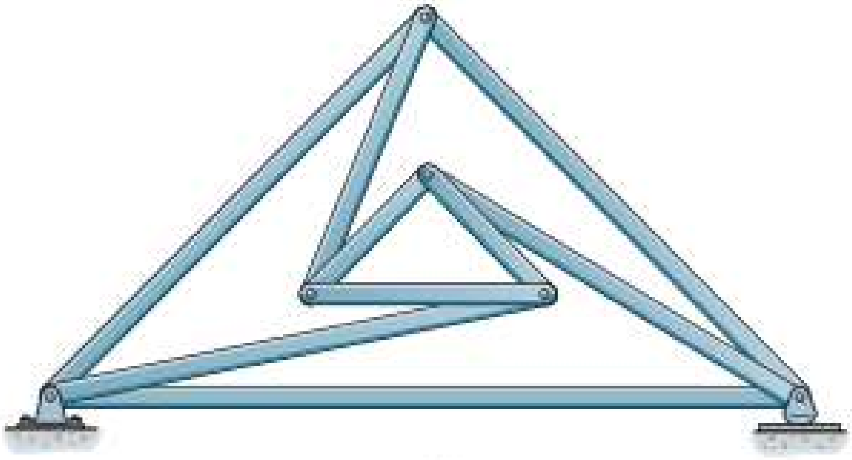
Fig. 3–18

Fig. 3–18*b*. *Externally stable.* Since $b = 15$, $r = 4$, $j = 9$, then $b + r > 2j$ or $19 > 18$. The truss is *statically indeterminate* to the first degree. By inspection the truss is *internally stable.*



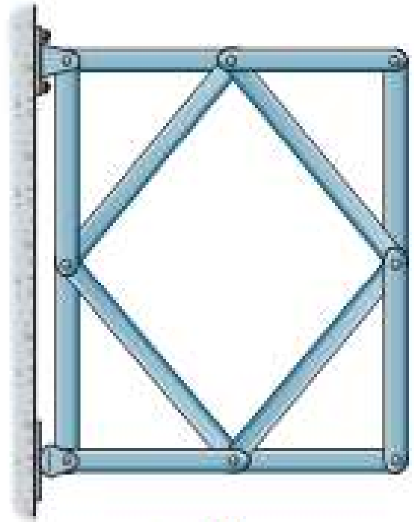
(b)

Fig. 3–18c. *Externally stable*. Since $b = 9$, $r = 3$, $j = 6$, then $b + r = 2j$ or $12 = 12$. The truss is *statically determinate*. By inspection the truss is *internally stable*.



(c)

Fig. 3–18*d*. *Externally stable*. Since $b = 12$, $r = 3$, $j = 8$, then $b + r < 2j$ or $15 < 16$. The truss is *internally unstable*.



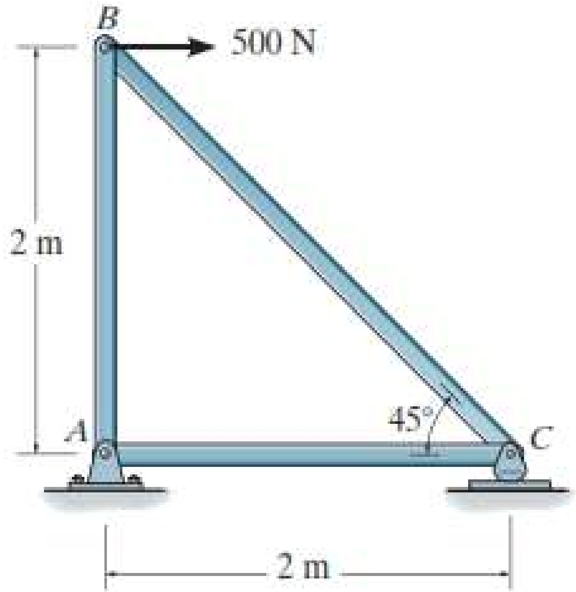
(d)

3.3 The Method of Joints

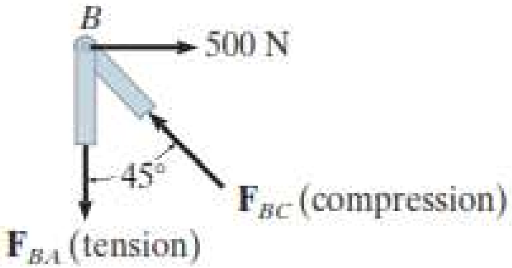
Procedure for Analysis

The following procedure provides a means for analyzing a truss using the method of joints.

- Draw the free-body diagram of a joint having at least one known force and at most two unknown forces. (If this joint is at one of the supports, it may be necessary to calculate the external reactions at the supports by drawing a free-body diagram of the entire truss.)
- Use one of the two methods previously described for establishing the sense of an unknown force.
- The x and y axes should be oriented such that the forces on the free-body diagram can be easily resolved into their x and y components. Apply the two force equilibrium equations $\Sigma F_x = 0$ and $\Sigma F_y = 0$, solve for the two unknown member forces, and verify their correct directional sense.
- Continue to analyze each of the other joints, where again it is necessary to choose a joint having at most two unknowns and at least one known force.
- Once the force in a member is found from the analysis of a joint at one of its ends, the result can be used to analyze the forces acting on the joint at its other end. Remember, a member in *compression* "pushes" on the joint and a member in *tension* "pulls" on the joint.

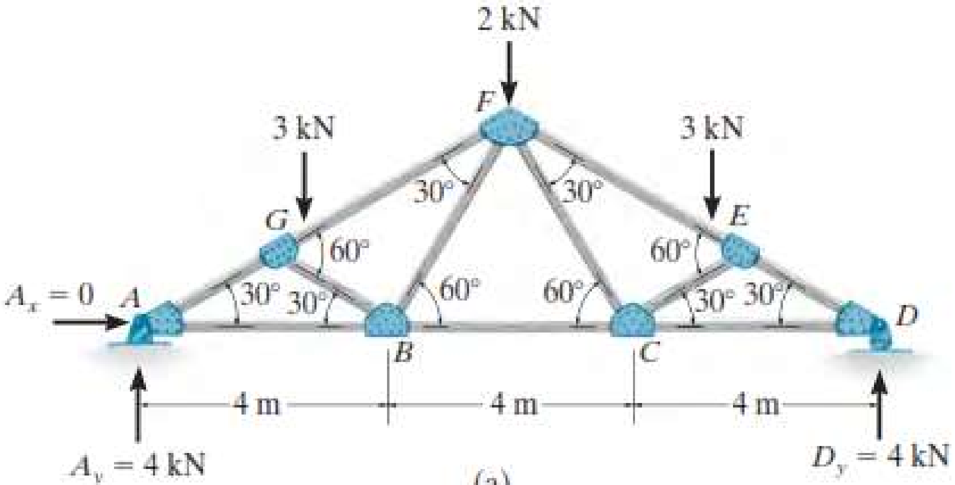


(b)

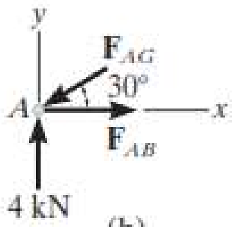


EXAMPLE 3.2

Determine the force in each member of the roof truss shown in the photo. The dimensions and loadings are shown in Fig. 3–20*a*. State whether the members are in tension or compression.



(a)



(b)

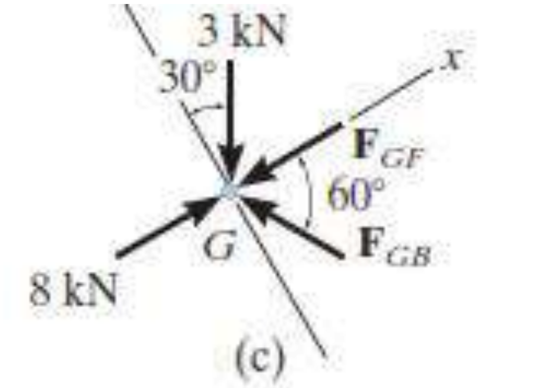
SOLUTION

Only the forces in half the members have to be determined, since the truss is symmetric with respect to *both* loading and geometry.

Joint *A*, Fig. 3–20*b*. We can start the analysis at joint *A*. Why? The free-body diagram is shown in Fig. 3–20*b*.

$$+\uparrow \Sigma F_y = 0; \quad 4 - F_{AG} \sin 30^\circ = 0 \qquad F_{AG} = 8 \text{ kN (C)} \qquad \textit{Ans.}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_{AB} - 8 \cos 30^\circ = 0 \qquad F_{AB} = 6.928 \text{ kN (T)} \qquad \textit{Ans.}$$



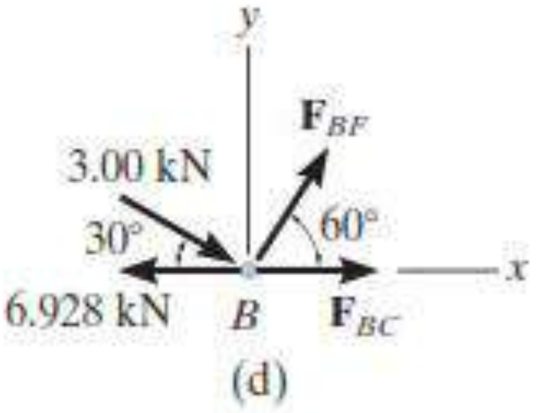


Fig. 3–20

Joint *G*, Fig. 3–20*c*. In this case note how the orientation of the x, y axes avoids simultaneous solution of equations.

$$+\nwarrow\Sigma F_y = 0; \quad F_{GB}\sin 60^\circ - 3\cos 30^\circ = 0$$

$$F_{GB} = 3.00 \text{ kN (C)} \qquad \textit{Ans.}$$

$$+\nearrow\Sigma F_x = 0; \quad 8 - 3\sin 30^\circ - 3.00\cos 60^\circ - F_{GF} = 0$$

$$F_{GF} = 5.00 \text{ kN (C)} \qquad \textit{Ans.}$$

Joint *B*, Fig. 3–20*d*.

$$+\uparrow\Sigma F_y = 0; \quad F_{BF}\sin 60^\circ - 3.00\sin 30^\circ = 0$$

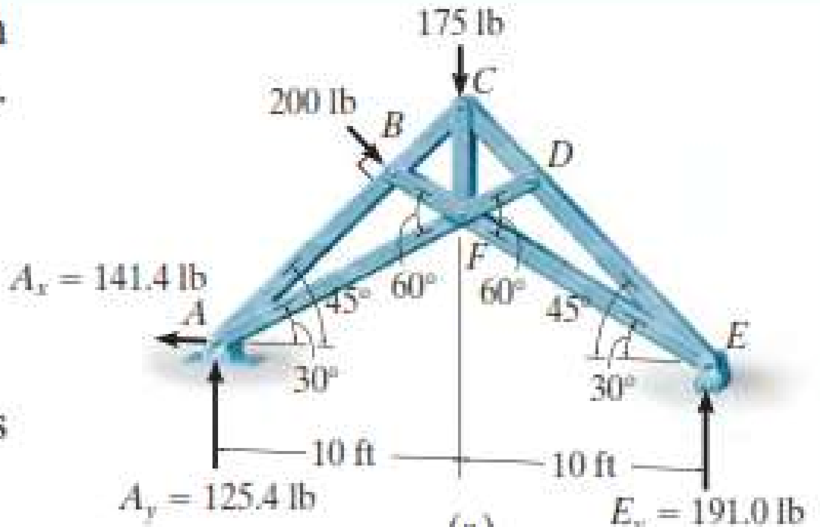
$$F_{BF} = 1.73 \text{ kN (T)} \qquad \textit{Ans.}$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad F_{BC} + 1.73\cos 60^\circ + 3.00\cos 30^\circ - 6.928 = 0$$

$$F_{BC} = 3.46 \text{ kN (T)} \qquad \textit{Ans.}$$

EXAMPLE 3.3

Determine the force in each member of the scissors truss shown in Fig. 3–21*a*. State whether the members are in tension or compression. The reactions at the supports are given.



(a)

SOLUTION

The truss will be analyzed in the following sequence:

Joint *E*, Fig. 3–21*b*. Note that simultaneous solution of equations is avoided by the x, y axes orientation.

$$+\nearrow\Sigma F_y = 0; \qquad 191.0 \cos 30^\circ - F_{ED} \sin 15^\circ = 0$$

$$F_{ED} = 639.1 \text{ lb (C)} \qquad \textit{Ans.}$$

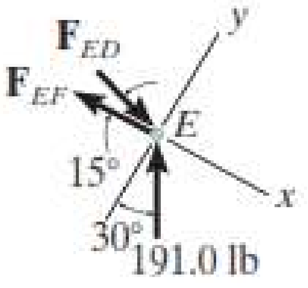
$$+\searrow\Sigma F_x = 0; \qquad 639.1 \cos 15^\circ - F_{EF} - 191.0 \sin 30^\circ = 0$$

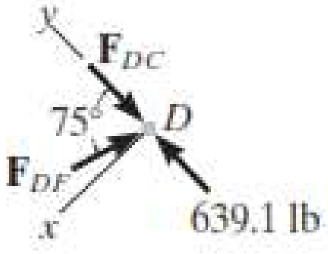
$$F_{EF} = 521.8 \text{ lb (T)} \qquad \textit{Ans.}$$

Joint *D*, Fig. 3–21*c*.

$$+\swarrow\Sigma F_x = 0; \qquad -F_{DF} \sin 75^\circ = 0 \qquad F_{DF} = 0 \qquad \textit{Ans.}$$

$$+\nwarrow\Sigma F_y = 0; \qquad -F_{DC} + 639.1 = 0 \qquad F_{DC} = 639.1 \text{ lb (C)} \qquad \textit{Ans.}$$





Joint *C*, Fig. 3–21*d*.

(b) (c)

Joint *C*, Fig. 3–21*d*.

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{CB} \sin 45^\circ - 639.1 \sin 45^\circ = 0$$

$$F_{CB} = 639.1 \text{ lb (C)} \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad -F_{CF} - 175 + 2(639.1) \cos 45^\circ = 0$$

$$F_{CF} = 728.8 \text{ lb (T)} \qquad \textit{Ans.}$$

Joint *B*, Fig. 3–21*e*.

$$+\nwarrow \Sigma F_y = 0; \qquad F_{BF} \sin 75^\circ - 200 = 0 \quad F_{BF} = 207.1 \text{ lb (C)} \qquad \textit{Ans.}$$

$$+\swarrow \Sigma F_x = 0; \qquad 639.1 + 207.1 \cos 75^\circ - F_{BA} = 0$$

$$F_{BA} = 692.7 \text{ lb (C)} \qquad \textit{Ans.}$$

Joint *A*, Fig. 3–21*f*.

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AF} \cos 30^\circ - 692.7 \cos 45^\circ - 141.4 = 0$$

$$F_{AF} = 728.9 \text{ lb (T)} \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad 125.4 - 692.7 \sin 45^\circ + 728.9 \sin 30^\circ = 0 \qquad \text{check}$$

Notice that since the reactions have been calculated, a further check of the calculations can be made by analyzing the last joint *F*. Try it and find out.

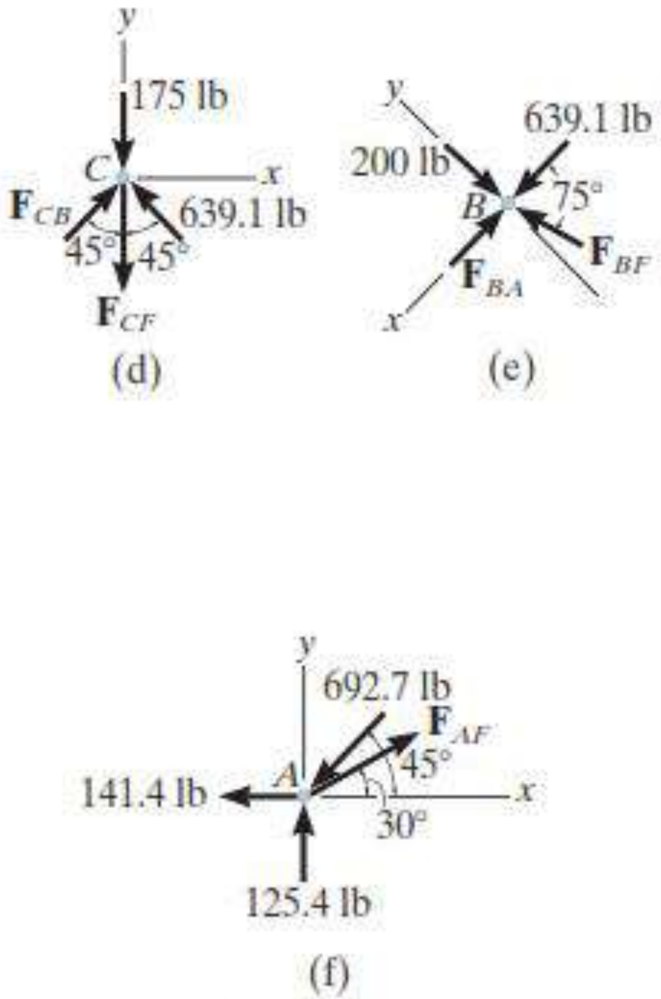
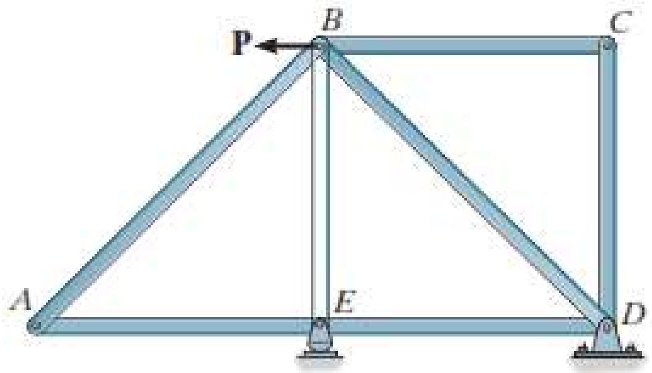


Fig. 3–21

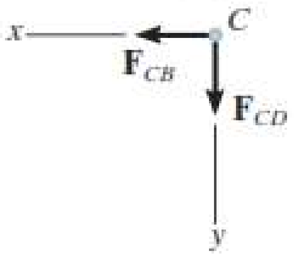
3.4 Zero-Force Members

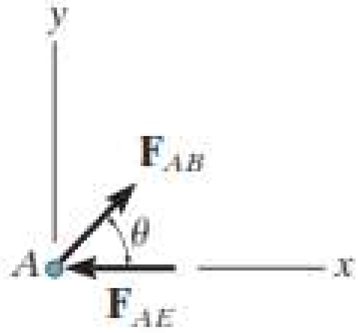
Truss analysis using the method of joints is greatly simplified if one is able to first determine those members that support *no loading*. These *zero-force members* may be necessary for the stability of the truss during construction and to provide support if the applied loading is changed. The zero-force members of a truss can generally be determined by inspection of the joints, and they occur in two cases.

Case 1. Consider the truss in Fig. 3–22*a*. The two members at joint C are connected together at a right angle *and* there is no external load on the joint. The free-body diagram of joint C, Fig. 3–22*b*, indicates that the force in each member must be zero in order to maintain equilibrium. Furthermore, as in the case of joint A, Fig. 3–22*c*, this must be true regardless of the angle, say θ, between the members.



(a)





Case 2. Zero-force members also occur at joints having a geometry as joint D in Fig. 3–23*a*. Here *no external load acts on the joint*, so that a force summation in the y direction, Fig. 3–23*b*, which is perpendicular to the two collinear members, requires that $F_{DF} = 0$. Using this result, FC is also a zero-force member, as indicated by the force analysis of joint F, Fig. 3–23*c*.

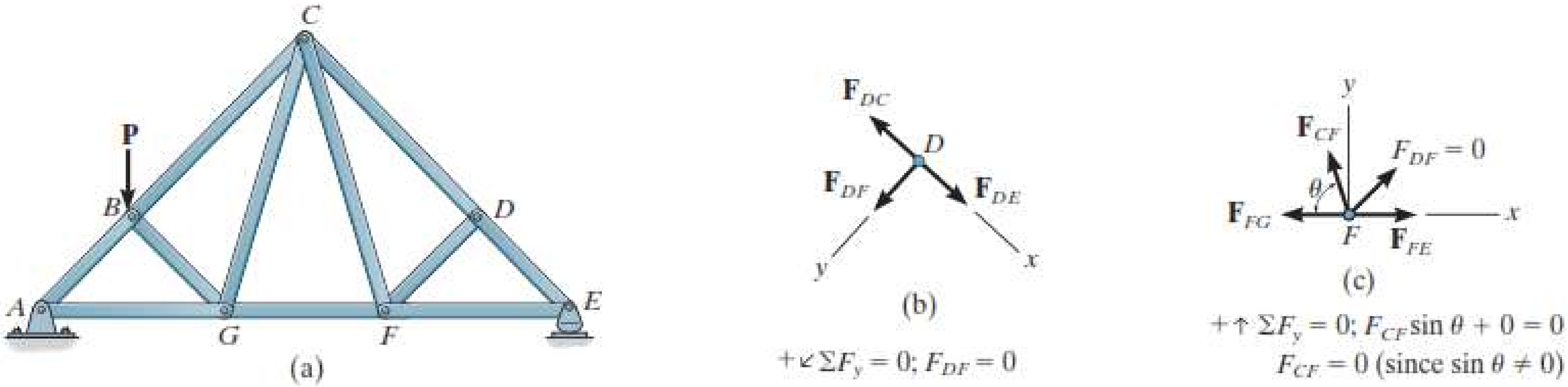
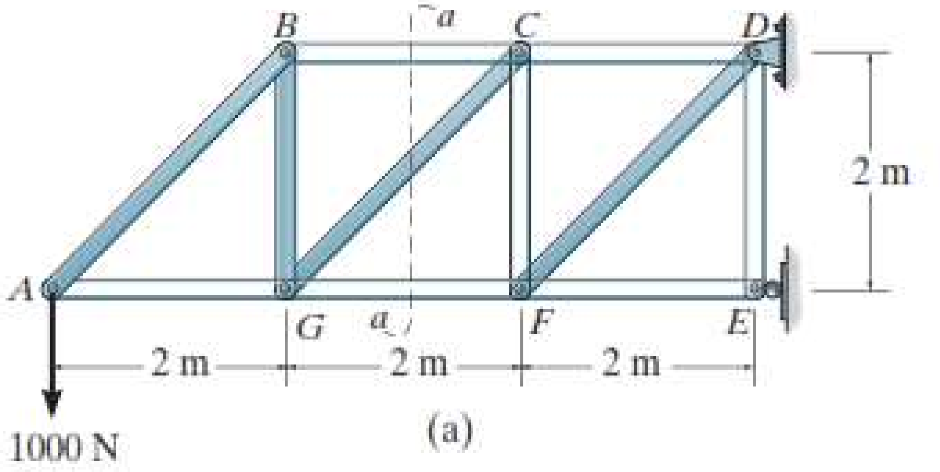


Fig. 3–23

3.5 The Method of Sections



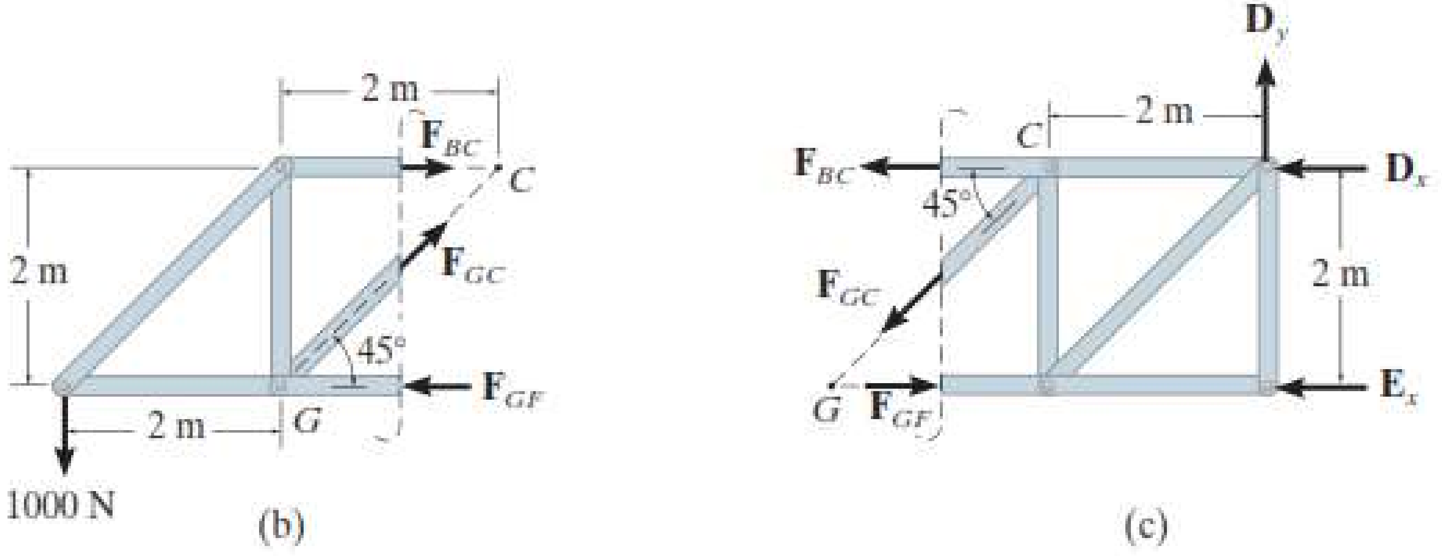


Fig. 3–25

Procedure for Analysis

The following procedure provides a means for applying the method of sections to determine the forces in the members of a truss.

Free-Body Diagram

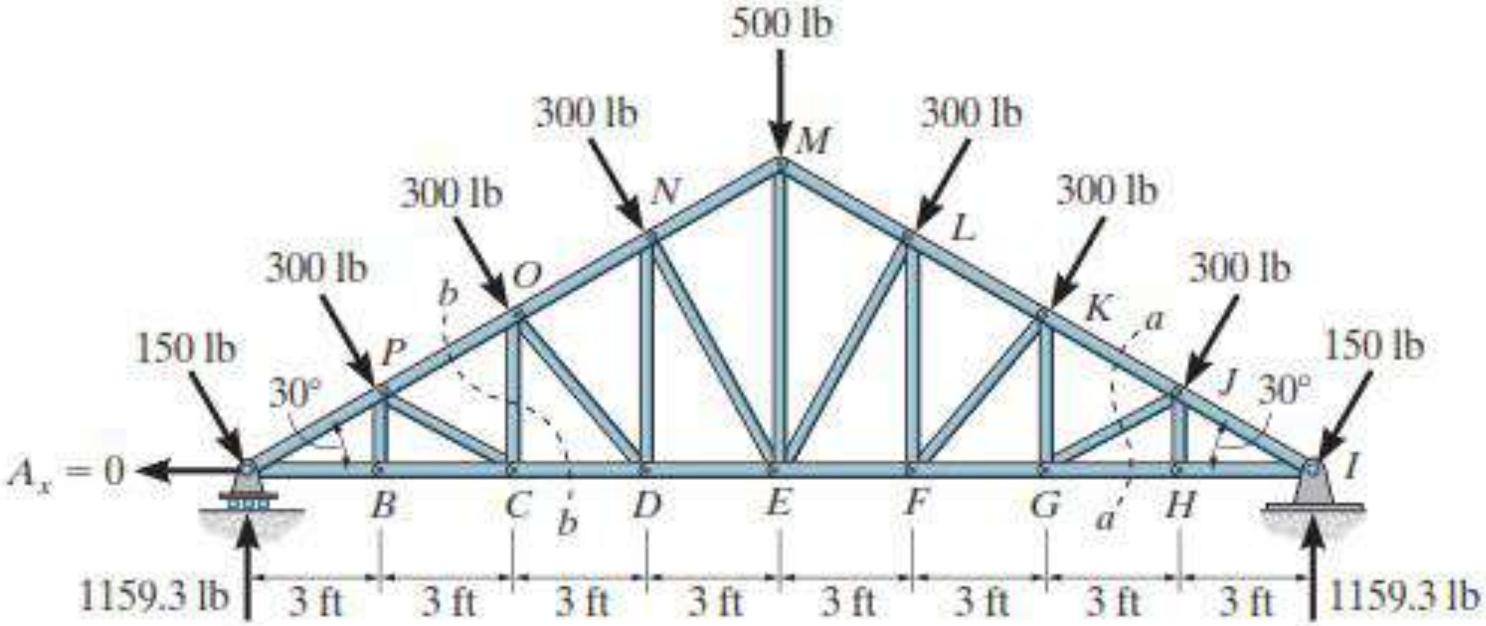
- Make a decision as to how to "cut" or section the truss through the members where forces are to be determined.
- Before isolating the appropriate section, it may first be necessary to determine the truss's *external* reactions, so that the three equilibrium equations are used *only* to solve for member forces at the cut section.
- Draw the free-body diagram of that part of the sectioned truss which has the least number of forces on it.
- Use one of the two methods described above for establishing the sense of an unknown force.

Equations of Equilibrium

- Moments should be summed about a point that lies at the intersection of the lines of action of two unknown forces; in this way, the third unknown force is determined directly from the equation.
- If two of the unknown forces are *parallel*, forces may be summed *perpendicular* to the direction of these unknowns to determine *directly* the third unknown force.

EXAMPLE 3.5

Determine the force in members *GJ* and *CO* of the roof truss shown in the photo. The dimensions and loadings are shown in Fig. 3–26*a*. State whether the members are in tension or compression. The reactions at the supports have been calculated.



(a)

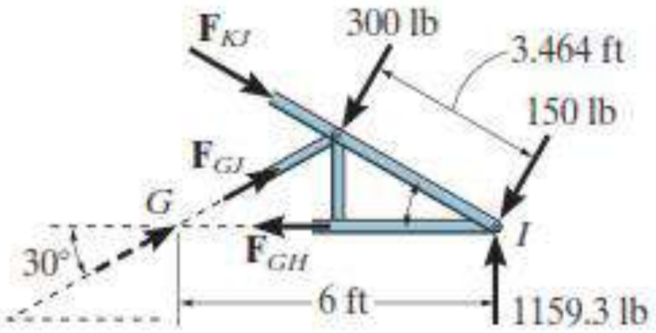
Fig. 3–26

SOLUTION

Member CF.

Free-Body Diagram. The force in member GJ can be obtained by considering the section aa in Fig. 3–26a. The free-body diagram of the right part of this section is shown in Fig. 3–26b.

Equations of Equilibrium. A direct solution for $\mathbf{F}_{GJ}$ can be obtained by applying $\Sigma M_I = 0$. Why? For simplicity, slide $\mathbf{F}_{GJ}$ to point G (principle of transmissibility), Fig. 3–26b. Thus,



(b)

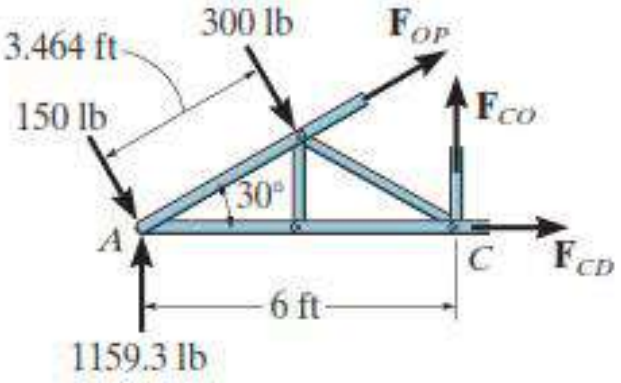
$$\circlearrowleft + \Sigma M_I = 0; \qquad -F_{GJ}\sin 30^\circ(6) + 300(3.464) = 0$$

$$F_{GJ} = 346 \text{ lb (C)} \qquad \textit{Ans.}$$

Member GC.

Free-Body Diagram. The force in CO can be obtained by using section bb in Fig. 3–26a. The free-body diagram of the left portion of the section is shown in Fig. 3–26c.

Equations of Equilibrium. Moments will be summed about point A in order to eliminate the unknowns $\mathbf{F}_{OP}$ and $\mathbf{F}_{CD}$.



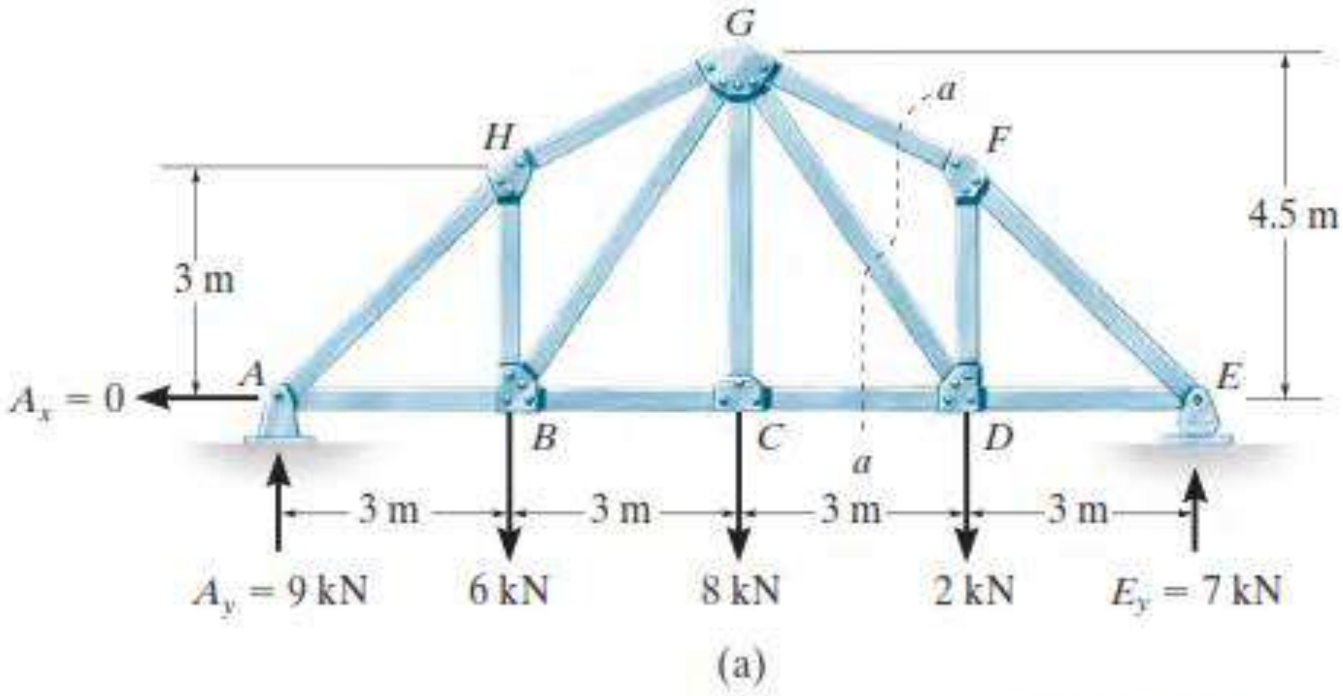
(c)

$$\circlearrowleft + \Sigma M_A = 0; \qquad -300(3.464) + F_{CO}(6) = 0$$

$$F_{CO} = 173 \text{ lb (T)} \qquad \textit{Ans.}$$

EXAMPLE 3.6

Determine the force in members *GF* and *GD* of the truss shown in Fig. 3–27*a*. State whether the members are in tension or compression. The reactions at the supports have been calculated.



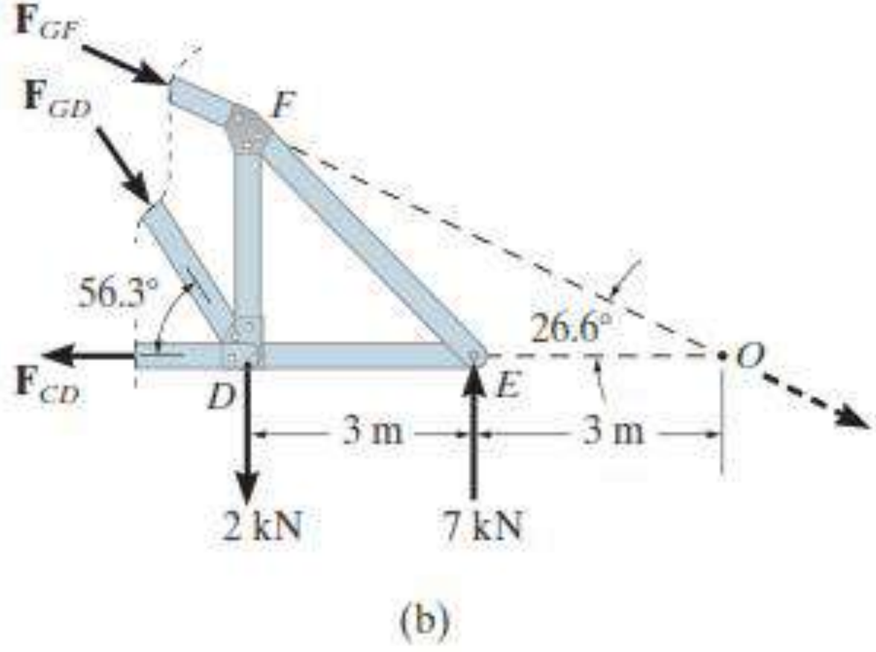


Fig. 3–27

SOLUTION

Free-Body Diagram. Section *aa* in Fig. 3–27*a* will be considered. Why? The free-body diagram to the right of this section is shown in Fig. 3–27*b*. The distance *EO* can be determined by proportional triangles or realizing that member *GF* drops vertically $4.5 - 3 = 1.5$ m in 3 m, Fig. 3–27*a*. Hence to drop 4.5 m from G the distance from *C* to *O* must be 9 m. Also, the angles that $\mathbf{F}_{GD}$ and $\mathbf{F}_{GF}$ make with the horizontal are $\tan^{-1}(4.5/3) = 56.3°$ and $\tan^{-1}(4.5/9) = 26.6°$, respectively.

Equations of Equilibrium. The force in *GF* can be determined directly by applying $\Sigma M_D = 0$. Why? For the calculation use the principle of transmissibility and slide $\mathbf{F}_{GF}$ to point *O*. Thus,

$$\circlearrowright + \Sigma M_D = 0; \qquad -F_{GF} \sin 26.6°(6) + 7(3) = 0$$

$$F_{GF} = 7.83 \text{ kN (C)} \qquad \textit{Ans.}$$

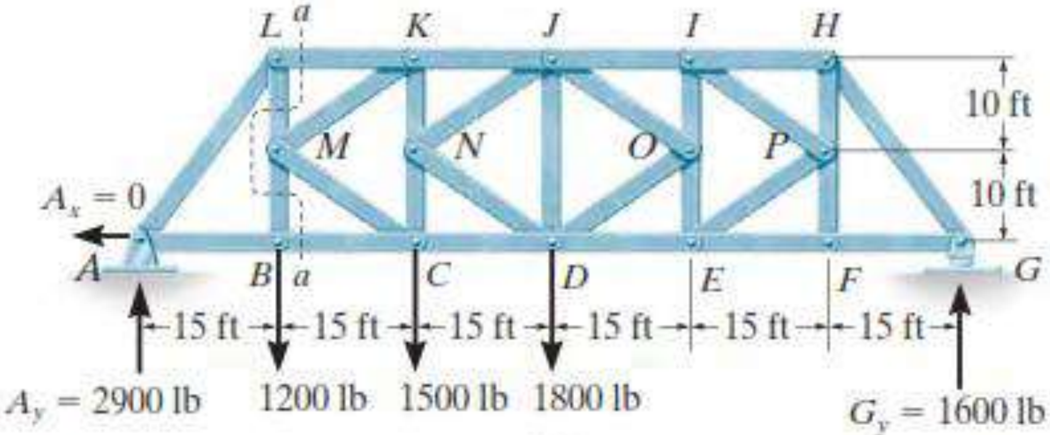
The force in *GD* is determined directly by applying $\Sigma M_O = 0$. For simplicity use the principle of transmissibility and slide $\mathbf{F}_{GD}$ to *D*. Hence,

$$\circlearrowright + \Sigma M_O = 0; \qquad -7(3) + 2(6) + F_{GD} \sin 56.3°(6) = 0$$

$$F_{GD} = 1.80 \text{ kN (C)} \qquad \textit{Ans.}$$

EXAMPLE 3.7

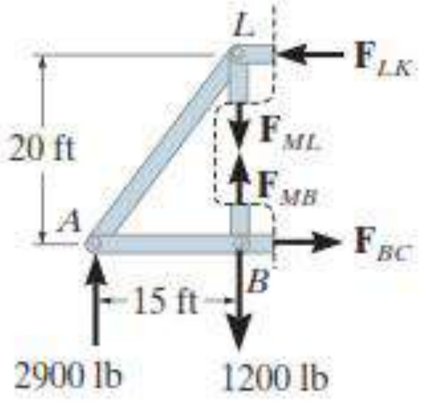
Determine the force in members *BC* and *MC* of the K-truss shown in Fig. 3–28*a*. State whether the members are in tension or compression. The reactions at the supports have been calculated.



(a)

SOLUTION

Free-Body Diagram. Although section *aa* shown in Fig. 3–28*a* cuts through four members, it is possible to solve for the force in member *BC* using this section. The free-body diagram of the left portion of the truss is shown in Fig. 3–28*b*.



(b)

Equations of Equilibrium. Summing moments about point *L* eliminates *three* of the unknowns, so that

$$\circlearrowleft + \Sigma M_L = 0; \qquad -2900(15) + F_{BC}(20) = 0$$

$$F_{BC} = 2175 \text{ lb (T)} \qquad \textit{Ans.}$$

Free-Body Diagrams. The force in MC can be obtained indirectly by first obtaining the force in MB from vertical force equilibrium of joint B, Fig. 3–28c, i.e., $F_{MB} = 1200$ lb (T). Then from the free-body diagram in Fig. 3–28b.

$$+\uparrow \Sigma F_y = 0; \qquad 2900 - 1200 + 1200 - F_{ML} = 0$$

$$F_{ML} = 2900 \text{ lb (T)}$$

Using these results, the free-body diagram of joint M is shown in Fig. 3–28d.

Equations of Equilibrium.

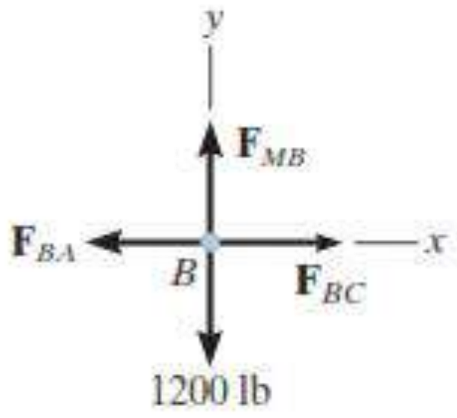
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad \left(\frac{3}{\sqrt{13}}\right)F_{MC} - \left(\frac{3}{\sqrt{13}}\right)F_{MK} = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad 2900 - 1200 - \left(\frac{2}{\sqrt{13}}\right)F_{MC} - \left(\frac{2}{\sqrt{13}}\right)F_{MK} = 0$$

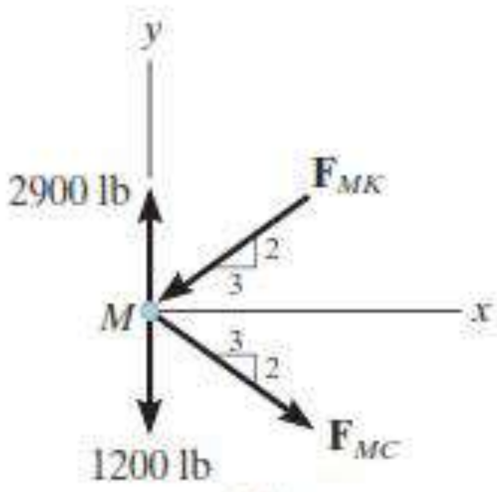
$$F_{MK} = 1532 \text{ lb (C)} \qquad F_{MC} = 1532 \text{ lb (T)} \qquad \textit{Ans.}$$

Sometimes, as in this example, application of both the method of sections and the method of joints leads to the most direct solution to the problem.

It is also possible to solve for the force in MC by using the result for $\mathbf{F}_{BC}$. In this case, pass a vertical section through LK, MK, MC, and BC, Fig. 3–28a. Isolate the left section and apply $\Sigma M_K = 0$.



(c)



(d)

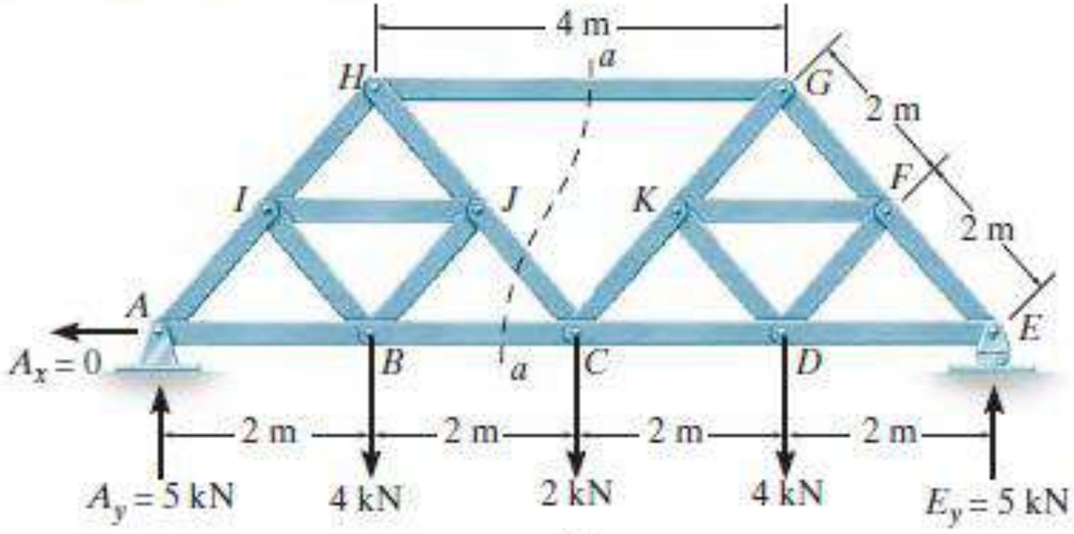
Fig. 3–28

3.6 Compound Trusses

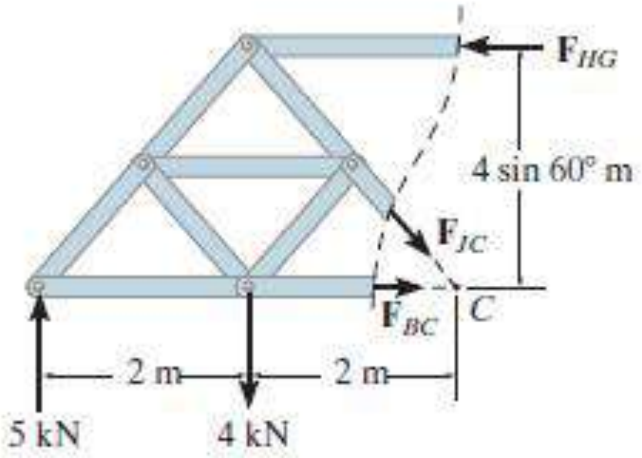
In Sec. 3–2 it was stated that compound trusses are formed by connecting two or more simple trusses together either by bars or by joints. Occasionally this type of truss is best analyzed by applying *both* the method of joints and the method of sections. It is often convenient to first recognize the type of construction as listed in Sec. 3–2 and then perform the analysis using the following procedure.

EXAMPLE 3.8

Indicate how to analyze the compound truss shown in Fig. 3–29*a*. The reactions at the supports have been calculated.



(a)



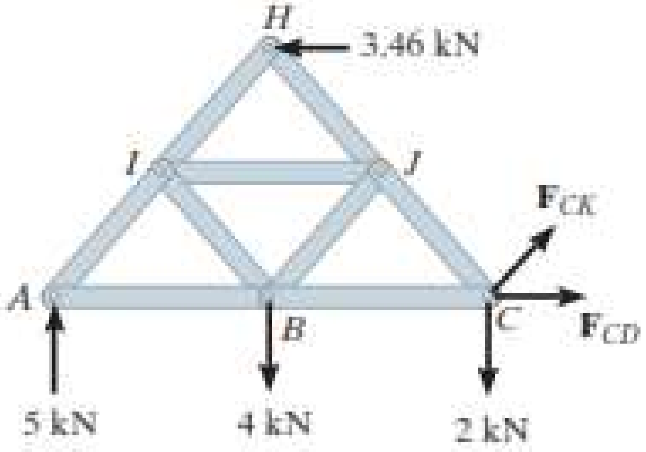
SOLUTION

The truss is a compound truss since the simple trusses *ACH* and *CEG* are connected by the pin at *C* and the bar *HG*.

Section *aa* in Fig. 3–29*a* cuts through bar *HG* and two other members having unknown forces. A free-body diagram for the left part is shown in Fig. 3–29*b*. The force in *HG* is determined as follows:

$$\circlearrowleft + \Sigma M_C = 0; \quad -5(4) + 4(2) + F_{HG}(4 \sin 60^\circ) = 0$$

(b)



(c)

Fig. 3–29

$$\circlearrowleft + \Sigma M_C = 0; \quad -5(4) + 4(2) + F_{HG}(4 \sin 60^\circ) = 0$$

$$F_{HG} = 3.46 \text{ kN (C)}$$

We can now proceed to determine the force in each member of the simple trusses using the method of joints. For example, the free-body diagram of ACH is shown in Fig. 3–29c. The joints of this truss can be analyzed in the following sequence:

Joint A: Determine the force in AB and AI.
Joint H: Determine the force in HI and HJ.
Joint I: Determine the force in IJ and IB.
Joint B: Determine the force in BC and BJ.
Joint J: Determine the force in JC.

EXAMPLE 3.9

Compound roof trusses are used in a garden center, as shown in the photo. They have the dimensions and loading shown in Fig. 3–30*a*. Indicate how to analyze this truss.

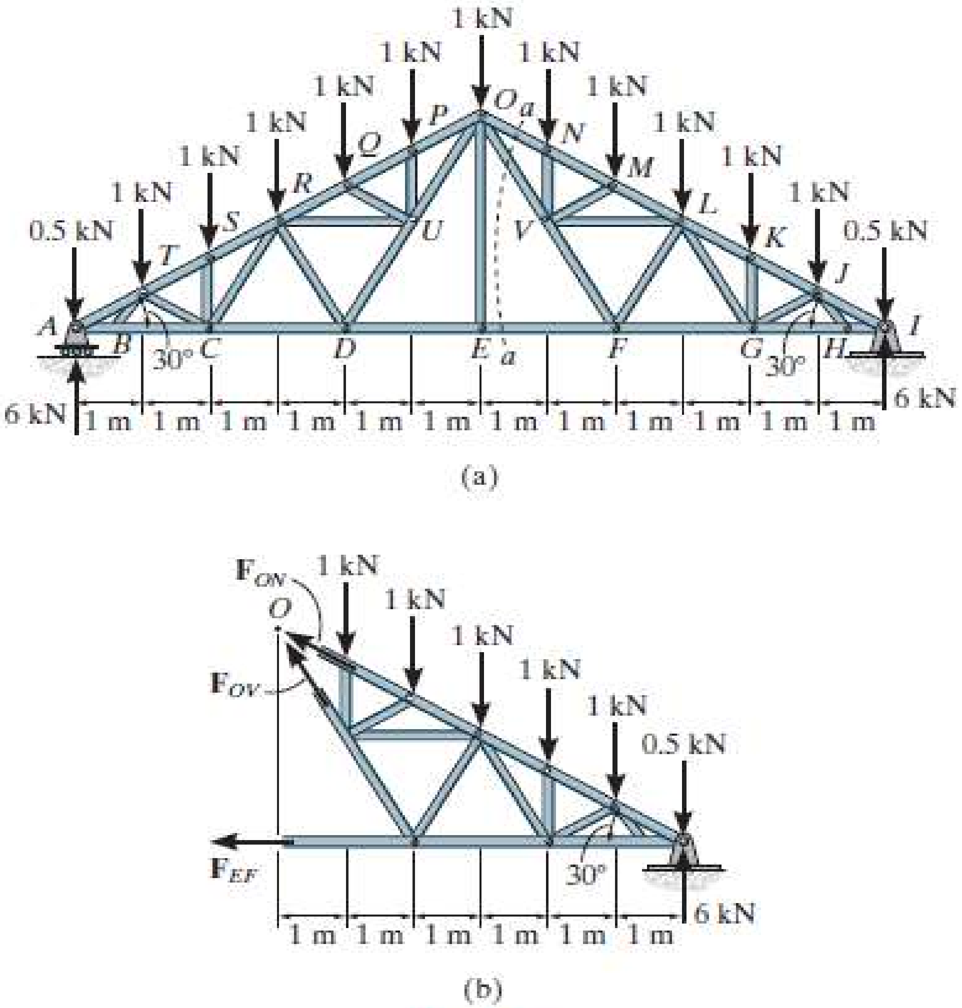


Fig. 3–30

SOLUTION

We can obtain the force in EF by using section aa in Fig. 3–30a. The free-body diagram of the right segment is shown in Fig. 3–30b

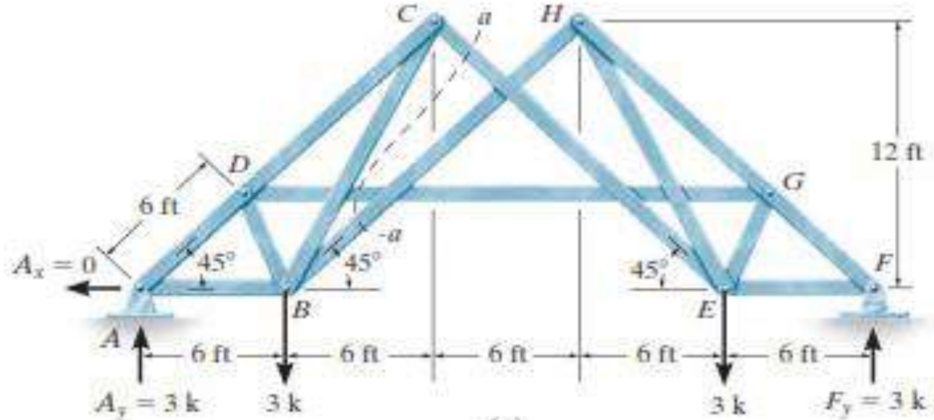
$$\circlearrowright + \Sigma M_O = 0; \quad -1(1) - 1(2) - 1(3) - 1(4) - 1(5) - 0.5(6) + 6(6) - F_{EF}(6 \tan 30^\circ) = 0$$

$$F_{EF} = 5.20 \text{ kN (T)} \qquad \textit{Ans.}$$

By inspection notice that BT, EO, and HJ are zero-force members since $+\uparrow \Sigma F_y = 0$ at joints B, E, and H, respectively. Also, by applying $+\nwarrow \Sigma F_y = 0$ (perpendicular to AO) at joints P, Q, S, and T, we can directly determine the force in members PU, QU, SC, and TC, respectively.

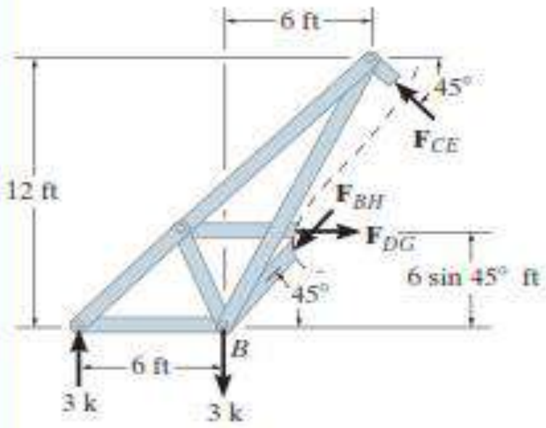
EXAMPLE 3.10

Indicate how to analyze the compound truss shown in Fig. 3–31*a*. The reactions at the supports have been calculated.



(a)

Fig. 3–31



(b)

SOLUTION

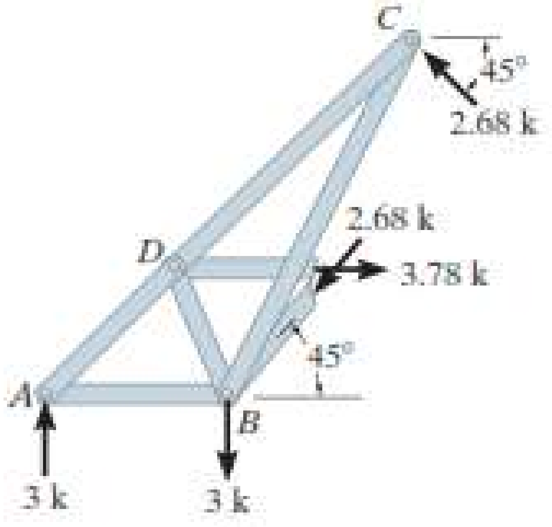
The truss may be classified as a type 2 compound truss since the simple trusses *ABCD* and *FEHG* are connected by three nonparallel or nonconcurrent bars, namely, *CE*, *BH*, and *DG*.

Using section *aa* in Fig. 3–31*a* we can determine the force in each connecting bar. The free-body diagram of the left part of this section is shown in Fig. 3–31*b*. Hence,

$$\circlearrowleft + \Sigma M_B = 0; \quad -3(6) - F_{DG}(6 \sin 45^\circ) + F_{CE} \cos 45^\circ(12) + F_{CE} \sin 45^\circ(6) = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \quad 3 - 3 - F_{BH} \sin 45^\circ + F_{CE} \sin 45^\circ = 0 \qquad (2)$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad -F_{BH} \cos 45^\circ + F_{DG} - F_{CE} \cos 45^\circ = 0 \qquad (3)$$



(c)

From Eq. (2), $F_{BH} = F_{CE}$; then solving Eqs. (1) and (3) simultaneously yields

$$F_{BH} = F_{CE} = 2.68 \text{ k (C)} \qquad F_{DG} = 3.78 \text{ k (T)}$$

Analysis of each connected simple truss can now be performed using the method of joints. For example, from Fig. 3–31c, this can be done in the following sequence.

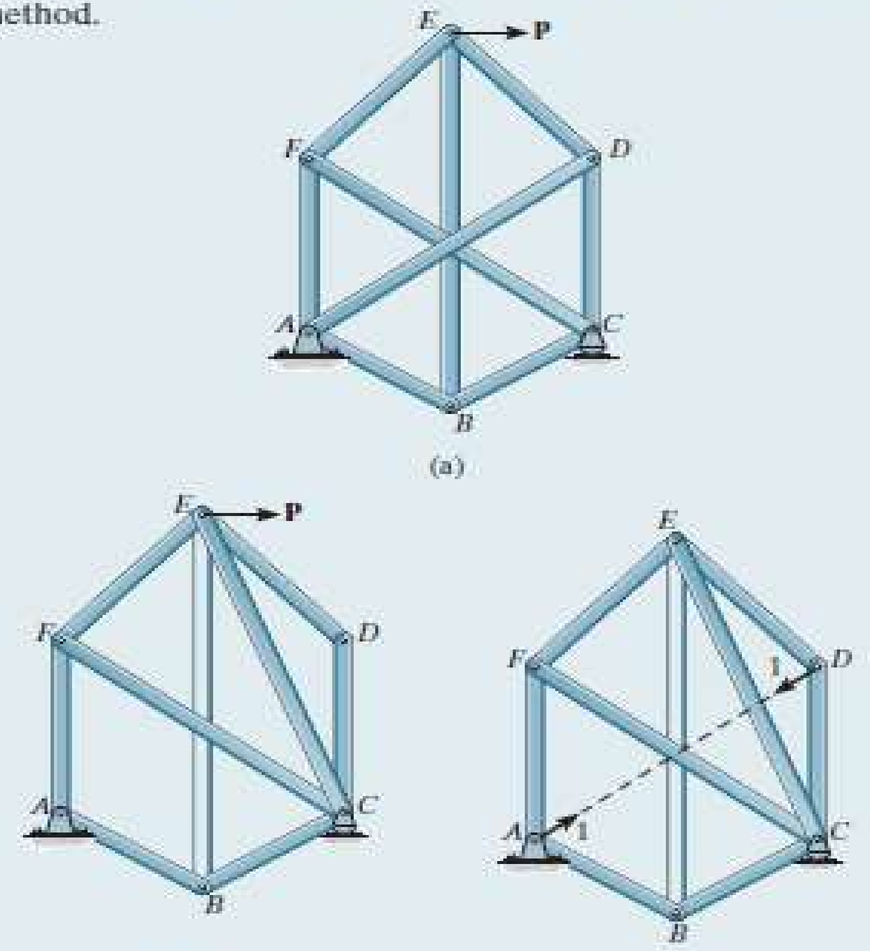
Joint A: Determine the force in AB and AD.
Joint D: Determine the force in DC and DB.
Joint C: Determine the force in CB.

3.7 Complex Trusses

The member forces in a complex truss can be determined using the method of joints; however, the solution will require writing the two equilibrium equations for each of the j joints of the truss and then solving the complete set of $2j$ equations *simultaneously*.* This approach may be impractical for hand calculations, especially in the case of large trusses. Therefore, a more direct method for analyzing a complex truss, referred to as the *method of substitute members*, will be presented here.

Procedure for Analysis

With reference to the truss in Fig. 3–32*a*, the following steps are necessary to solve for the member forces using the substitute-member method.



(a)

S_i' forces
(b)

s_i forces
(c)

Fig. 3–32

Reduction to Stable Simple Truss

Determine the reactions at the supports and begin by imagining how to analyze the truss by the method of joints, i.e., progressing from joint to joint and solving for each member force. If a joint is reached where there are *three unknowns*, remove one of the members at the joint and replace it by an *imaginary* member elsewhere in the truss. By doing this, reconstruct the truss to be a stable simple truss.

For example, in Fig. 3–32*a* it is observed that each joint will have three *unknown* member forces acting on it. Hence we will remove member *AD* and replace it with the imaginary member *EC*, Fig. 3–32*b*. This truss can now be analyzed by the method of joints for the two types of loading that follow.

External Loading on Simple Truss

Load the simple truss with the actual loading **P**, then determine the force S_i' in each member *i*. In Fig. 3–32*b*, provided the reactions have been determined, one could start at joint *A* to determine the forces in *AB* and *AF*, then joint *F* to determine the forces in *FE* and *FC*, then joint *D* to determine the forces in *DE* and *DC* (both of which are zero), then joint *E* to determine *EB* and *EC*, and finally joint *B* to determine the force in *BC*.

Remove External Loading from Simple Truss

Consider the simple truss without the external load **P**. Place equal but opposite collinear *unit loads* on the truss at the two joints from which the member was removed. If these forces develop a force s_i in the ith truss member, then by proportion an unknown force x in the removed member would exert a force xs_i in the ith member.

From Fig. 3–32c the equal but opposite unit loads will create *no reactions* at A and C when the equations of equilibrium are applied to the entire truss. The s_i forces can be determined by analyzing the joints in the same sequence as before, namely, joint A, then joints F, D, E, and finally B.

Superposition

If the effects of the above two loadings are combined, the force in the ith member of the truss will be

$$S_i = S_i' + xs_i \qquad (1)$$

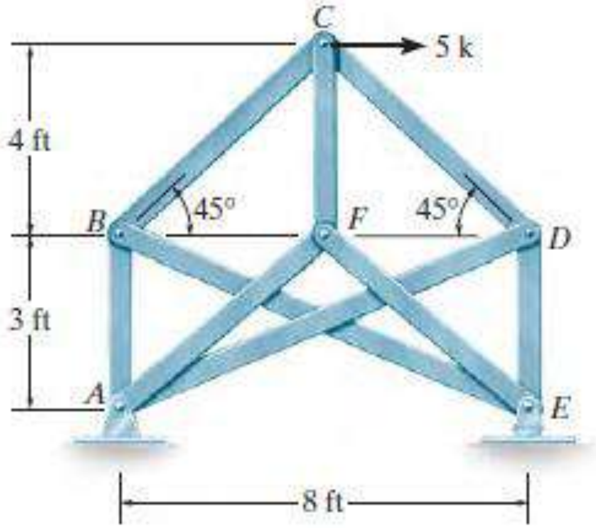
In particular, for the substituted member EC in Fig. 3–32b the force $S_{EC} = S'_{EC} + xs_{EC}$. Since member EC does not actually exist on the original truss, we will choose x to have a magnitude such that it yields *zero force* in EC. Hence,

$$S'_{EC} + xs_{EC} = 0 \qquad (2)$$

or $x = -S'_{EC}/s_{EC}$. Once the value of x has been determined, the force in the other members i of the complex truss can be determined from Eq. (1).

EXAMPLE 3.11

Determine the force in each member of the complex truss shown in Fig. 3–33*a*. Assume joints *B*, *F*, and *D* are on the same horizontal line. State whether the members are in tension or compression.

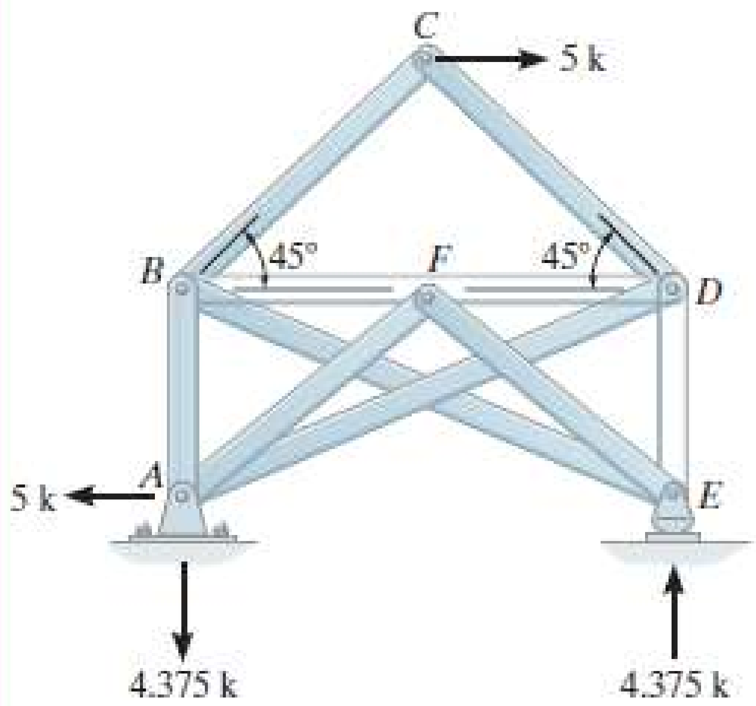


(a)

Fig. 3–33

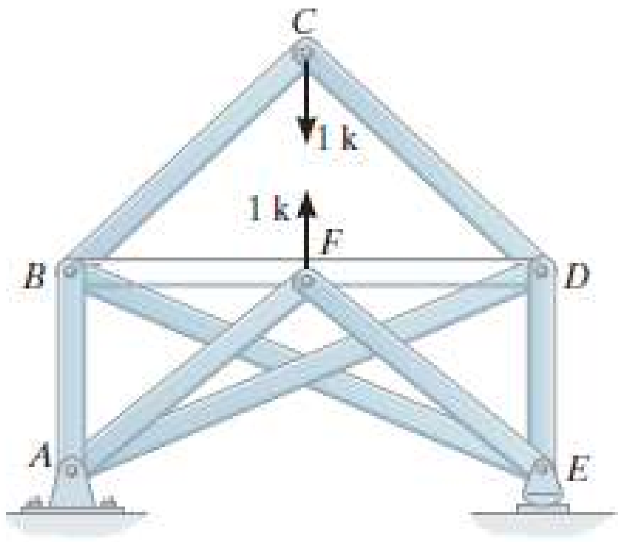
SOLUTION

Reduction to Stable Simple Truss. By inspection, each joint has three unknown member forces. A joint analysis can be performed by hand if, for example, member *CF* is removed and member *DB* substituted, Fig. 3–33*b*. The resulting truss is stable and will not collapse.



(b)

External Loading on Simple Truss. As shown in Fig. 3–33*b*, the support reactions on the truss have been determined. Using the method of joints, we can first analyze joint *C* to find the forces in members *CB* and *CD*; then joint *F*, where it is seen that *FA* and *FE* are zero-force members; then joint *E* to determine the forces in members *EB* and *ED*; then joint *D* to determine the forces in *DA* and *DB*; then finally joint *B* to determine the force in *BA*. Considering tension as positive and compression as negative, these S_i' forces are recorded in column 2 of Table 1.



(c)

Remove External Loading from Simple Truss. The unit load acting on the truss is shown in Fig. 3–33*c*. These equal but opposite forces create no external reactions on the truss. The joint analysis follows the same sequence as discussed previously, namely, joints C, F, E, D, and B. The results of the s_i force analysis are recorded in column 3 of Table 1.

Superposition. We require

$$S_{DB} = S'_{DB} + xs_{DB} = 0$$

Substituting the data for S'_{DB} and s_{DB}, where S'_{DB} is negative since the force is compressive, we have

$$-2.50 + x(1.167) = 0 \qquad x = 2.143$$

The values of xs_i are recorded in column 4 of Table 1, and the actual member forces $S_i = S'_i + xs_i$ are listed in column 5.

TABLE 1

Member	S'_i	s_i	xs_i	S_i
CB	3.54	−0.707	−1.52	2.02 (T)
CD	−3.54	−0.707	−1.52	5.05 (C)
FA	0	0.833	1.79	1.79 (T)
FE	0	0.833	1.79	1.79 (T)
EB	0	−0.712	−1.53	1.53 (C)
ED	−4.38	−0.250	−0.536	4.91 (C)
DA	5.34	−0.712	−1.53	3.81 (T)
DB	−2.50	1.167	2.50	0
BA	2.50	−0.250	−0.536	1.96 (T)
CB				2.14 (T)

3.8 Space Trusses

A *space truss* consists of members joined together at their ends to form a stable three-dimensional structure. In Sec. 3–2 it was shown that the simplest form of a stable two-dimensional truss consists of the members arranged in the form of a triangle. We then built up the simple plane truss from this basic triangular element by adding two members at a time to form further elements. In a similar manner, the simplest element of a stable space truss is a *tetrahedron*, formed by connecting six members together with four joints as shown in Fig. 3–34. Any additional members added to this basic element would be redundant in supporting the force **P**. A simple space truss can be built from this basic tetrahedral element by adding three additional members and another joint forming multiconnected tetrahedrons.

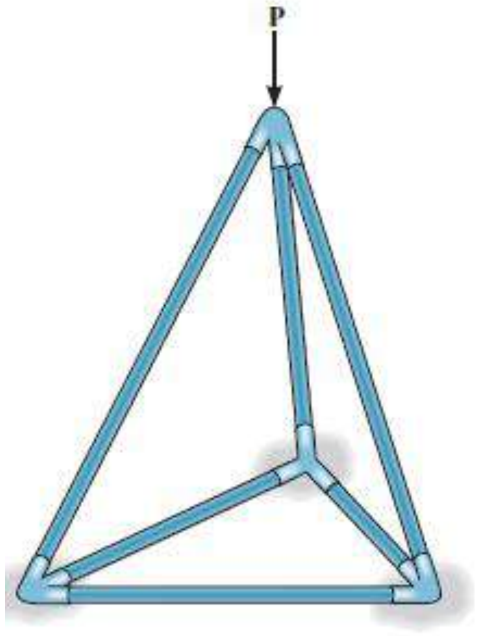


Fig. 3–34

Determinacy and Stability. Realizing that in three dimensions there are three equations of equilibrium available for each joint ($\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$), then for a space truss with j number of joints, $3j$ equations are available. If the truss has b number of bars and r number of reactions, then like the case of a planar truss (Eqs. 3–1 and 3–2) we can write

$b + r < 3j$	unstable truss	
$b + r = 3j$	statically determinate—check stability	(3–3)
$b + r > 3j$	statically indeterminate—check stability	

The *external stability* of the space truss requires that the support reactions keep the truss in force and moment equilibrium about any and all axes. This can sometimes be checked by inspection, although if the truss is unstable a solution of the equilibrium equations will give inconsistent results. *Internal stability* can sometimes be checked by careful inspection of the member arrangement. Provided each joint is held fixed by its supports or connecting members, so that it cannot move with respect to the other joints, the truss can be classified as internally stable. Also, if we do a force analysis of the truss and obtain inconsistent results, then the truss configuration will be unstable or have a "critical form."

The roof of this pavilion is supported using a system of space trusses.

Assumptions for Design. The members of a space truss may be treated as axial-force members provided the external loading is applied at the joints and the joints consist of ball-and-socket connections. This assumption is justified provided the joined members at a connection intersect at a common point and the weight of the members can be neglected. In cases where the weight of a member is to be included in the analysis, it is generally satisfactory to apply it as a vertical force, half of its magnitude applied to each end of the member.

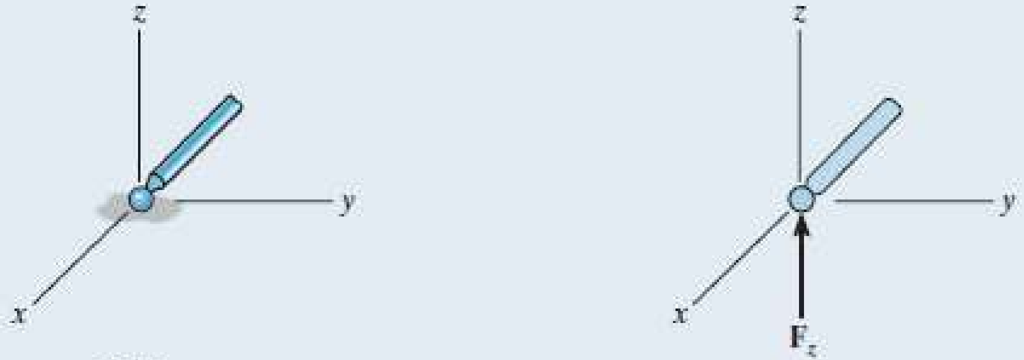
For the force analysis the supports of a space truss are generally modeled as a short link, plane roller joint, slotted roller joint, or a ball-and-socket joint. Each of these supports and their reactive force components are shown in Table 3–1.

TABLE 3–1 Supports and Their Reactive Force Components

(1)

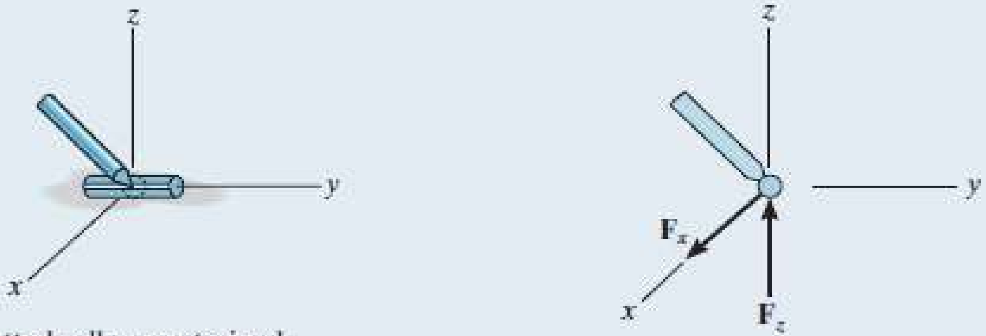
short link

(2)



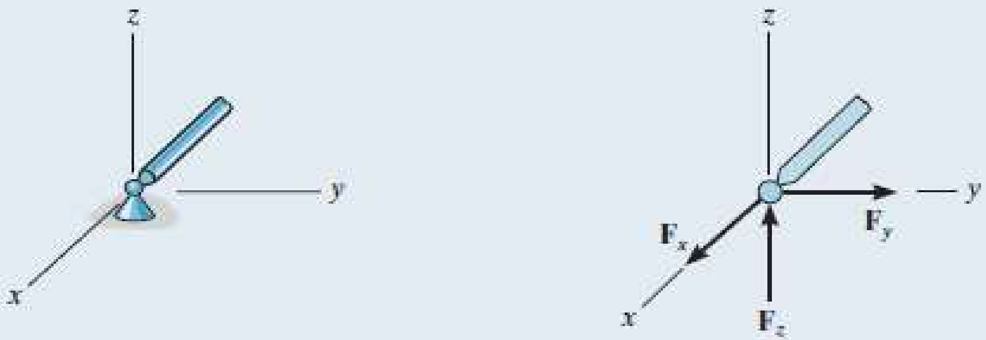
roller

(3)



slotted roller constrained
in a cylinder

(4)



ball-and-socket

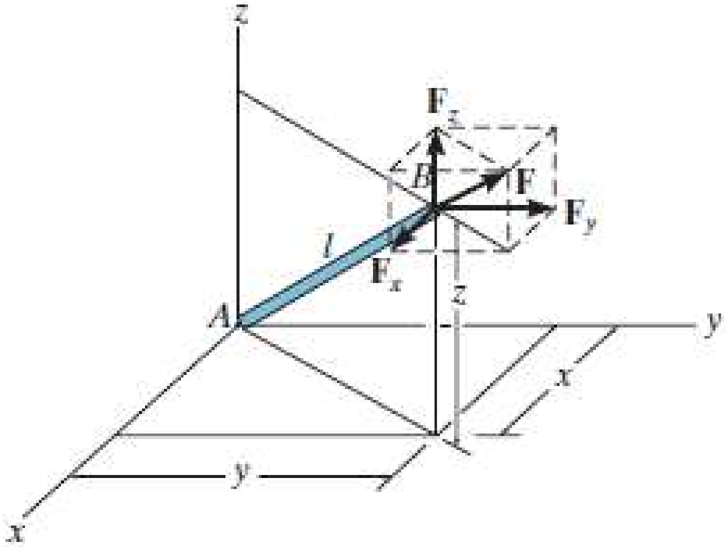


Fig. 3–35

x, y, z, Force Components. Since the analysis of a space truss is three-dimensional, it will often be necessary to resolve the force **F** in a member into components acting along the x, y, z axes. For example, in Fig. 3–35 member AB has a length l and *known* projections x, y, z along the coordinate axes. These projections can be related to the member's length by the equation

$$l = \sqrt{x^2 + y^2 + z^2} \tag{3–4}$$

Since the force **F** acts along the axis of the member, then the components of **F** can be determined by *proportion* as follows:

$$F_x = F\left(\frac{x}{l}\right) \qquad F_y = F\left(\frac{y}{l}\right) \qquad F_z = F\left(\frac{z}{l}\right) \tag{3–5}$$

Because of their cost effectiveness, towers such as these are often used to support multiple electric transmission lines.

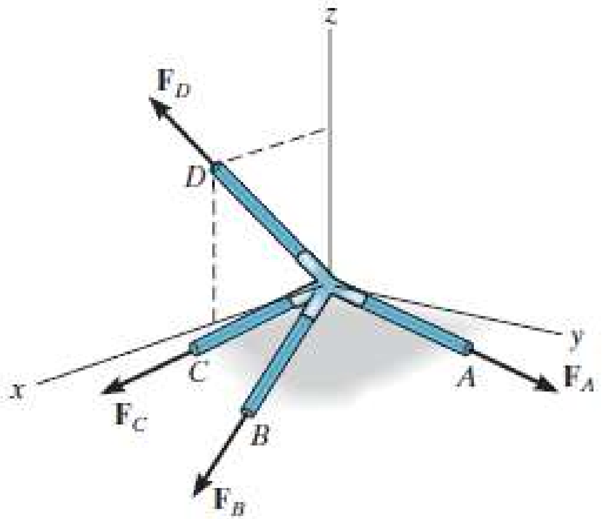
Notice that this requires

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} \tag{3–6}$$

Use of these equations will be illustrated in Example 3–12.

Zero-Force Members. In some cases the joint analysis of a truss can be simplified if one is able to spot the zero-force members by recognizing two common cases of joint geometry.

Case 1. If all but one of the members connected to a joint lie in the same plane, and provided no external load acts on the joint, then the member not lying in the plane of the other members must be subjected to zero force. The proof of this statement is shown in Fig. 3–36, where members A, B, C lie in the x–y plane. Since the z component of $\mathbf{F}_D$ must be zero to satisfy $\Sigma F_z = 0$, member D must be a zero-force member. By the same reasoning, member D will carry a load that can be determined from $\Sigma F_z = 0$ if an external force acts on the joint and has a component acting along the z axis.



Case 2. If it has been determined that all but two of several members connected at a joint support zero force, then the two remaining members must also support zero force, provided they do not lie along the same line. This situation is illustrated in Fig. 3–37, where it is known that A and C are zero-force members. Since $\mathbf{F}_D$ is collinear with the y axis, then application of $\Sigma F_x = 0$ or $\Sigma F_z = 0$ requires the x or z component of $\mathbf{F}_B$ to be zero. Consequently, $F_B = 0$. This being the case, $F_D = 0$ since $\Sigma F_y = 0$.

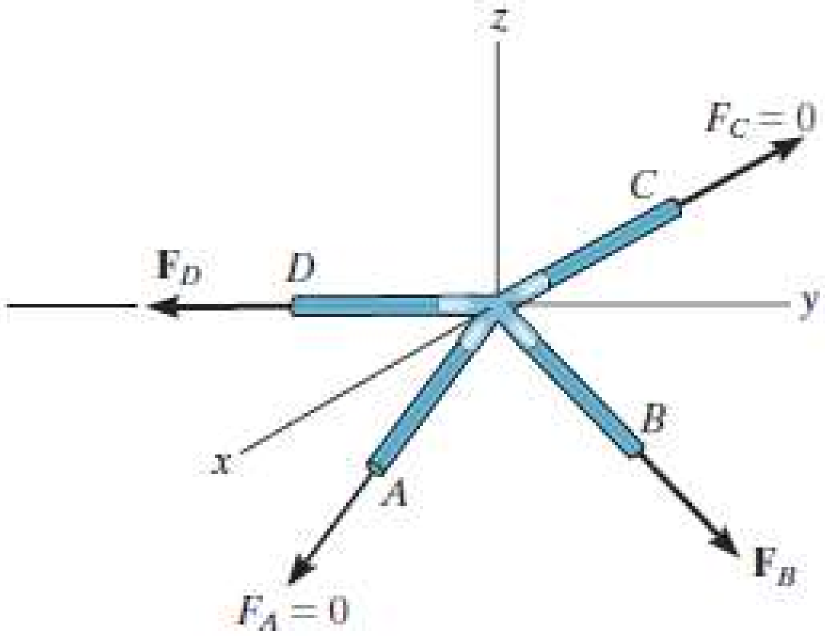


Fig. 3–37

Particular attention should be directed to the foregoing two cases of joint geometry and loading, since the analysis of a space truss can be considerably simplified by first spotting the zero-force members.

Procedure for Analysis

Either the method of sections or the method of joints can be used to determine the forces developed in the members of a space truss.

Method of Sections

If only a *few* member forces are to be determined, the method of sections may be used. When an imaginary section is passed through a truss and the truss is separated into two parts, the force system acting on either one of the parts must satisfy the six scalar equilibrium equations: $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$, $\Sigma M_x = 0$, $\Sigma M_y = 0$, $\Sigma M_z = 0$. By proper choice of the section and axes for summing forces and moments, many of the unknown member forces in a space truss can be computed *directly*, using a single equilibrium equation. In this regard, recall that the *moment* of a force about an axis is *zero* provided *the force is parallel to the axis or its line of action passes through a point on the axis*.

Method of Joints

Generally, if the forces in *all* the members of the truss must be determined, the method of joints is most suitable for the analysis. When using the method of joints, it is necessary to solve the three scalar equilibrium equations $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$ at each joint. Since it is relatively easy to draw the free-body diagrams and apply the equations of equilibrium, the method of joints is very consistent in its application.

EXAMPLE 3.12

Determine the force in each member of the space truss shown in Fig. 3–38*a*. The truss is supported by a ball-and-socket joint at *A*, a slotted roller joint at *B*, and a cable at *C*.

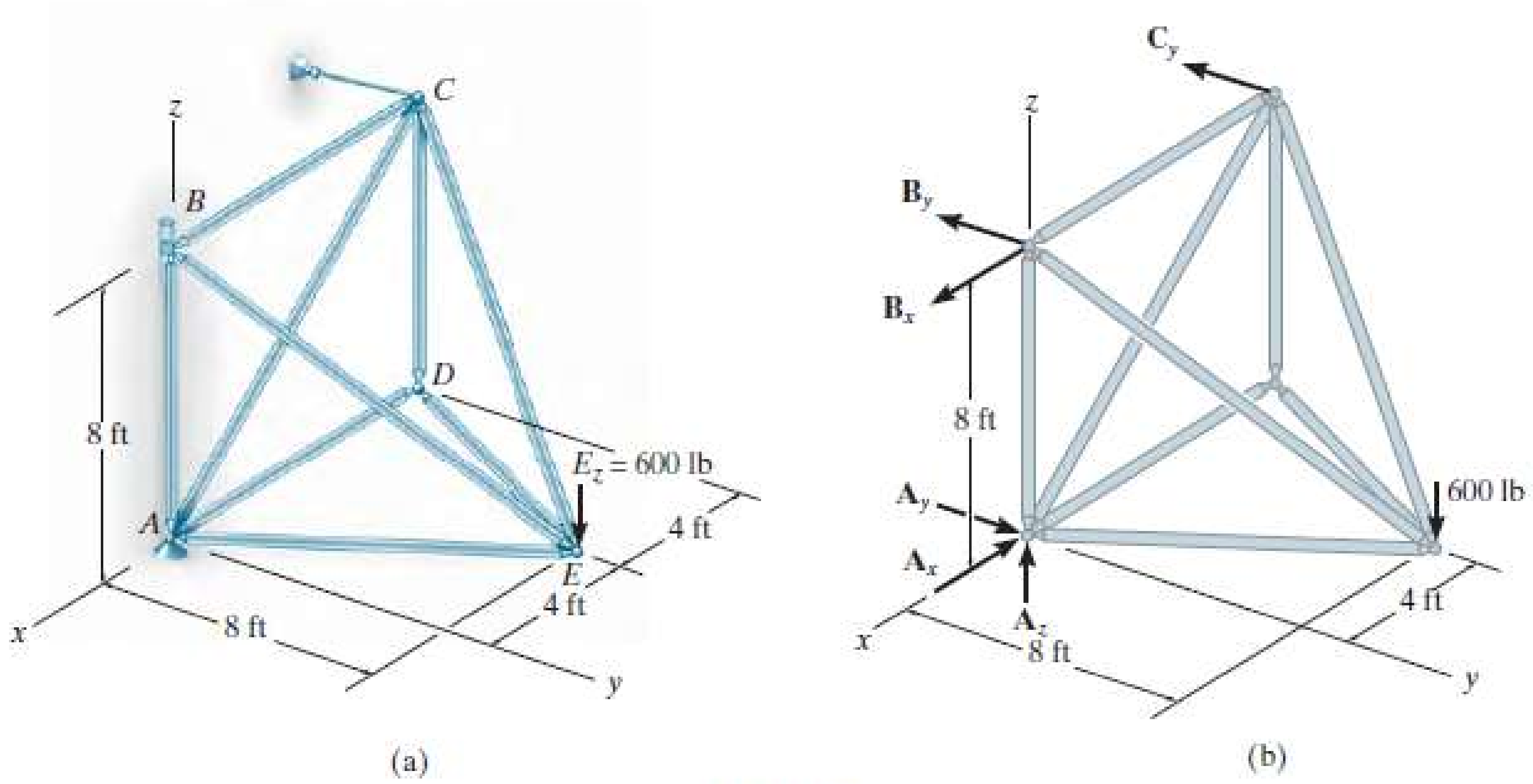


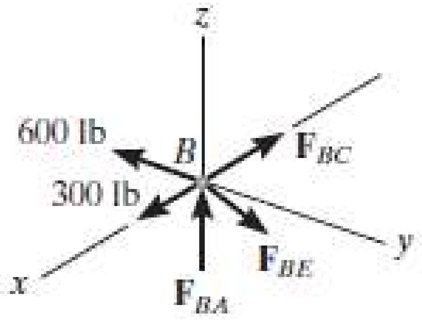
Fig. 3–38

SOLUTION

The truss is statically determinate since $b + r = 3j$ or $9 + 6 = 3(5)$, Fig. 3–38*b*.

Support Reactions. We can obtain the support reactions from the free-body diagram of the entire truss, Fig. 3–38*b*, as follows:

$$
\begin{array}{lrl}
\Sigma M_y = 0; & -600(4) + B_x(8) = 0 & B_x = 300 \text{ lb} \\
\Sigma M_z = 0; & C_y = 0 & \\
\Sigma M_x = 0; & B_y(8) - 600(8) = 0 & B_y = 600 \text{ lb} \\
\Sigma F_x = 0; & 300 - A_x = 0 & A_x = 300 \text{ lb} \\
\Sigma F_y = 0; & A_y - 600 = 0 & A_y = 600 \text{ lb} \\
\Sigma F_z = 0; & A_z - 600 = 0 & A_z = 600 \text{ lb}
\end{array}
$$



(c)

Joint B. We can begin the method of joints at B since there are three unknown member forces at this joint, Fig. 3–38*c*. The components of $\mathbf{F}_{BE}$ can be determined by proportion to the length of member BE, as indicated by Eqs. 3–5. We have

$$\Sigma F_y = 0; \qquad -600 + F_{BE}\left(\tfrac{8}{12}\right) = 0 \qquad F_{BE} = 900 \text{ lb (T)} \qquad \textit{Ans.}$$

$$\Sigma F_x = 0; \qquad 300 - F_{BC} - 900\left(\tfrac{4}{12}\right) = 0 \qquad F_{BC} = 0 \qquad \textit{Ans.}$$

$$\Sigma F_z = 0; \qquad F_{BA} - 900\left(\tfrac{8}{12}\right) = 0 \qquad F_{BA} = 600 \text{ lb (C)} \qquad \textit{Ans.}$$

z

Joint A. Using the result for $F_{BA} = 600$ lb (C), the free-body diagram of joint A is shown in Fig. 3–38d. We have

$$\Sigma F_z = 0; \qquad 600 - 600 + F_{AC} \sin 45^\circ = 0$$

$$F_{AC} = 0 \qquad \textit{Ans.}$$

$$\Sigma F_y = 0; \qquad -F_{AE}\left(\tfrac{2}{\sqrt{5}}\right) + 600 = 0$$

$$F_{AE} = 670.8 \text{ lb (C)} \qquad \textit{Ans.}$$

$$\Sigma F_x = 0; \qquad -300 + F_{AD} + 670.8\left(\tfrac{1}{\sqrt{5}}\right) = 0$$

$$F_{AD} = 0 \qquad \textit{Ans.}$$

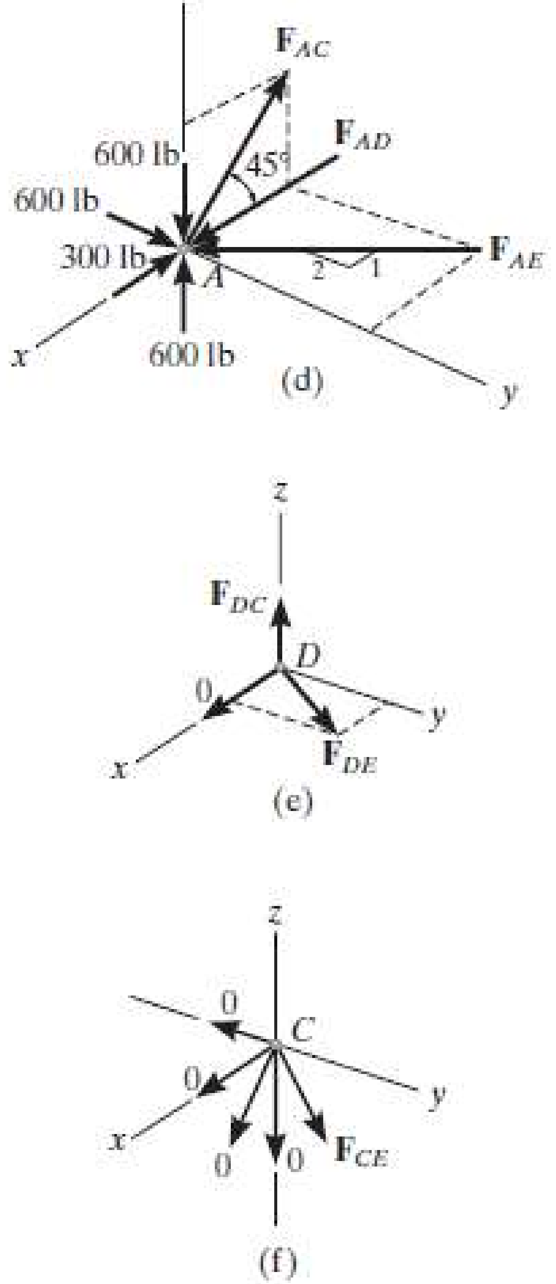
Joint D. By inspection the members at joint D, Fig. 3–38a, support zero force, since the arrangement of the members is similar to either of the two cases discussed in reference to Figs. 3–36 and 3–37. Also, from Fig. 3–38e,

$$\Sigma F_x = 0; \qquad F_{DE} = 0 \qquad \textit{Ans.}$$

$$\Sigma F_z = 0; \qquad F_{DC} = 0 \qquad \textit{Ans.}$$

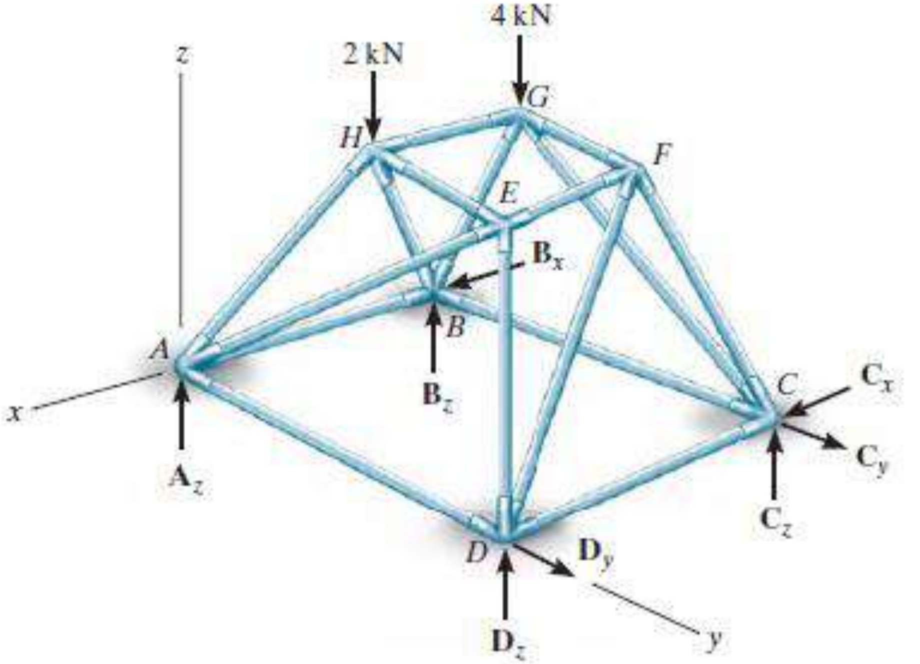
Joint C. By observation of the free-body diagram, Fig. 3–38f,

$$F_{CE} = 0 \qquad \textit{Ans.}$$



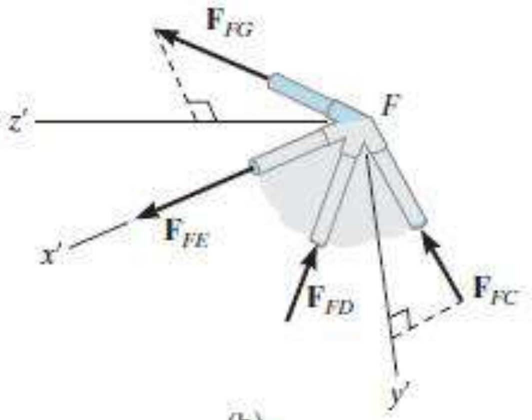
EXAMPLE 3.13

Determine the zero-force members of the truss shown in Fig. 3–39a. The supports exert components of reaction on the truss as shown.



(a)

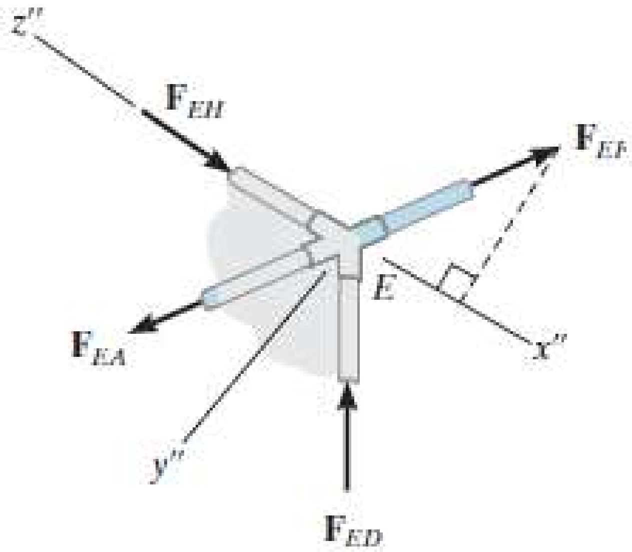
Fig. 3–39



(b)

SOLUTION

The free-body diagram, Fig. 3–39a, indicates there are eight unknown reactions for which only six equations of equilibrium are available for solution. Although this is the case, the reactions can be determined, since $b + r = 3j$ or $16 + 8 = 3(8)$.



(c)

To spot the zero-force members, we must compare the conditions of joint geometry and loading to those of Figs. 3–36 and 3–37. Consider joint F, Fig. 3–39b. Since members FC, FD, FE lie in the x'–y' plane and FG is not in this plane, FG is a zero-force member. ($\Sigma F_{z'} = 0$ must be satisfied.) In the same manner, from joint E, Fig. 3–39c, *EF is a zero-force member*, since it does not lie in the y''–z'' plane. ($\Sigma F_{x''} = 0$ must be satisfied.) Returning to joint F, Fig. 3–39b, it can be seen that $F_{FD} = F_{FC} = 0$, since $F_{FE} = F_{FG} = 0$, and there are no external forces acting on the joint. Use this procedure to show that AB is a zero force member.

The numerical force analysis of the joints can now proceed by analyzing joint G ($F_{GF} = 0$) to determine the forces in GH, GB, GC. Then analyze joint H to determine the forces in HE, HB, HA; joint E to determine the forces in EA, ED; joint A to determine the forces in AB, AD, and A_z; joint B to determine the force in BC and B_x, B_z; joint D to determine the force in DC and D_y, D_z; and finally, joint C to determine C_x, C_y, C_z.

Internal Loadings Developed in Structural Members

4

4.1 Internal Loadings at a Specified Point

The internal load at a specified point in a member can be determined by using the *method of sections*. In general, this loading for a coplanar structure will consist of a normal force **N**, shear force **V**, and bending moment **M**.

Sign Convention. Before presenting a method for finding the internal normal force, shear force, and bending moment, we will need to establish a sign convention to define their "positive" and "negative" values.

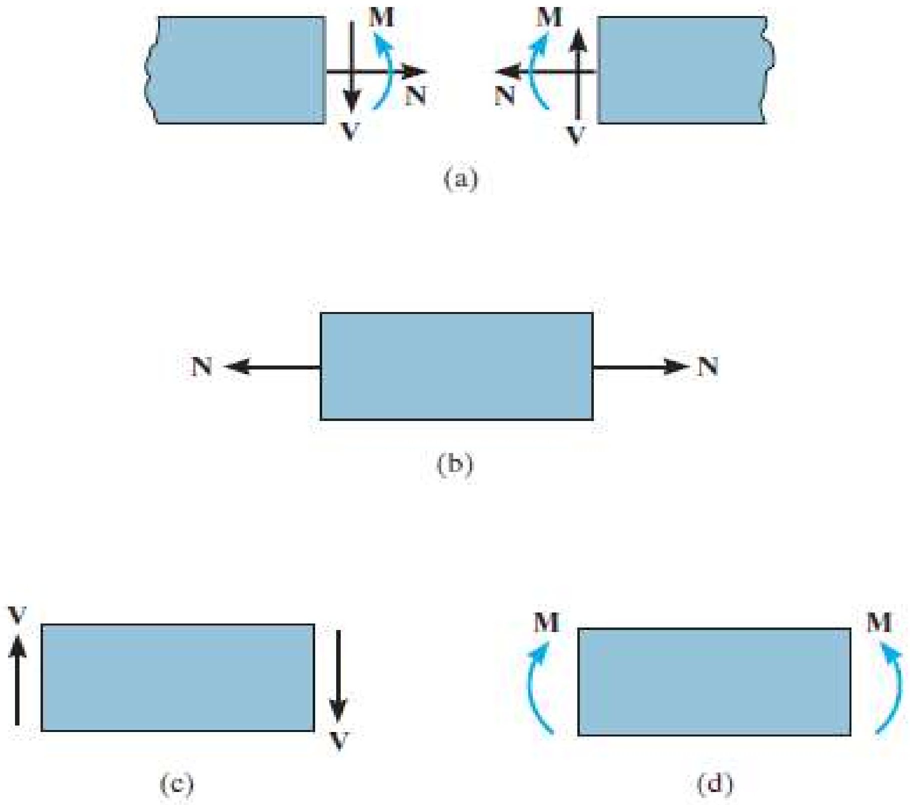


Fig. 4–1

Procedure for Analysis

The following procedure provides a means for applying the method of sections to determine the internal normal force, shear force, and bending moment at a specific location in a structural member.

Support Reactions

- Before the member is "cut" or sectioned, it may be necessary to determine the member's support reactions so that the equilibrium equations are used only to solve for the internal loadings when the member is sectioned.
- If the member is part of a pin-connected structure, the pin reactions can be determined using the methods of Sec. 2–5.

Free-Body Diagram

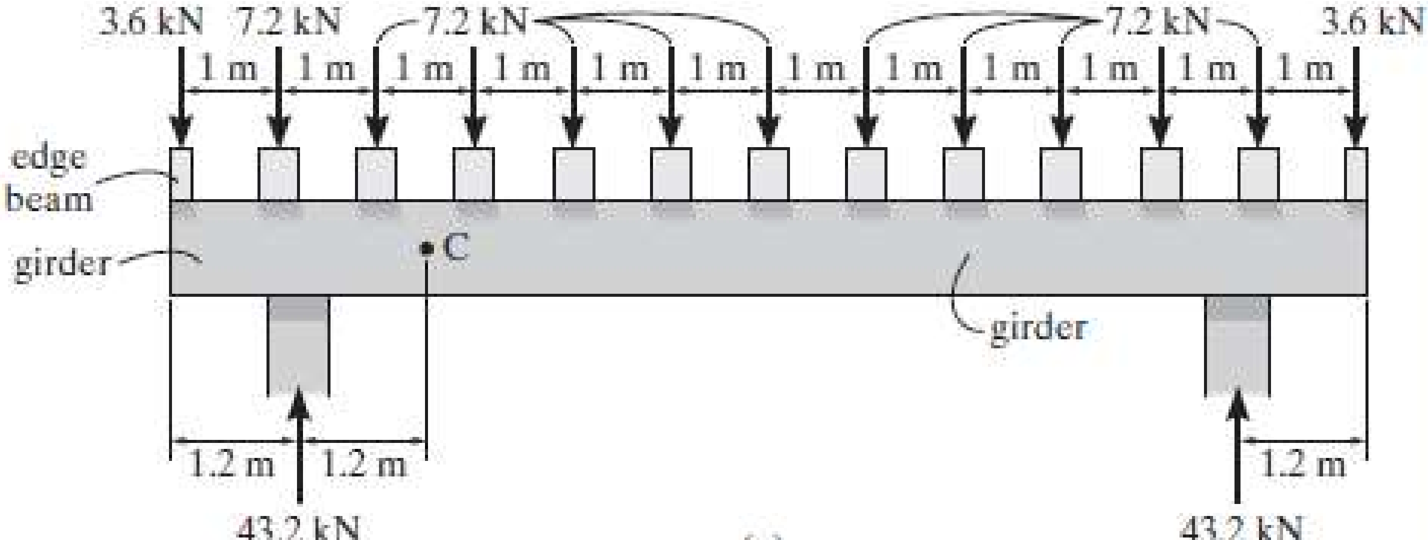
- Keep all distributed loadings, couple moments, and forces acting on the member in their *exact location*, then pass an imaginary section through the member, perpendicular to its axis at the point where the internal loading is to be determined.
- After the section is made, draw a free-body diagram of the segment that has the least number of loads on it. At the section indicate the unknown resultants **N**, **V**, and **M** acting in their *positive* directions (Fig. 4–1*a*).

Equations of Equilibrium

- Moments should be summed at the section about axes that pass through the *centroid* of the member's cross-sectional area, in order to eliminate the unknowns **N** and **V** and thereby obtain a direct solution for **M**.
- If the solution of the equilibrium equations yields a quantity having a negative magnitude, the assumed directional sense of the quantity is opposite to that shown on the free-body diagram.

EXAMPLE 4.1

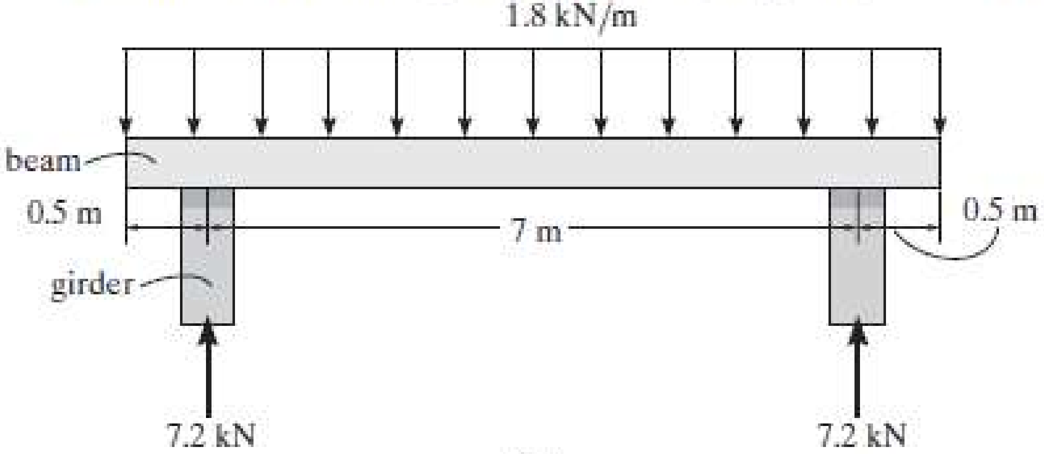
The building roof shown in the photo has a weight of 1.8 kN/m^2 and is supported on 8-m long simply supported beams that are spaced 1 m apart. Each beam, shown in Fig. 4–2*b* transmits its loading to two girders, located at the front and back of the building. Determine the internal shear and moment in the front girder at point *C*, Fig. 4–2*a*. Neglect the weight of the members.



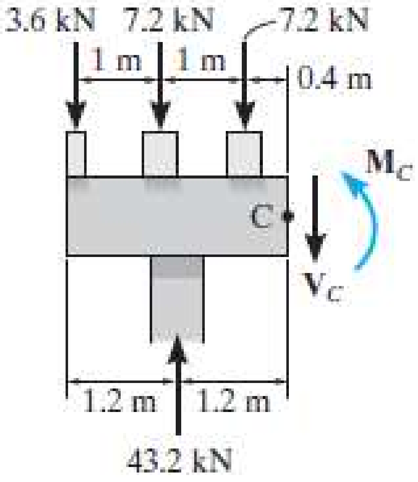
(a)

SOLUTION

Support Reactions. The roof loading is transmitted to each beam as a one-way slab ($L_2/L_1 = 8\text{ m}/1\text{ m} = 8 > 2$). The tributary loading on each interior beam is therefore $(1.8\text{ kN/m}^2)(1\text{ m}) = 1.8\text{ kN/m}$. (The two edge beams support 0.9 kN/m.) From Fig. 4–2*b*, the reaction of each interior beam on the girder is $(1.8\text{ kN/m})(8\text{ m})/2 = 7.2\text{ kN}$.



(b)



(c)

Fig. 4–2

Free-Body Diagram. The free-body diagram of the girder is shown in Fig. 4–2*a*. Notice that each column reaction is

$$[(2(3.6\text{ kN}) + 11(7.2\text{ kN})]/2 = 43.2\text{ kN}$$

The free-body diagram of the left girder segment is shown in Fig. 4–2*c*. Here the internal loadings are assumed to act in their positive directions.

Equations of Equilibrium

$$+\uparrow \Sigma F_y = 0; \quad 43.2 - 3.6 - 2(7.2) - V_C = 0 \qquad V_C = 25.2\text{ kN} \qquad \textit{Ans.}$$

$$\circlearrowleft + \Sigma M_C = 0; \quad M_C + 7.2(0.4) + 7.2(1.4) + 3.6(2.4) - 43.2(1.2) = 0 \qquad M_C = 30.2\text{ kN}\cdot\text{m} \qquad \textit{Ans.}$$

EXAMPLE 4.2

Determine the internal shear and moment acting at a section passing through point C in the beam shown in Fig. 4–3a.

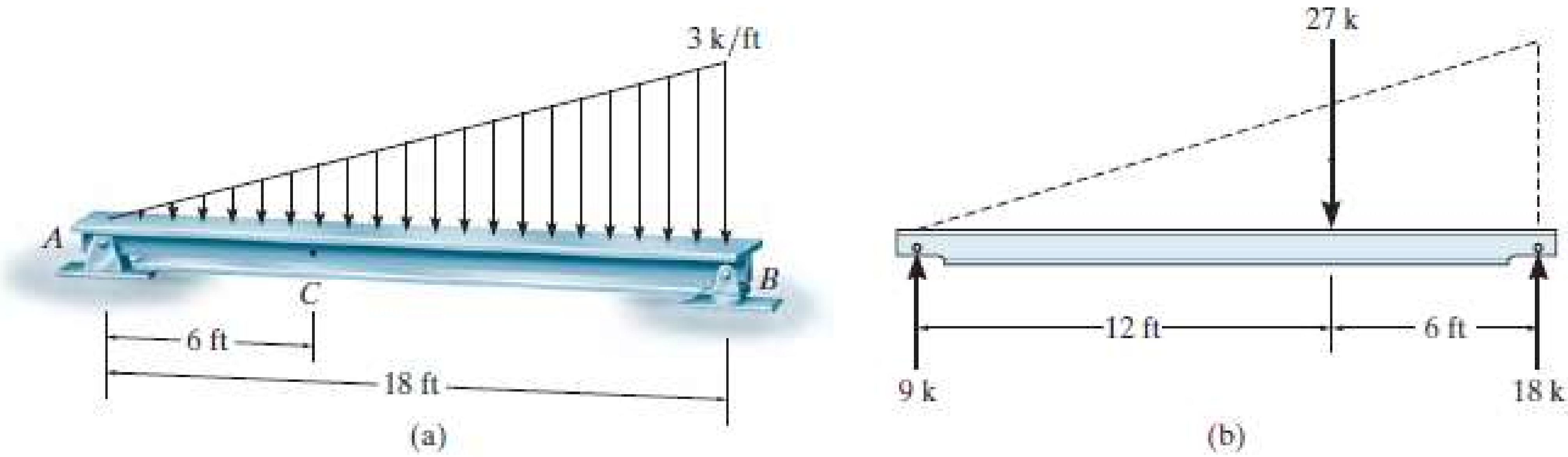


Fig. 4–3

SOLUTION

Support Reactions. Replacing the distributed load by its resultant force and computing the reactions yields the results shown in Fig. 4–3*b*.

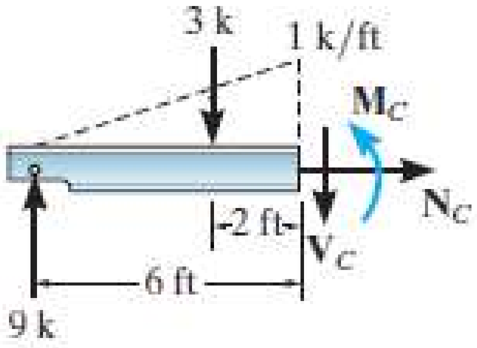
Free-Body Diagram. Segment *AC* will be considered since it yields the simplest solution, Fig. 4–3*c*. The distributed load intensity at *C* is computed by proportion, that is,

$$w_C = (6\text{ ft}/18\text{ ft})(3\text{ k/ft}) = 1\text{ k/ft}$$

Equations of Equilibrium.

$+\uparrow \Sigma F_y = 0; \qquad 9 - 3 - V_C = 0 \qquad V_C = 6\text{ k}$ *Ans.*

$\circlearrowleft + \Sigma M_C = 0; \qquad -9(6) + 3(2) + M_C = 0 \qquad M_C = 48\text{ k}\cdot\text{ft}$ *Ans.*

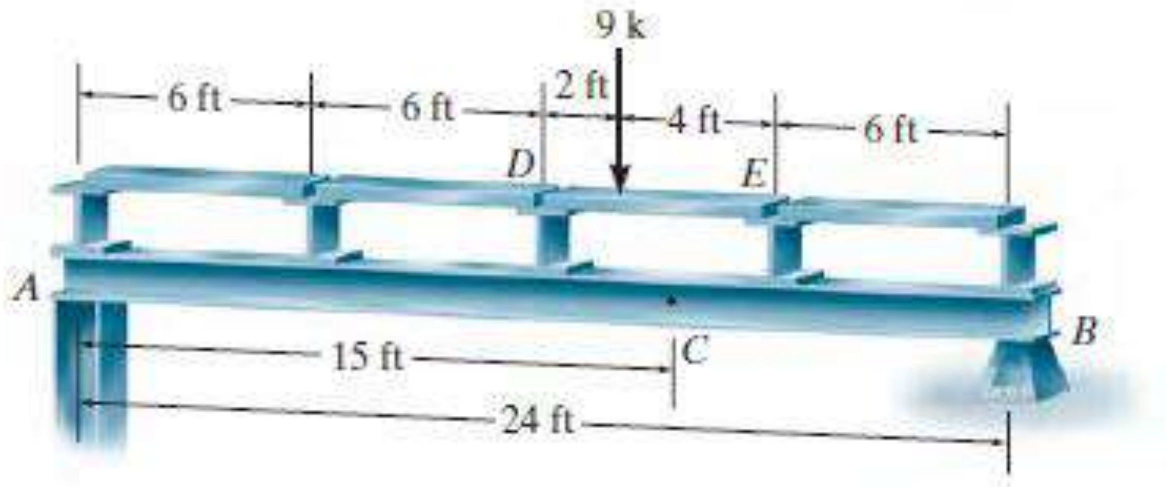


(c)

This problem illustrates the importance of *keeping* the distributed loading on the beam until *after* the beam is sectioned. If the beam in Fig. 4–3*b* were sectioned at *C*, the effect of the distributed load on segment *AC* would not be recognized, and the result $V_C = 9\text{ k}$ and $M_C = 54\text{ k}\cdot\text{ft}$ would be wrong.

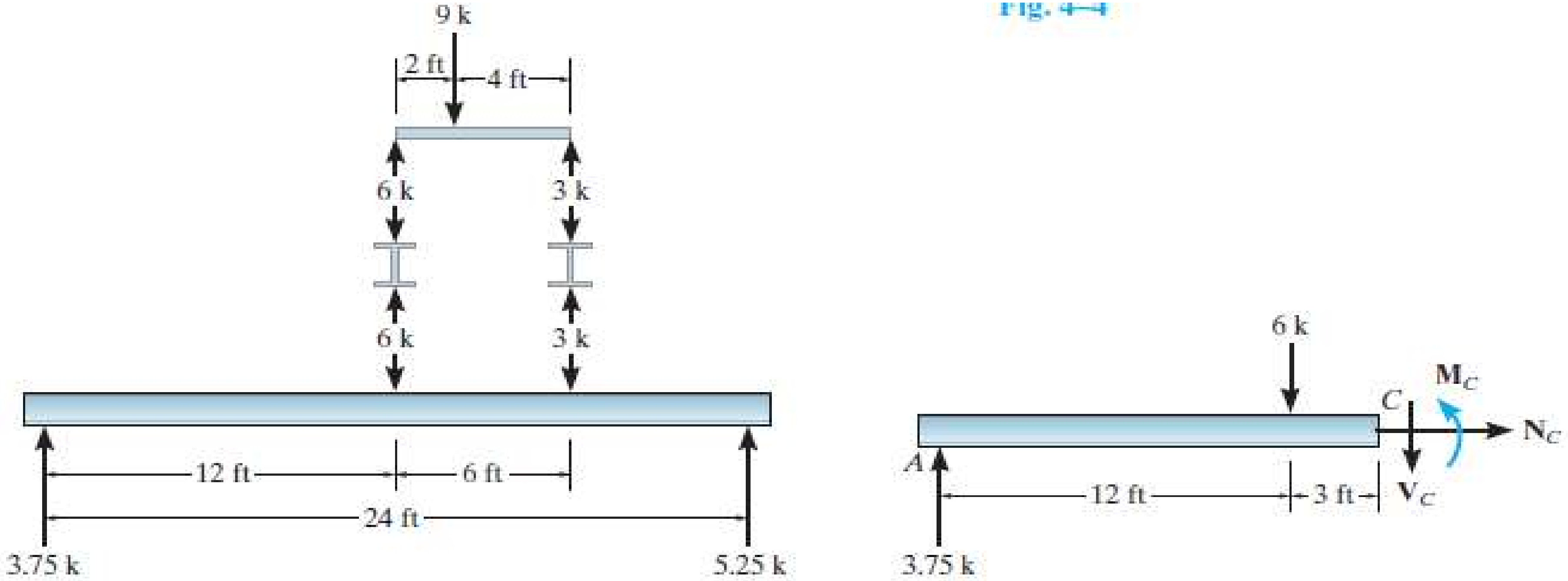
EXAMPLE 4.3

The 9-k force in Fig. 4–4*a* is supported by the floor panel *DE*, which in turn is simply supported at its ends by floor beams. These beams transmit their loads to the simply supported girder *AB*. Determine the internal shear and moment acting at point *C* in the girder.



(a)

Fig. 4–4



(b) (c)

SOLUTION

Support Reactions. Equilibrium of the floor panel, floor beams, and girder is shown in Fig. 4–4*b*. It is advisable to check these results.

Free-Body Diagram. The free-body diagram of segment *AC* of the girder will be used since it leads to the simplest solution, Fig. 4–4*c*. Note that there are *no loads* on the floor beams supported by *AC*.

Equations of Equilibrium.

$$+\uparrow \Sigma F_y = 0; \qquad 3.75 - 6 - V_C = 0 \qquad V_C = -2.25 \text{ k} \qquad \textit{Ans.}$$

$$\circlearrowleft + \Sigma M_C = 0; \qquad -3.75(15) + 6(3) + M_C = 0 \qquad M_C = 38.25 \text{ k} \cdot \text{ft} \qquad \textit{Ans.}$$

4.2 Shear and Moment Functions

Procedure for Analysis

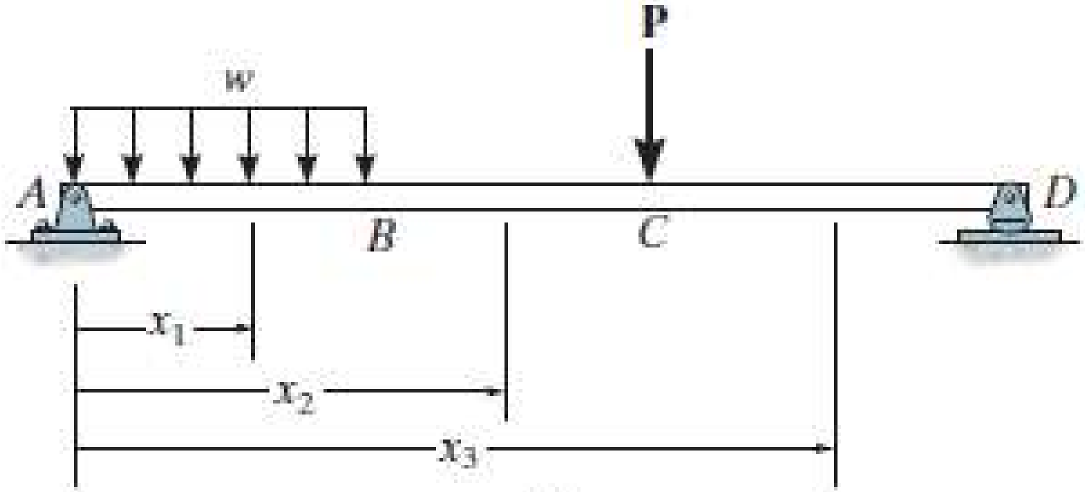
The following procedure provides a method for determining the variation of shear and moment in a beam as a function of position x.

Support Reactions

- Determine the support reactions on the beam and resolve all the external forces into components acting perpendicular and parallel to the beam's axis.

Shear and Moment Functions

- Specify separate coordinates x and associated origins, extending into regions of the beam between concentrated forces and/or couple moments, or where there is a discontinuity of distributed loading.
- Section the beam perpendicular to its axis at each distance x, and from the free-body diagram of one of the segments determine the unknowns V and M at the cut section as functions of x. On the free-body diagram, V and M should be shown acting in their *positive directions*, in accordance with the sign convention given in Fig. 4–1.
- V is obtained from $\Sigma F_y = 0$ and M is obtained by summing moments about the point S located at the cut section, $\Sigma M_S = 0$.
- The results can be checked by noting that $dM/dx = V$ and $dV/dx = w$, where w is positive when it acts upward, away from the beam. These relationships are developed in Sec. 4–3.



(a)

EXAMPLE 4.4

Determine the shear and moment in the beam shown in Fig. 4–6*a* as a function of x.

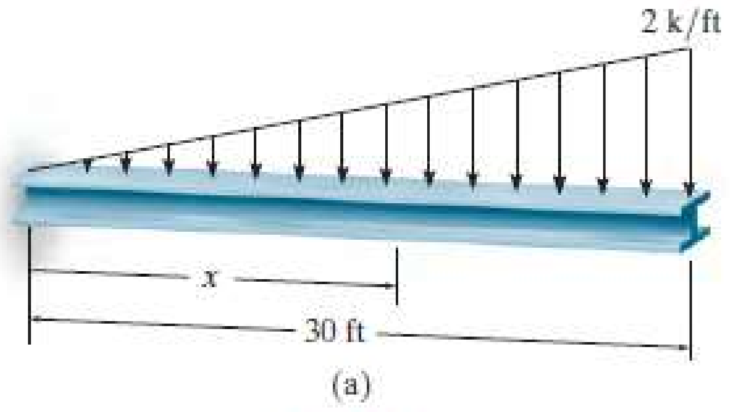
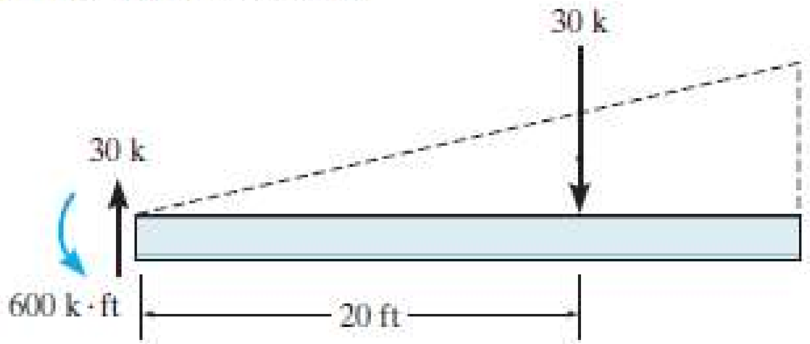


Fig. 4–6

SOLUTION

Support Reactions. For the purpose of computing the support reactions, the distributed load is replaced by its resultant force of 30 k, Fig. 4–6*b*. It is important to remember, however, that this resultant is not the actual load on the beam.



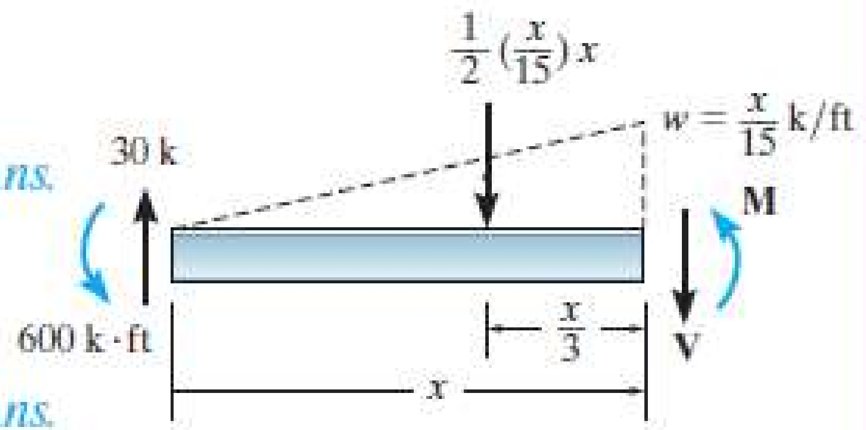
Shear and Moment Functions. A free-body diagram of the beam segment of length x is shown in Fig. 4–6c. Note that the intensity of the triangular load at the section is found by proportion; that is, $w/x = 2/30$ or $w = x/15$. With the load intensity known, the resultant of the distributed loading is found in the usual manner as shown in the figure. Thus,

$$+\uparrow\Sigma F_y = 0; \qquad 30 - \frac{1}{2}\left(\frac{x}{15}\right)x - V = 0$$

$$V = 30 - 0.0333x^2 \qquad \textit{Ans.}$$

$$\circlearrowleft + \Sigma M_S = 0; \qquad 600 - 30x + \left[\frac{1}{2}\left(\frac{x}{15}\right)x\right]\frac{x}{3} + M = 0$$

$$M = -600 + 30x - 0.0111x^3 \qquad \textit{Ans.}$$



(c)

Note that $dM/dx = V$ and $dV/dx = -x/15 = w$, which serves as a check of the results.

EXAMPLE 4.5

Determine the shear and moment in the beam shown in Fig. 4–7a as a function of x.

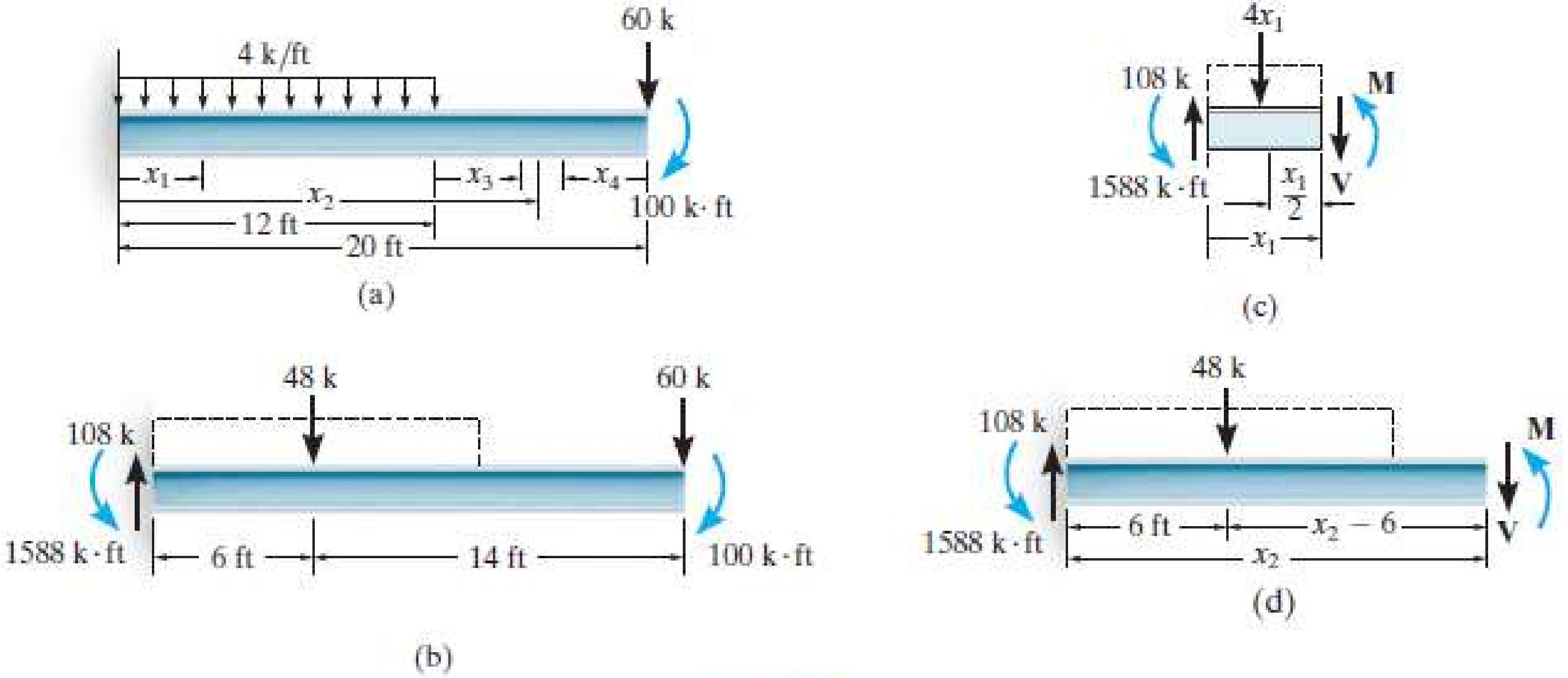


Fig. 4–7

SOLUTION

Support Reactions. The reactions at the fixed support are $V = 108$ k and $M = 1588$ k · ft, Fig. 4–7*b*.

Shear and Moment Functions. Since there is a discontinuity of distributed load at $x = 12$ ft, two regions of x must be considered in order to describe the shear and moment functions for the entire beam. Here x_1 is appropriate for the left 12 ft and x_2 can be used for the remaining segment.

$0 \le x_1 \le 12$ ft. Notice that V and M are shown in the positive directions, Fig. 4–7*c*.

$+\uparrow \Sigma F_y = 0; \qquad 108 - 4x_1 - V = 0, \quad V = 108 - 4x_1$ *Ans.*

$\circlearrowleft + \Sigma M_S = 0; \qquad 1588 - 108x_1 + 4x_1\left(\frac{x_1}{2}\right) + M = 0$

$$M = -1588 + 108x_1 - 2x_1^2 \qquad \textit{Ans.}$$

$12 \text{ ft} \le x_2 \le 20$ ft, Fig. 4–7*d*.

$+\uparrow \Sigma F_y = 0; \qquad 108 - 48 - V = 0, \quad V = 60$ *Ans.*

$\circlearrowleft + \Sigma M_S = 0; \qquad 1588 - 108x_2 + 48(x_2 - 6) + M = 0$

$$M = 60x_2 - 1300 \qquad \textit{Ans.}$$

These results can be partially checked by noting that when $x_2 = 20$ ft, then $V = 60$ k and $M = -100$ k · ft. Also, note that $dM/dx = V$ and $dV/dx = w$.

EXAMPLE 4.6

Determine the shear and moment in the beam shown in Fig. 4–8*a* as a function of x.

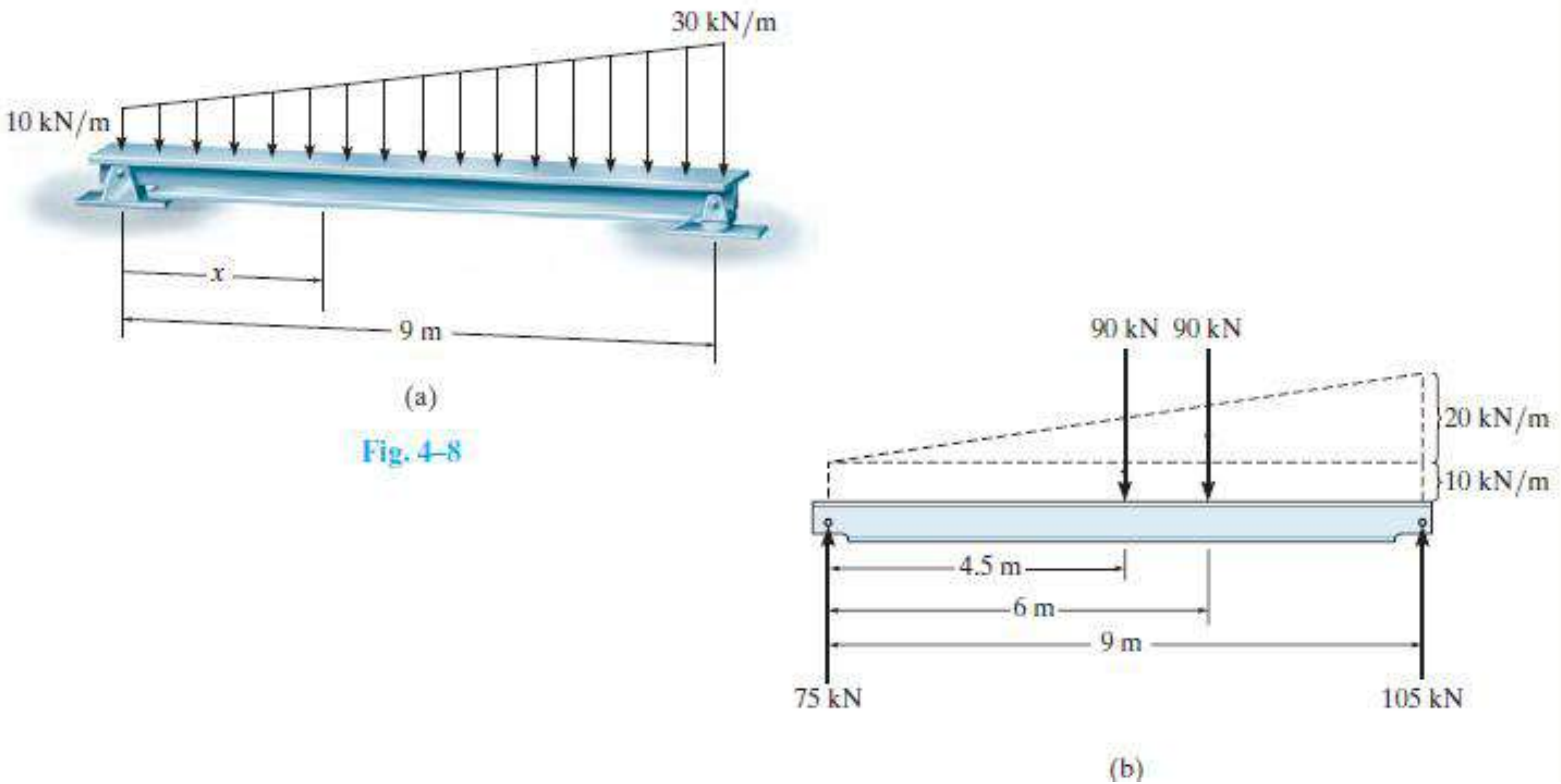
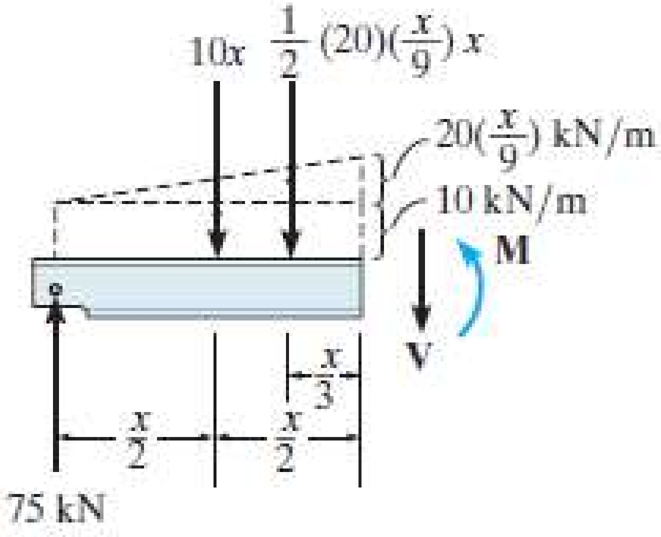


Fig. 4–8

SOLUTION

Support Reactions. To determine the support reactions, the distributed load is divided into a triangular and rectangular loading, and these loadings are then replaced by their resultant forces. These reactions have been computed and are shown on the beam's free-body diagram, Fig. 4–8*b*.

Shear and Moment Functions. A free-body diagram of the cut section is shown in Fig. 4–8*c*. As above, the trapezoidal loading is replaced by rectangular and triangular distributions. Note that the intensity of the triangular load at the cut is found by proportion. The resultant force of each distributed loading and its location are indicated. Applying the equilibrium equations, we have



(c)

$$+\uparrow \Sigma F_y = 0; \qquad 75 - 10x - \left[\frac{1}{2}(20)\left(\frac{x}{9}\right)x\right] - V = 0$$

$$V = 75 - 10x - 1.11x^2 \qquad \textit{Ans.}$$

$$\circlearrowleft + \Sigma M_S = 0; \qquad -75x + (10x)\left(\frac{x}{2}\right) + \left[\frac{1}{2}(20)\left(\frac{x}{9}\right)x\right]\frac{x}{3} + M = 0$$

$$M = 75x - 5x^2 - 0.370x^3 \qquad \textit{Ans.}$$

4.3 Shear and Moment Diagrams for a Beam

$$+\uparrow \Sigma F_y = 0; \qquad V + w(x)\,\Delta x - (V + \Delta V) = 0$$
$$\Delta V = w(x)\,\Delta x$$

$$\circlearrowright\!+ \Sigma M_O = 0; \qquad -V\,\Delta x - M - w(x)\,\Delta x\,\epsilon(\Delta x) + (M + \Delta M) = 0$$
$$\Delta M = V\,\Delta x + w(x)\,\epsilon(\Delta x)^2$$

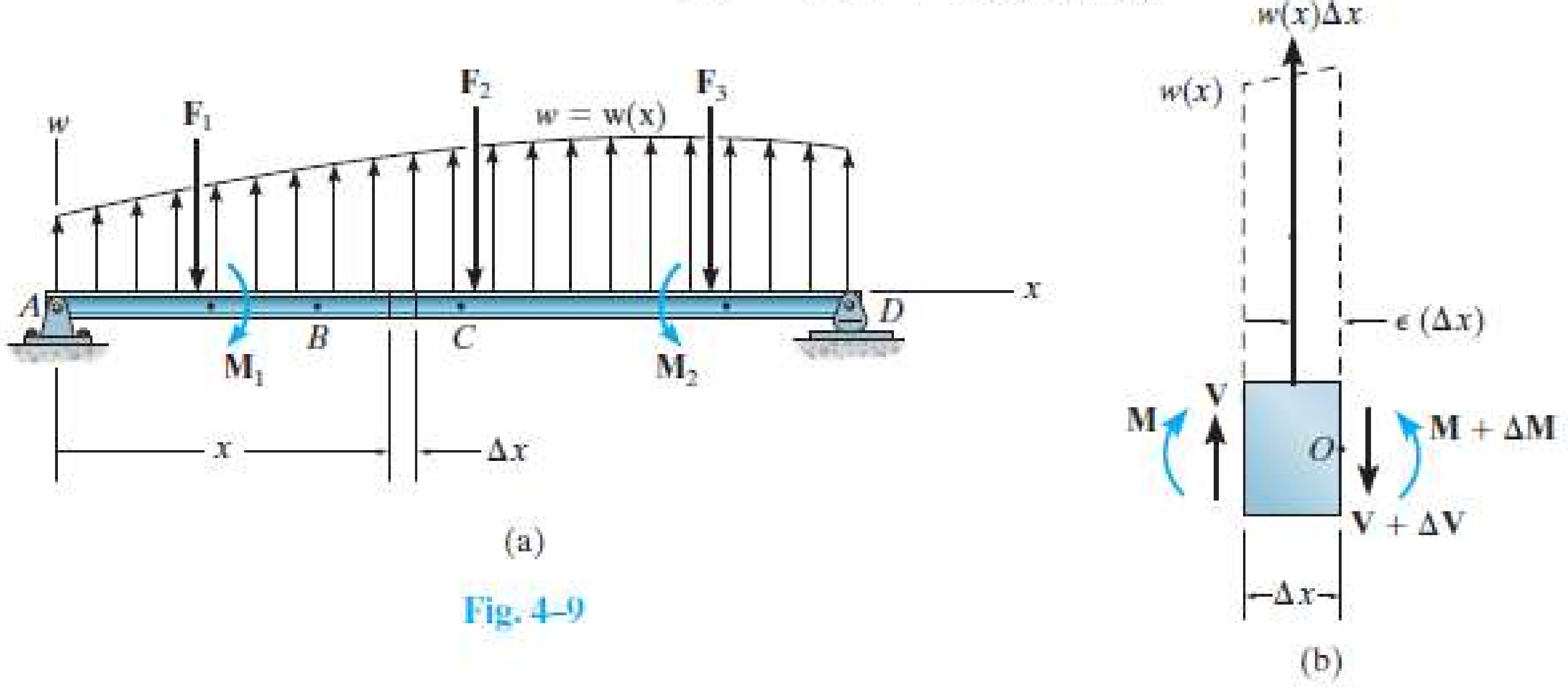


Fig. 4–9

Dividing by Δx and taking the limit as $\Delta x \rightarrow 0$, these equations become

$$\frac{dV}{dx} = w(x)$$

$$\left.\begin{array}{r}\text{Slope of}\\ \text{Shear Diagram}\end{array}\right\} = \left\{\begin{array}{l}\text{Intensity of}\\ \text{Distributed Load}\end{array}\right. \tag{4–1}$$

$$\frac{dM}{dx} = V$$

$$\left.\begin{array}{r}\text{Slope of}\\ \text{Moment Diagram}\end{array}\right\} = \{\text{Shear} \tag{4–2}$$

Equations 4–1 and 4–2 can be "integrated" from one point to another between concentrated forces or couples (such as from B to C in Fig. 4–9a), in which case

$$\Delta V = \int w(x)\,dx$$

$$\left.\begin{array}{r}\text{Change in}\\ \text{Shear}\end{array}\right\} = \left\{\begin{array}{l}\text{Area under}\\ \text{Distributed Loading}\\ \text{Diagram}\end{array}\right. \qquad (4\text{–}3)$$

and

$$\Delta M = \int V(x)\,dx$$

$$\left.\begin{array}{r}\text{Change in}\\ \text{Moment}\end{array}\right\} = \left\{\begin{array}{l}\text{Area under}\\ \text{Shear Diagram}\end{array}\right. \qquad (4\text{–}4)$$

Procedure for Analysis

The following procedure provides a method for constructing the shear and moment diagrams for a beam using Eqs. 4–1 through 4–6.

Support Reactions

- Determine the support reactions and resolve the forces acting on the beam into components which are perpendicular and parallel to the beam's axis.

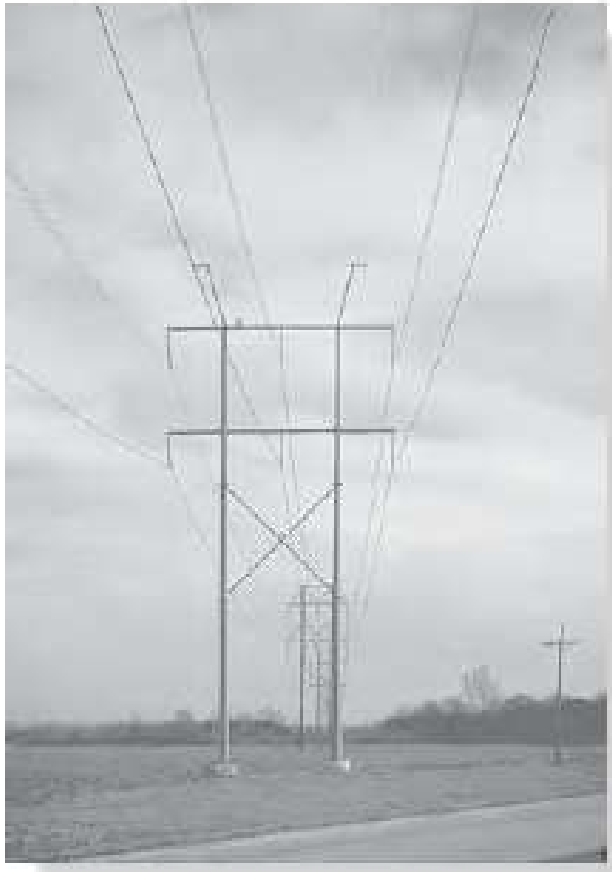
Shear Diagram

- Establish the V and x axes and plot the values of the shear at the two *ends* of the beam.
- Since $dV/dx = w$, the *slope* of the *shear diagram* at any point is equal to the intensity of the *distributed loading* at the point. (Note that w is positive when it acts upward.)
- If a numerical value of the shear is to be determined at the point, one can find this value either by using the method of sections as discussed in Sec. 4–1 or by using Eq. 4–3, which states that the *change in the shear force* is equal to the *area under the distributed loading diagram*.
- Since $w(x)$ is *integrated* to obtain V, if $w(x)$ is a curve of degree n, then $V(x)$ will be a curve of degree $n + 1$. For example, if $w(x)$ is uniform, $V(x)$ will be linear.

Moment Diagram

- Establish the M and x axes and plot the values of the moment at the ends of the beam.
- Since $dM/dx = V$, the *slope* of the *moment diagram* at any point is equal to the intensity of the *shear* at the point.
- At the point where the shear is zero, $dM/dx = 0$, and therefore this may be a point of maximum or minimum moment.
- If the numerical value of the moment is to be determined at a point, one can find this value either by using the method of sections as discussed in Sec. 4–1 or by using Eq. 4–4, which states that the *change in the moment* is equal to the *area under the shear diagram*.
- Since $V(x)$ is *integrated* to obtain M, if $V(x)$ is a curve of degree n, then $M(x)$ will be a curve of degree $n + 1$. For example, if $V(x)$ is linear, $M(x)$ will be parabolic.

EXAMPLE 4.7

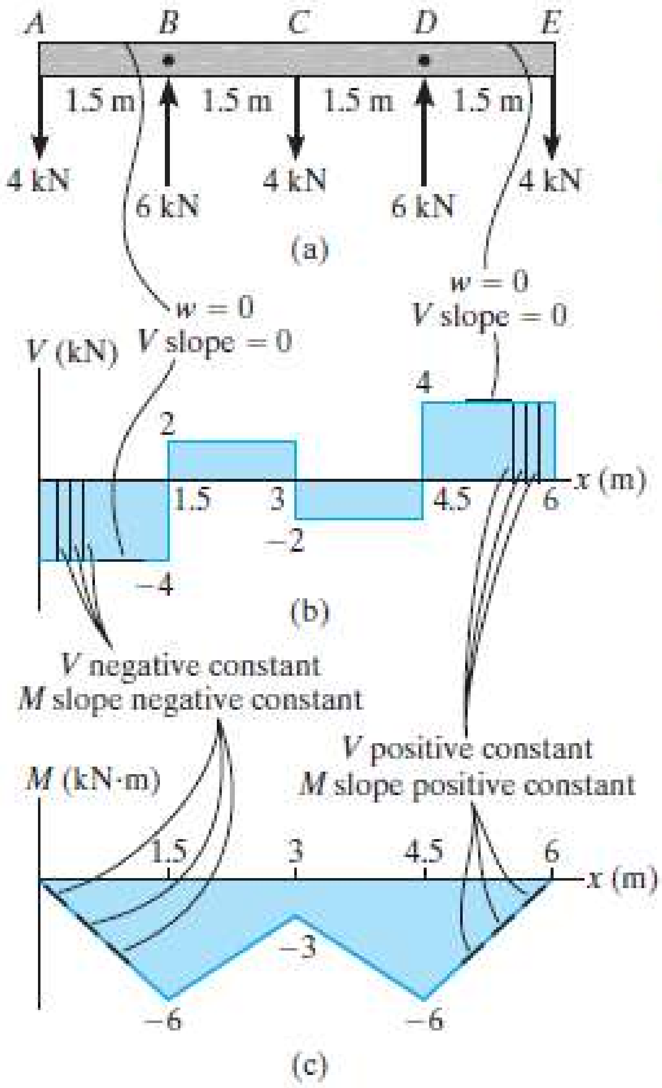


The two horizontal members of the power line support frame are subjected to the cable loadings shown in Fig. 4–11*a*. Draw the shear and moment diagrams for each member.

SOLUTION

Support Reactions. Each pole exerts a force of 6 kN on each member as shown on the free-body diagram.

Shear Diagram. The end points $x = 0, V = -4$ kN and $x = 6$ m, $V = 4$ kN are plotted first, Fig. 4–11*b*. As indicated, the shear between each concentrated force is *constant* since $w = dV/dx = 0$. The shear just to the right of point B (or C and D) can be determined by the method of sections, Fig. 4–11*d*. The shear diagram can also be established by "following the load" on the free-body diagram. Beginning at A the 4 kN load acts downward so $V_A = -4$ kN. No load acts between A and B so the shear is constant. At B the 6 kN force acts upward, so the shear jumps up 6 kN, from -4 kN to $+2$ kN, etc.



Moment Diagram. The moment at the end points $x = 0$, $M = 0$ and $x = 6$ m, $M = 0$ is plotted first, Fig. 4–11*c*. The slope of the moment diagram within each 1.5-m-long region is constant because V is constant. Specific values of the moment, such as at C, can be determined by the method of sections, Fig. 4–11*d*, or by finding the change in moment by the area under the shear diagram. For example, since $M_A = 0$ at A, then at C, $M_C = M_A + \Delta M_{AC} = 0 + (-4)(1.5) + 2(1.5) = -3\ \text{kN}\cdot\text{m}$.

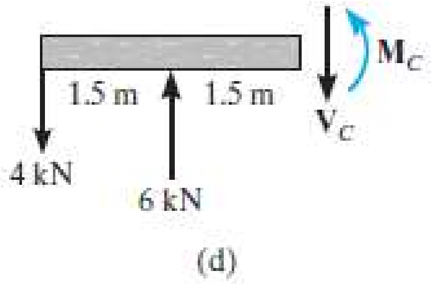


Fig. 4–11

EXAMPLE 4.8

Draw the shear and moment diagrams for the beam in Fig. 4–12*a*.

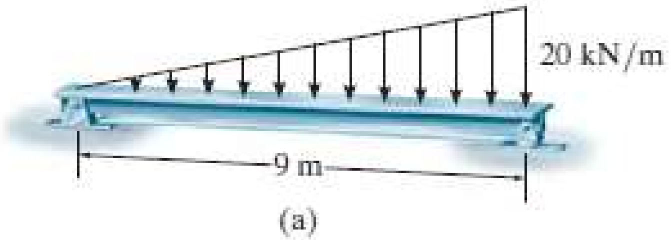
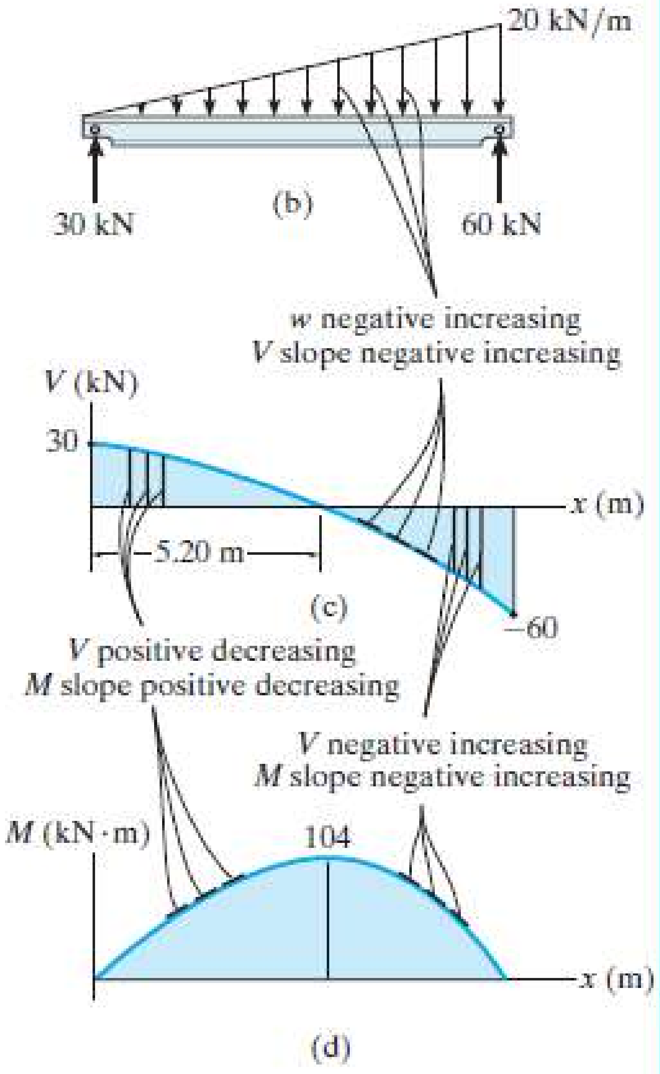


Fig. 4–12



SOLUTION

Support Reactions. The reactions have been calculated and are shown on the free-body diagram of the beam, Fig. 4–12*b*.

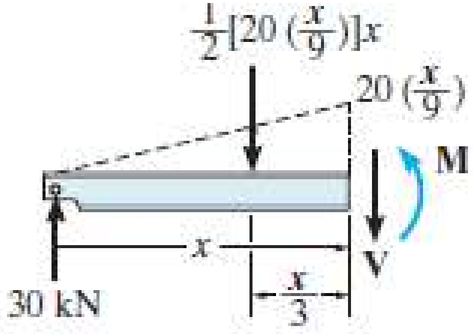
Shear Diagram. The end points $x = 0$, $V = +30$ kN and $x = 9$ m, $V = -60$ kN are first plotted. Note that the shear diagram *starts* with zero slope since $w = 0$ at $x = 0$, and ends with a slope of $w = -20$ kN/m.

The point of zero shear can be found by using the method of sections from a beam segment of length x, Fig. 4–12*e*. We require $V = 0$, so that

$$+\uparrow \Sigma F_y = 0; \qquad 30 - \frac{1}{2}\left[20\left(\frac{x}{9}\right)\right]x = 0 \qquad x = 5.20\text{ m}$$

Moment Diagram. For $0 < x < 5.20$ m the value of shear is positive but decreasing and so the slope of the moment diagram is also positive and decreasing ($dM/dx = V$). At $x = 5.20$ m, $dM/dx = 0$. Likewise for $5.20 \text{ m} < x < 9$ m, the shear and so the slope of the moment diagram are negative increasing as indicated.

The maximum value of moment is at $x = 5.20$ m since $dM/dx = V = 0$ at this point, Fig. 4–12d. From the free-body diagram in Fig. 4–12e we have

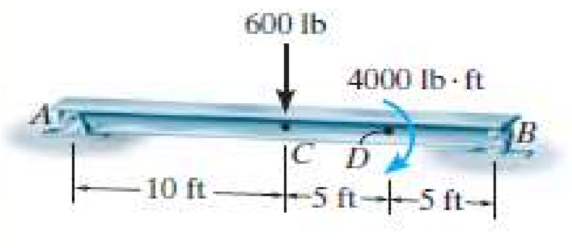


(e)

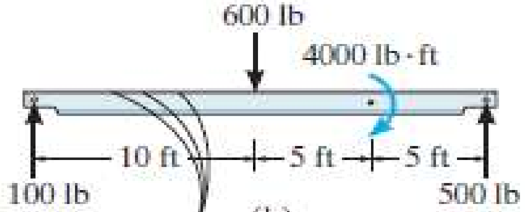
$$\circlearrowleft + \Sigma M_S = 0; \qquad -30(5.20) + \frac{1}{2}\left[20\left(\frac{5.20}{9}\right)\right](5.20)\left(\frac{5.20}{3}\right) + M = 0$$

$$M = 104 \text{ kN} \cdot \text{m}$$

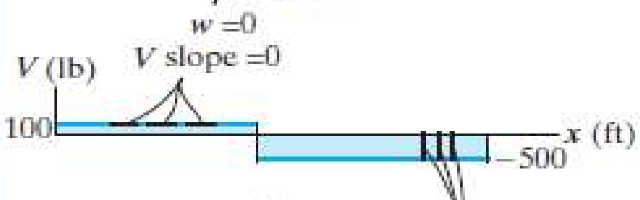
EXAMPLE 4.9



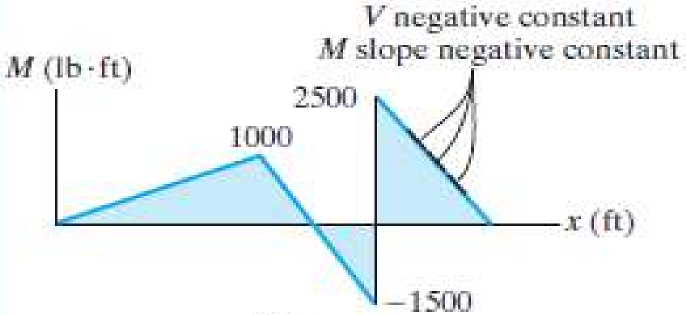
(a)



(b)



(c)



(d)

Draw the shear and moment diagrams for the beam shown in Fig. 4–13*a*.

SOLUTION

Support Reactions. The reactions are calculated and indicated on the free-body diagram.

Shear Diagram. The values of the shear at the end points A ($V_A = +100$ lb) and B ($V_B = -500$ lb) are plotted. At C the shear is *discontinuous* since there is a *concentrated force* of 600 lb there. The value of the shear just to the right of C can be found by sectioning the beam at this point. This yields the free-body diagram shown in equilibrium in Fig. 4–13*e*. This point ($V = -500$ lb) is plotted on the shear diagram. Notice that no jump or discontinuity in shear occurs at D, the point where the 4000-lb · ft couple moment is applied, Fig. 4–13*b*.

Moment Diagram. The moment at each end of the beam is zero, Fig. 4–13*d*. The value of the moment at C can be determined by the method of sections, Fig. 4–13*e*, or by finding the area under the shear diagram between A and C. Since $M_A = 0$,

$$M_C = M_A + \Delta M_{AC} = 0 + (100\text{ lb})(10\text{ ft})$$
$$M_C = 1000\text{ lb}\cdot\text{ft}$$

Also, since $M_C = 1000$ lb · ft, the moment at D is

$$M_D = M_C + \Delta M_{CD} = 1000\text{ lb}\cdot\text{ft} + (-500\text{ lb})(5\text{ ft})$$
$$M_D = -1500\text{ lb}\cdot\text{ft}$$

A jump occurs at point D due to the couple moment of 4000 lb · ft. The method of sections, Fig. 4–13f, gives a value of +2500 lb · ft just to the right of D.

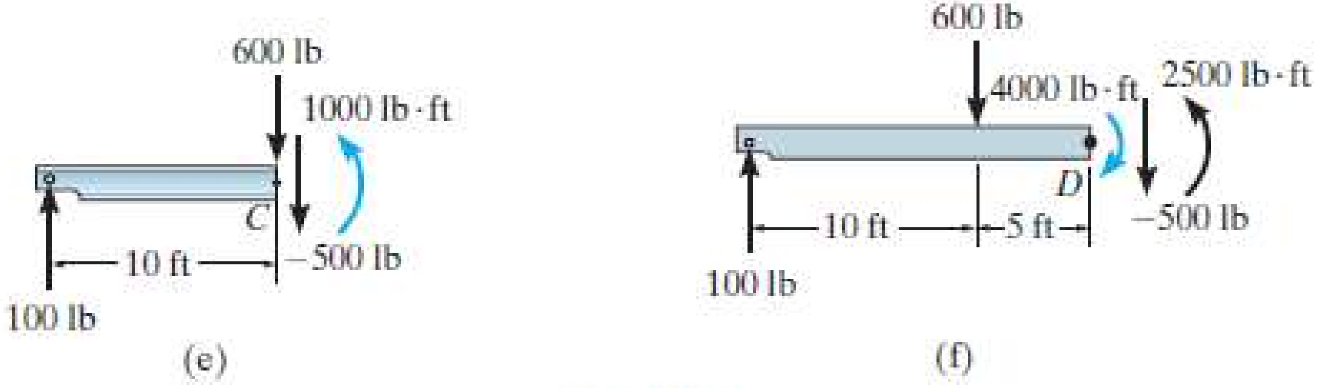
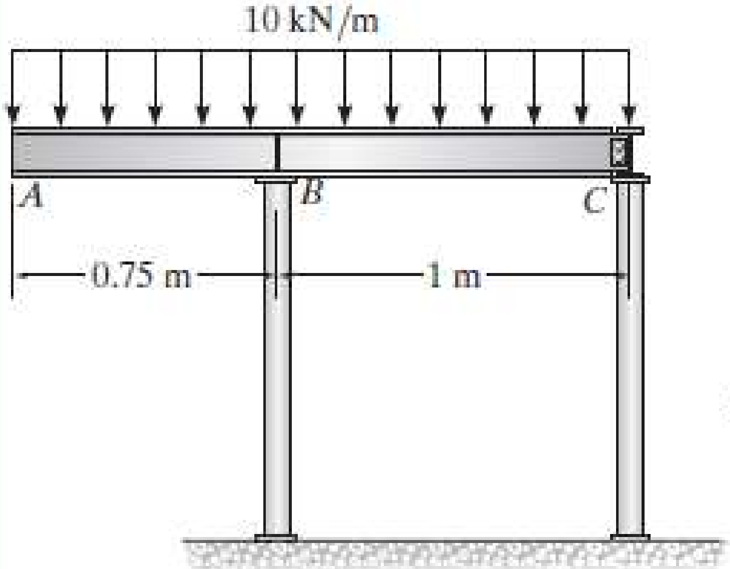


Fig. 4–13

EXAMPLE 4.11

The beam shown in the photo is used to support a portion of the overhang for the entranceway of the building. The idealized model for the beam with the load acting on it is shown in Fig. 4–15*a*. Assume *B* is a roller and *C* is pinned. Draw the shear and moment diagrams for the beam.



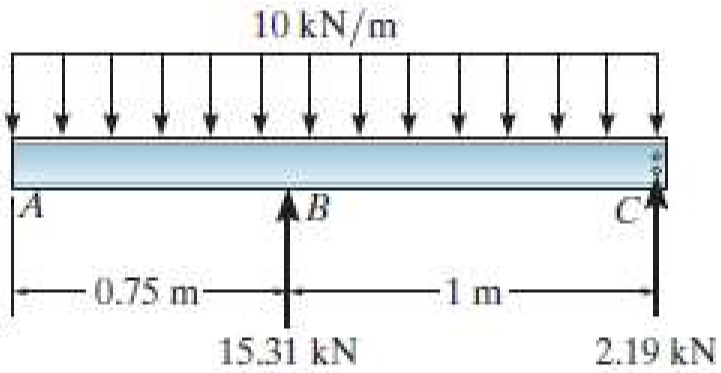
SOLUTION

Support Reactions. The reactions are calculated in the usual manner. The results are shown in Fig. 4–15*b*.

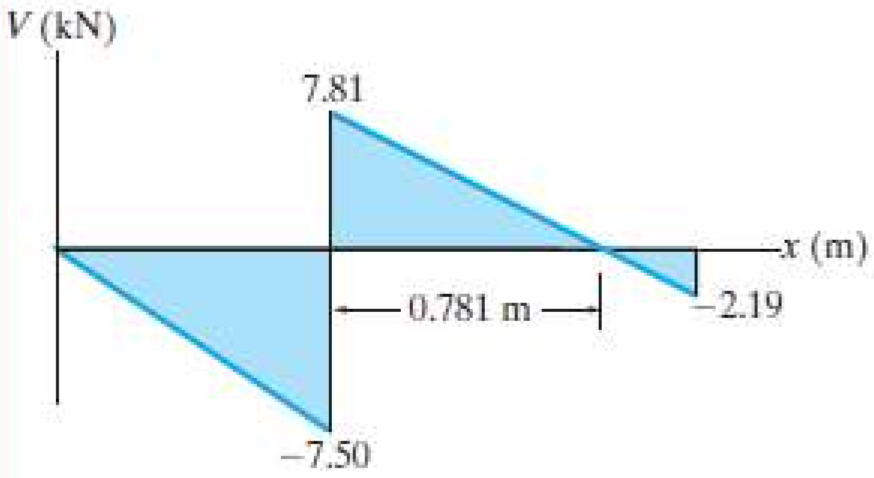
Shear Diagram. The shear at the ends of the beam is plotted first, i.e., $V_A = 0$ and $V_C = -2.19$ kN, Fig. 4–15*c*. To find the shear to the left of *B* use the method of sections for segment *AB*, or calculate the area under the distributed loading diagram, i.e., $\Delta V = V_B - 0 = -10(0.75)$, $V_{B^-} = -7.50$ kN. The support reaction causes the shear to jump up $-7.50 + 15.31 = 7.81$ kN. The point of zero shear can be determined from the slope -10 kN/m, or by proportional triangles, $7.81/x = 2.19/(1 - x)$, $x = 0.781$ m. Notice how the *V* diagram follows the negative slope, defined by the constant negative distributed loading.

Moment Diagram. The moment at the end points is plotted first, $M_A = M_C = 0$, Fig. 4–15*d*. The values of -2.81 and 0.239 on the moment diagram can be calculated by the method of sections, or by finding the areas under the shear diagram. For example, $\Delta M = M_B - 0 = \frac{1}{2}(-7.50)(0.75) = -2.81$, $M_B = -2.81\ \text{kN}\cdot\text{m}$. Likewise, show that the maximum positive moment is $0.239\ \text{kN}\cdot\text{m}$. Notice how the M diagram is formed, by following the slope, defined by the V diagram.

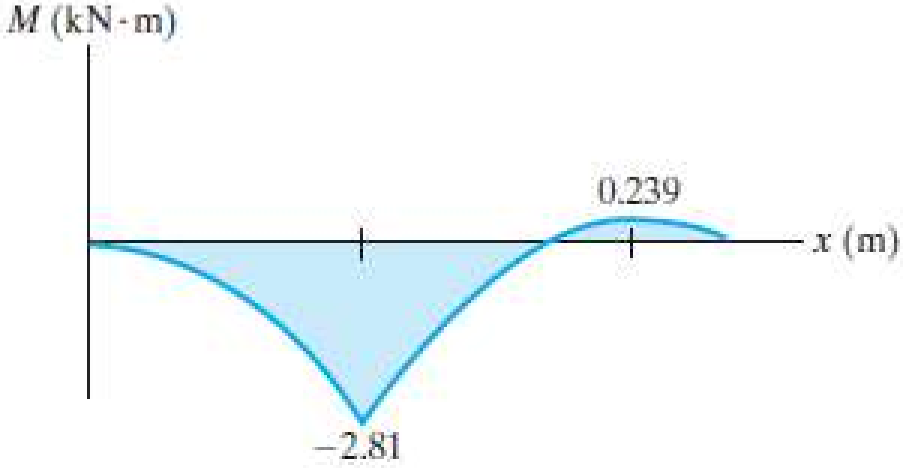
(a)



(b)



(c)

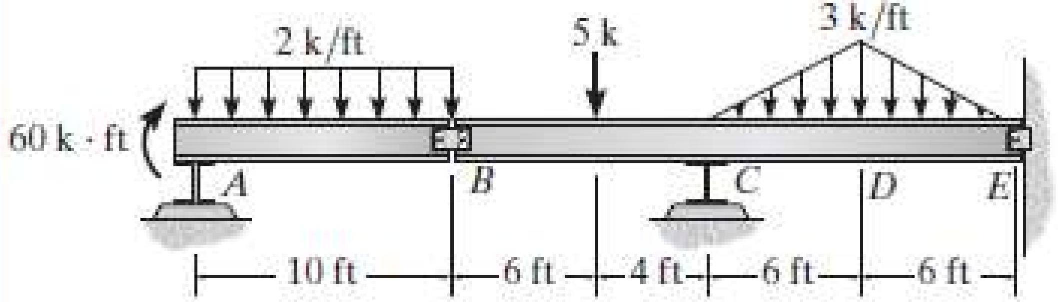


(d)

Fig. 4–15

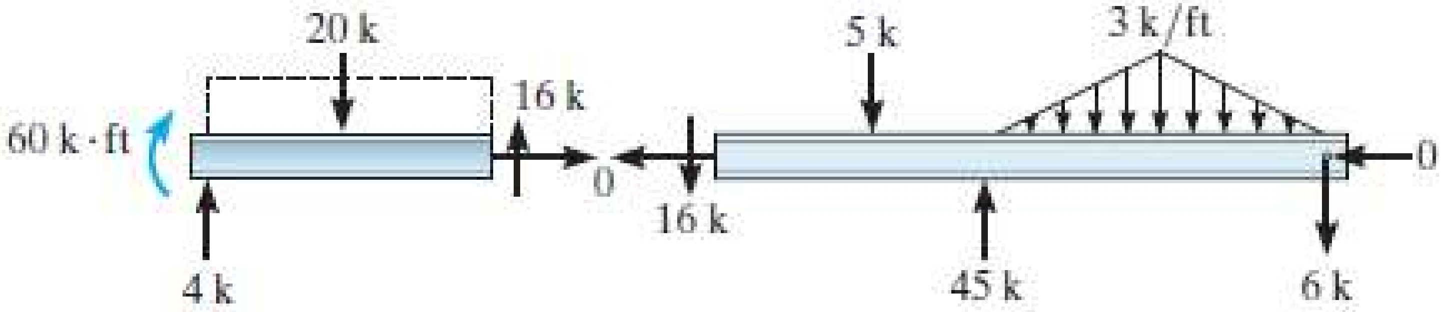
EXAMPLE 4.12

Draw the shear and moment diagrams for the compound beam shown in Fig. 4–16*a*. Assume the supports at *A* and *C* are rollers and *B* and *E* are pin connections.



SOLUTION

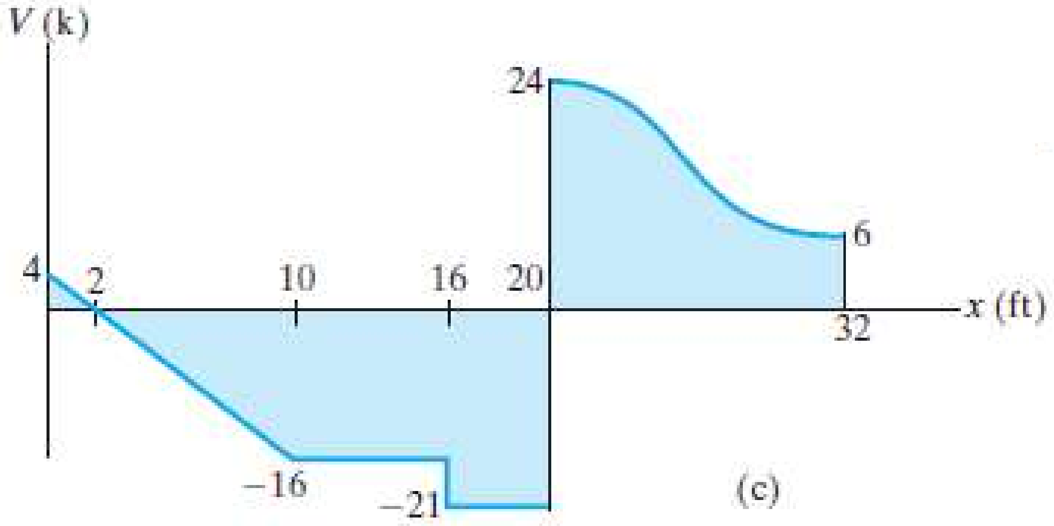
Support Reactions. Once the beam segments are disconnected from the pin at *B*, the support reactions can be calculated as shown in Fig. 4–16*b*.



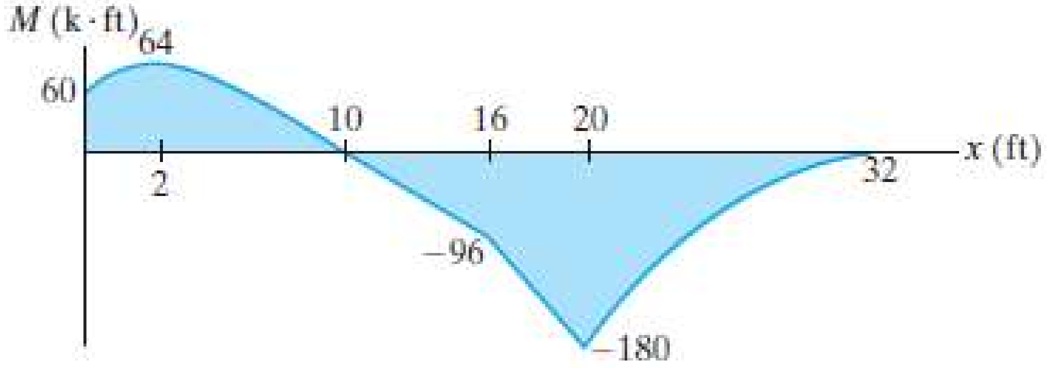
(b)

Shear Diagram. As usual, we start by plotting the end shear at A and E, Fig. 4–16c. The shape of the V diagram is formed by following its slope, defined by the loading. Try to establish the values of shear using the appropriate areas under the load diagram (w curve) to find the change in shear. The zero value for shear at $x = 2$ ft can either be found by proportional triangles, or by using statics, as was done in Fig. 4–12e of Example 4–8.

Moment Diagram. The end moments $M_A = 60\ \text{k}\cdot\text{ft}$ and $M_E = 0$ are plotted first, Fig. 4–16d. Study the diagram and note how the various curves are established using $dM/dx = V$. Verify the numerical values for the peaks using statics or by calculating the appropriate areas under the shear diagram to find the change in moment.



(c)



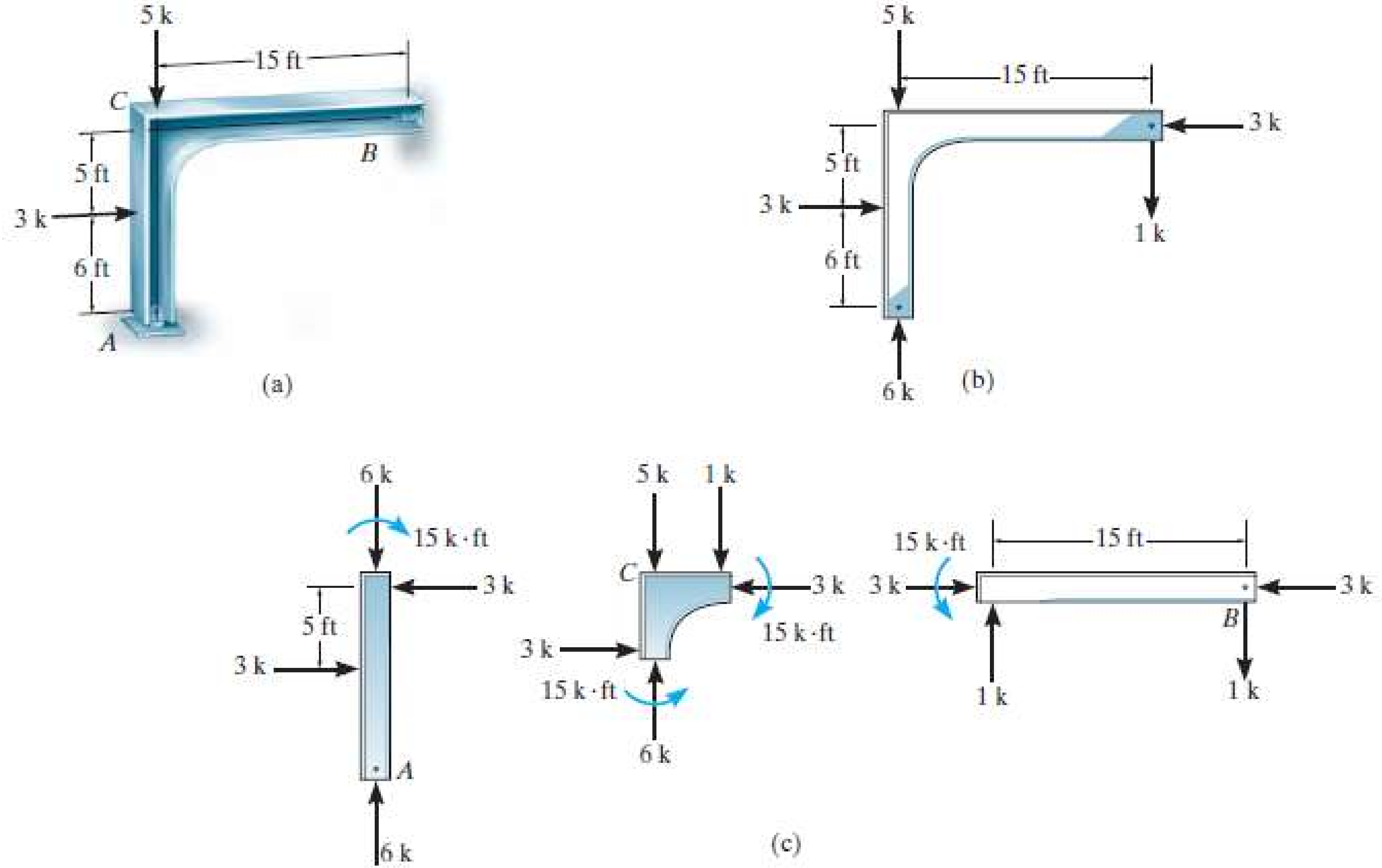
4.4 Shear and Moment Diagrams for a Frame

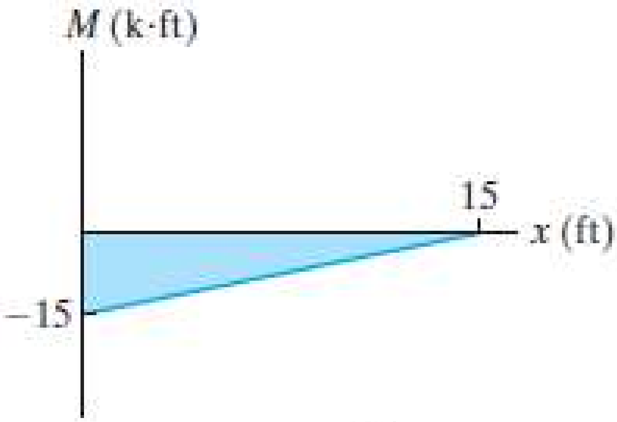
Recall that a *frame* is composed of several connected members that are either fixed or pin connected at their ends. The design of these structures often requires drawing the shear and moment diagrams for each of the members.

The simply supported girder of this concrete building frame was designed by first drawing its shear and moment diagrams.

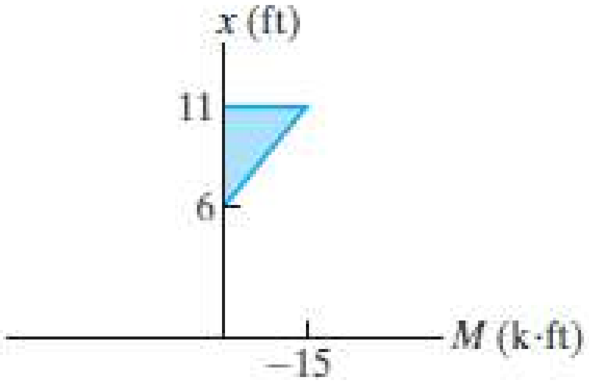
EXAMPLE 4.13

Draw the moment diagram for the tapered frame shown in Fig. 4–17*a*. Assume the support at *A* is a roller and *B* is a pin.





member *CB*



member *AC*

(d)

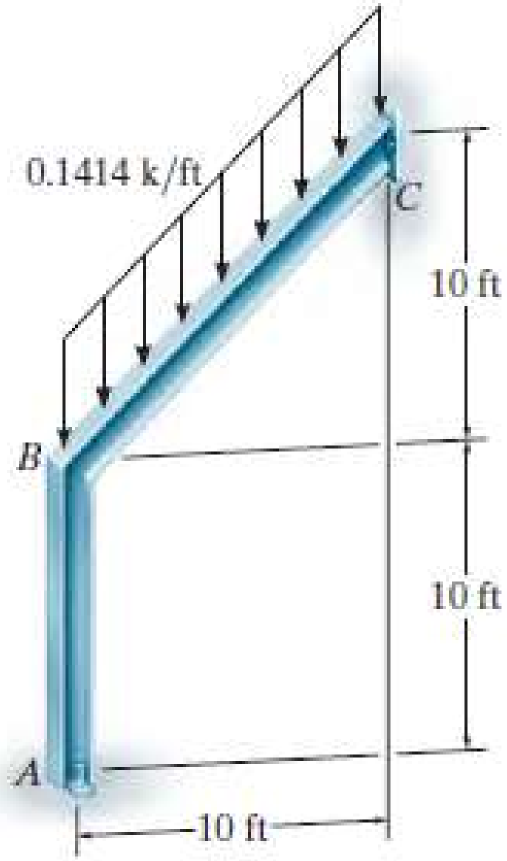
SOLUTION

Support Reactions. The support reactions are shown on the free-body diagram of the entire frame, Fig. 4–17*b*. Using these results, the frame is then sectioned into two members, and the internal reactions at the joint ends of the members are determined, Fig. 4–17*c*. Note that the external 5-k load is shown only on the free-body diagram of the joint at *C*.

Moment Diagram. In accordance with our positive sign convention, and using the techniques discussed in Sec. 4–3, the moment diagrams for the frame members are shown in Fig. 4–17*d*.

EXAMPLE 4.14

Draw the shear and moment diagrams for the frame shown in Fig. 4–18*a*. Assume *A* is a pin, *C* is a roller, and *B* is a fixed joint. Neglect the thickness of the members.

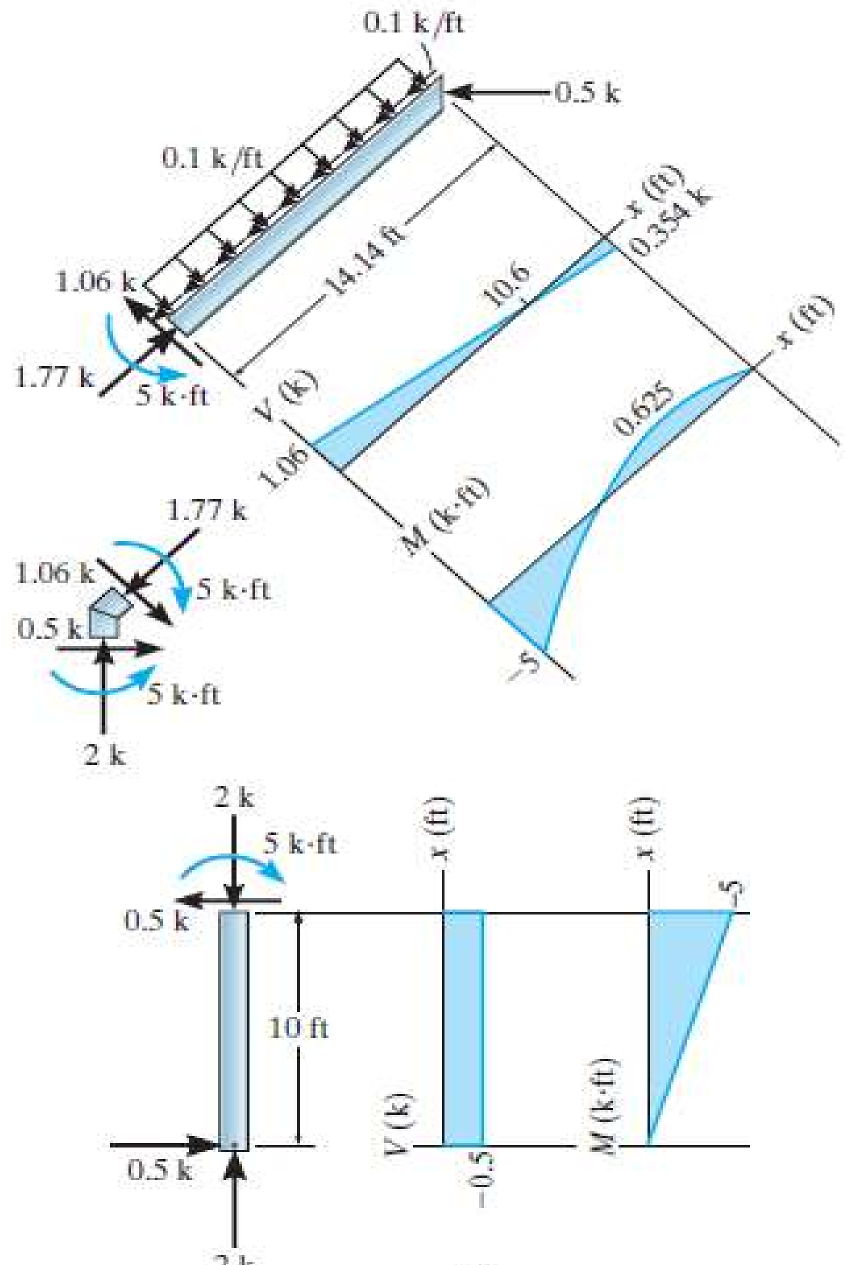


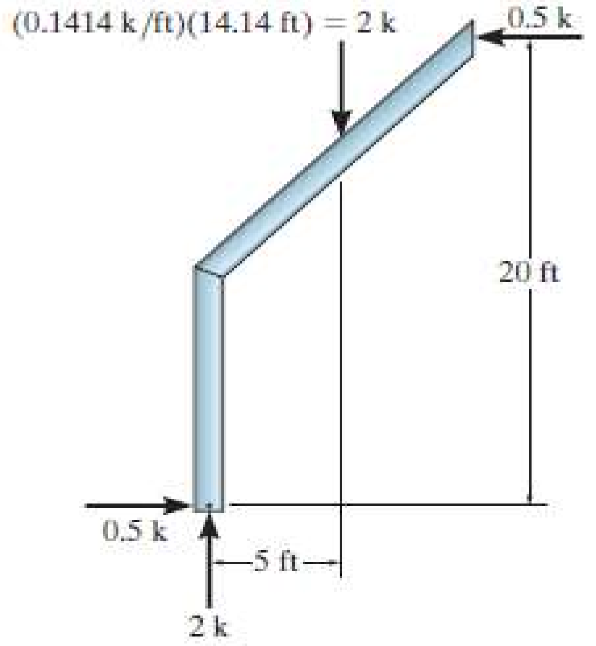
(a)

Fig. 4–18

SOLUTION

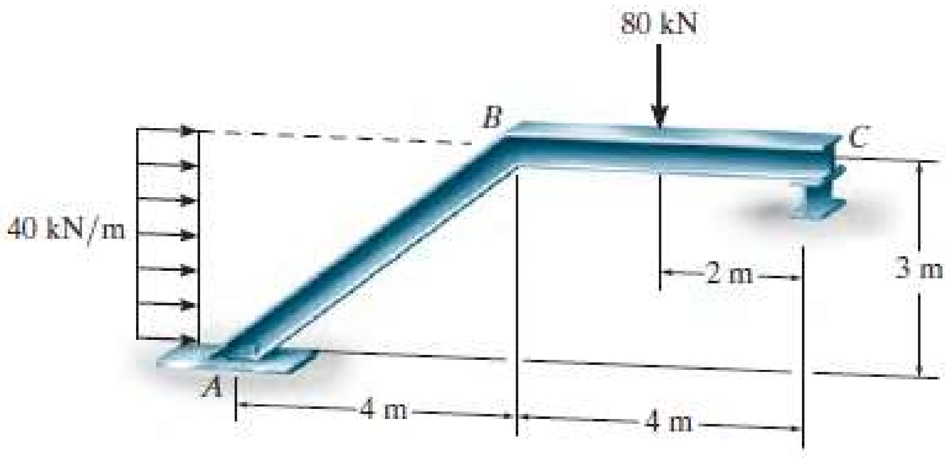
Notice that the distributed load acts over a length of $10 \text{ ft}\sqrt{2} = 14.14$ ft. The reactions on the entire frame are calculated and shown on its free-body diagram, Fig. 4–18*b*. From this diagram the free-body diagrams of each member are drawn, Fig. 4–18*c*. The distributed loading on *BC* has components along *BC* and perpendicular to its axis of $(0.1414 \text{ k/ft}) \cos 45° = (0.1414 \text{ k/ft}) \sin 45° = 0.1 \text{ k/ft}$ as shown. Using these results, the shear and moment diagrams are also shown in Fig. 4–18*c*.


0.1 k/ft
0.5 k
0.1 k/ft
14.14 ft
x (ft)
0.354 k
1.06 k
10.6
1.77 k
5 k·ft
V (k)
1.06
x (ft)
0.625
1.77 k
M (k·ft)
1.06 k
5 k·ft
0.5 k
5 k·ft
−5
2 k
2 k
5 k·ft
0.5 k
x (ft)
x (ft)
−5
10 ft
V (k)
M (k·ft)
0.5 k
−0.5
2 k
(c)

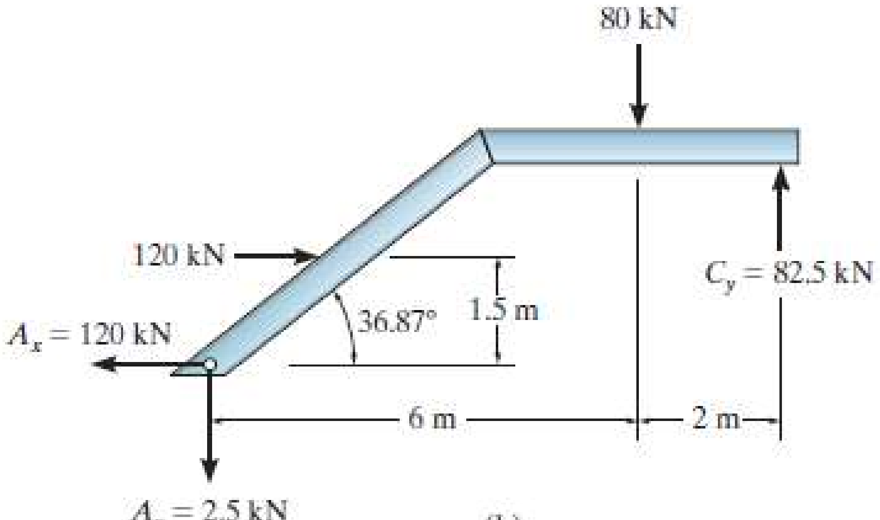

(0.1414 k/ft)(14.14 ft) = 2 k
0.5 k
20 ft
0.5 k
5 ft
2 k
(b)

EXAMPLE 4.15

Draw the shear and moment diagrams for the frame shown in Fig. 4–19*a*. Assume *A* is a pin, *C* is a roller, and *B* is a fixed joint.



(a)

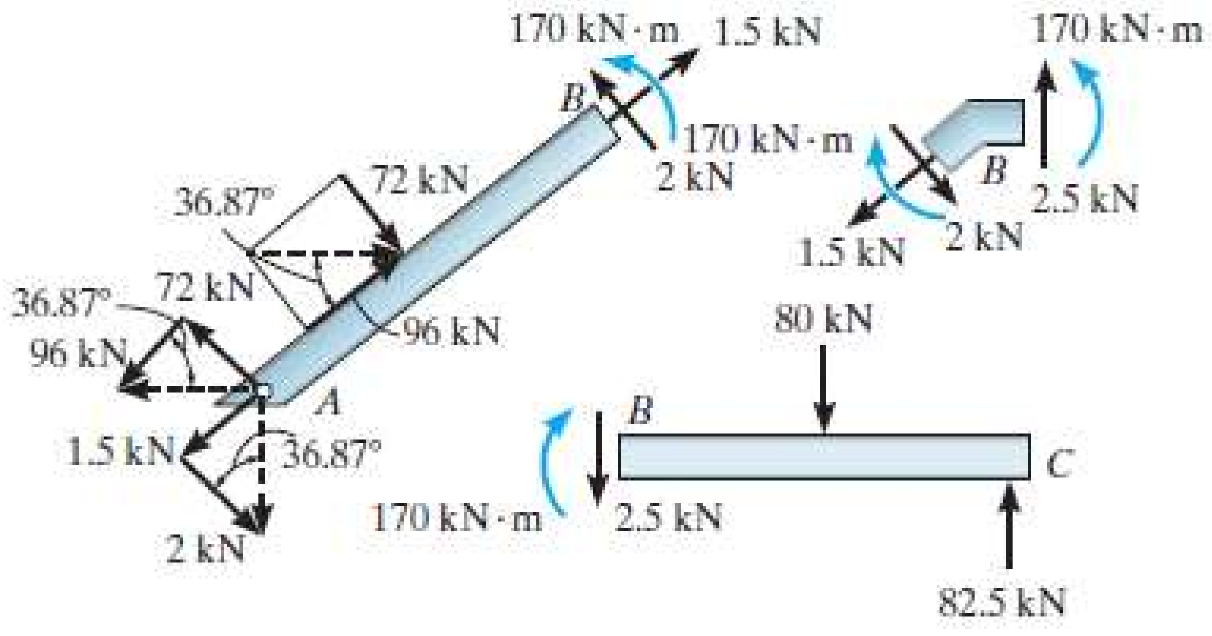


(b)

SOLUTION

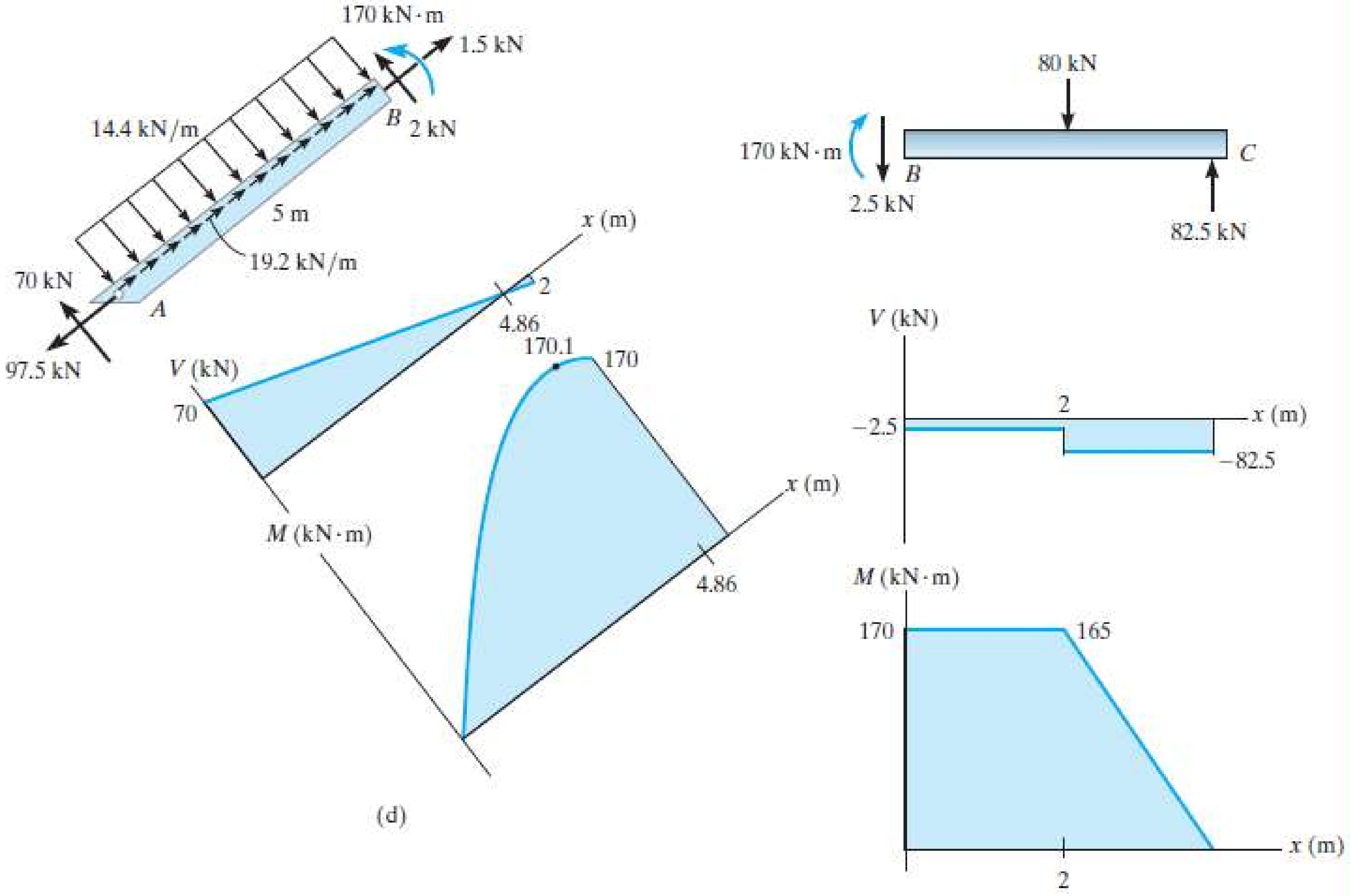
Support Reactions. The free-body diagram of the entire frame is shown in Fig. 4–19*b*. Here the distributed load, which represents wind loading, has been replaced by its resultant, and the reactions have been computed. The frame is then sectioned at joint B and the internal loadings at B are determined, Fig. 4–19*c*. As a check, equilibrium is satisfied at joint B, which is also shown in the figure.

Shear and Moment Diagrams. The components of the distributed load, $(72\text{ kN})/(5\text{ m}) = 14.4\text{ kN/m}$ and $(96\text{ kN})/(5\text{ m}) = 19.2\text{ kN/m}$, are shown on member AB, Fig. 4–19*d*. The associated shear and moment diagrams are drawn for each member as shown in Figs. 4–19*d* and 4–19*e*.



170 kN · m
1.5 kN
B
170 kN · m
2 kN
72 kN
36.87°
72 kN
36.87°
96 kN
96 kN
A
1.5 kN
36.87°
2 kN
170 kN · m
B
170 kN · m
2.5 kN
1.5 kN
2 kN
80 kN
B
C
170 kN · m
2.5 kN
82.5 kN

(c)



170 kN · m
1.5 kN
14.4 kN/m
B
2 kN
5 m
19.2 kN/m
70 kN
A
97.5 kN
x (m)
2
4.86
V (kN)
70
170.1
170
x (m)
M (kN · m)
4.86
80 kN
170 kN · m
B
C
2.5 kN
82.5 kN
V (kN)
2
x (m)
−2.5
−82.5
M (kN · m)
170
165
x (m)
2

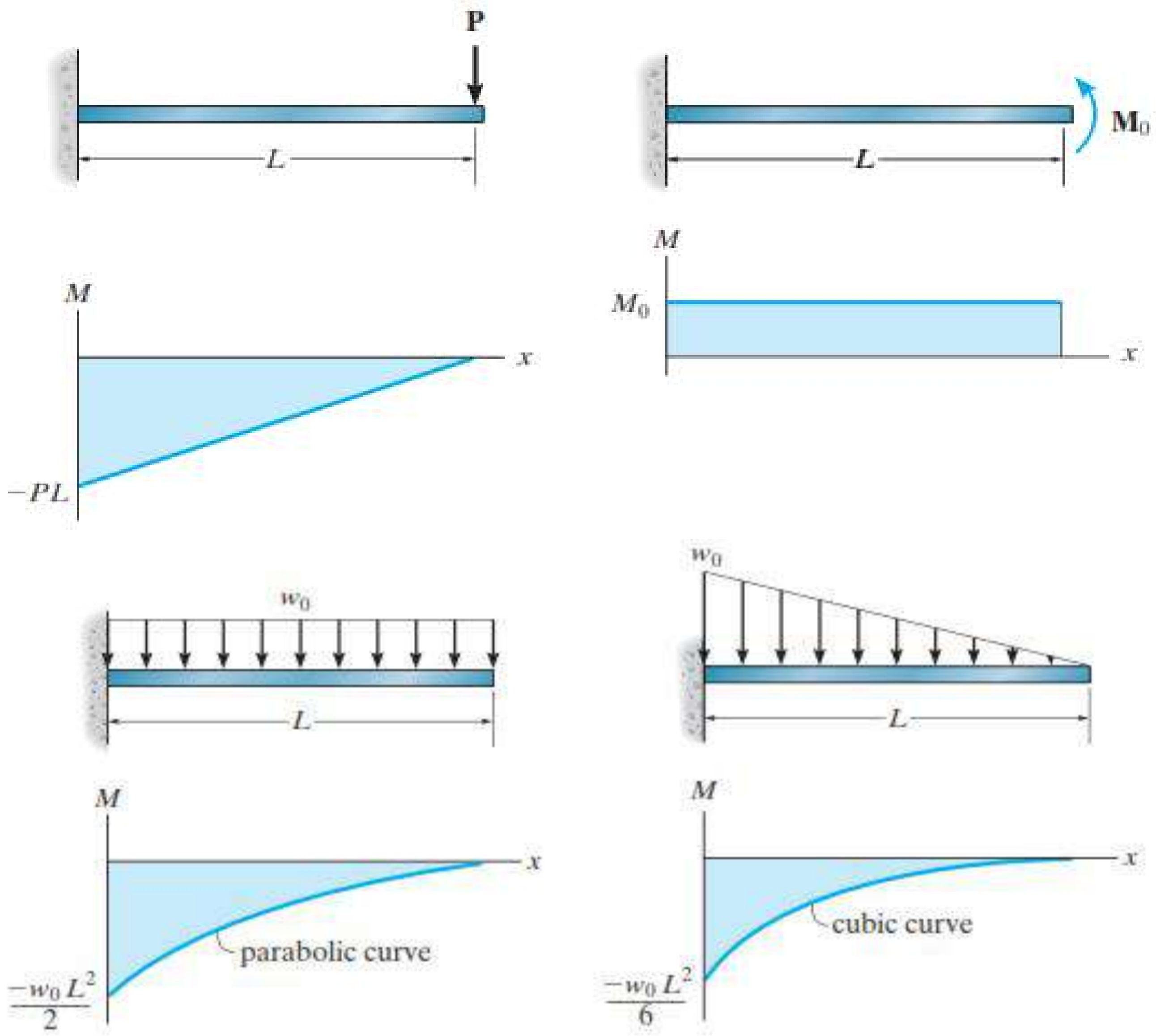
(d)

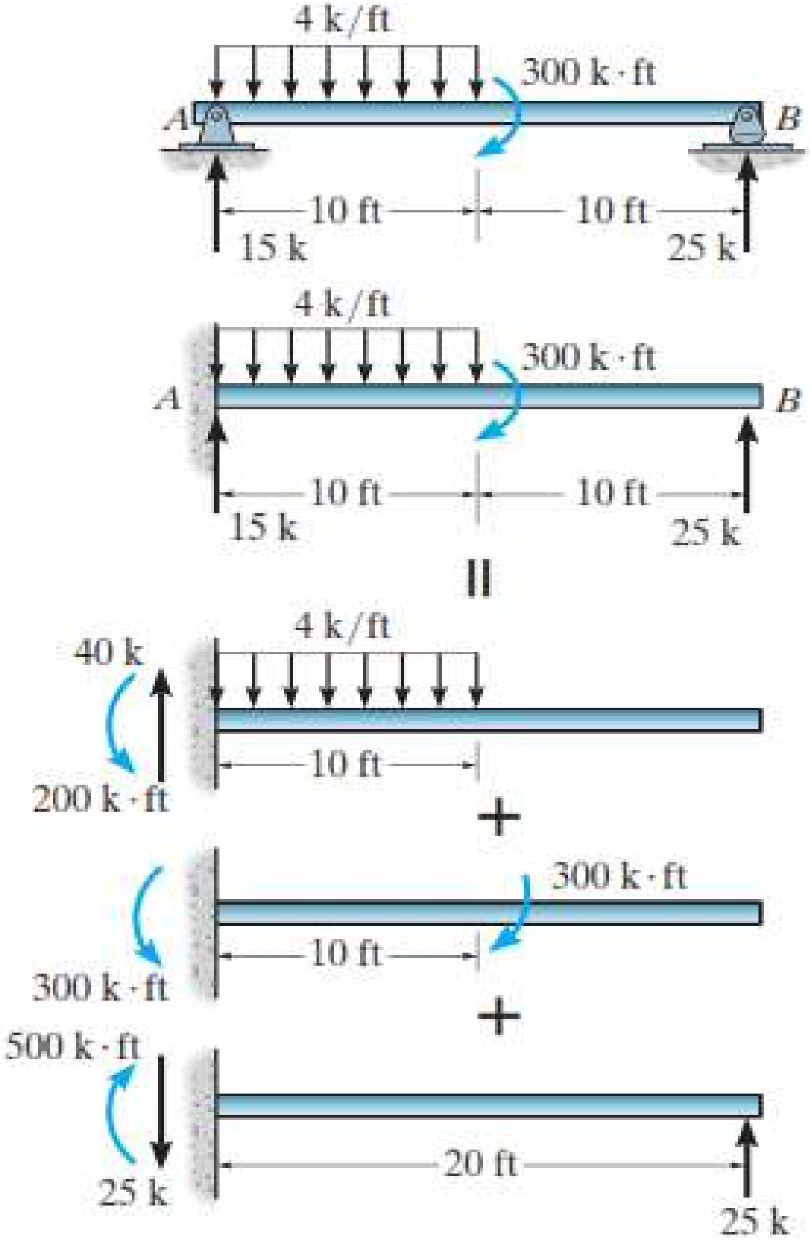
4.5 Moment Diagrams Constructed by the Method of Superposition

Since beams are used primarily to resist bending stress, it is important that the moment diagram accompany the solution for their design. In Sec. 4–3 the moment diagram was constructed by *first* drawing the shear diagram. If we use the principle of superposition, however, each of the loads on the beam can be treated separately and the moment diagram can then be constructed in a series of parts rather than a single and sometimes complicated shape. It will be shown later in the text that this can be particularly advantageous when applying geometric deflection methods to determine both the deflection of a beam and the reactions on statically indeterminate beams.

Most loadings on beams in structural analysis will be a combination of the loadings shown in Fig. 4–20. Construction of the associated moment diagrams has been discussed in Example 4–8. To understand how to use

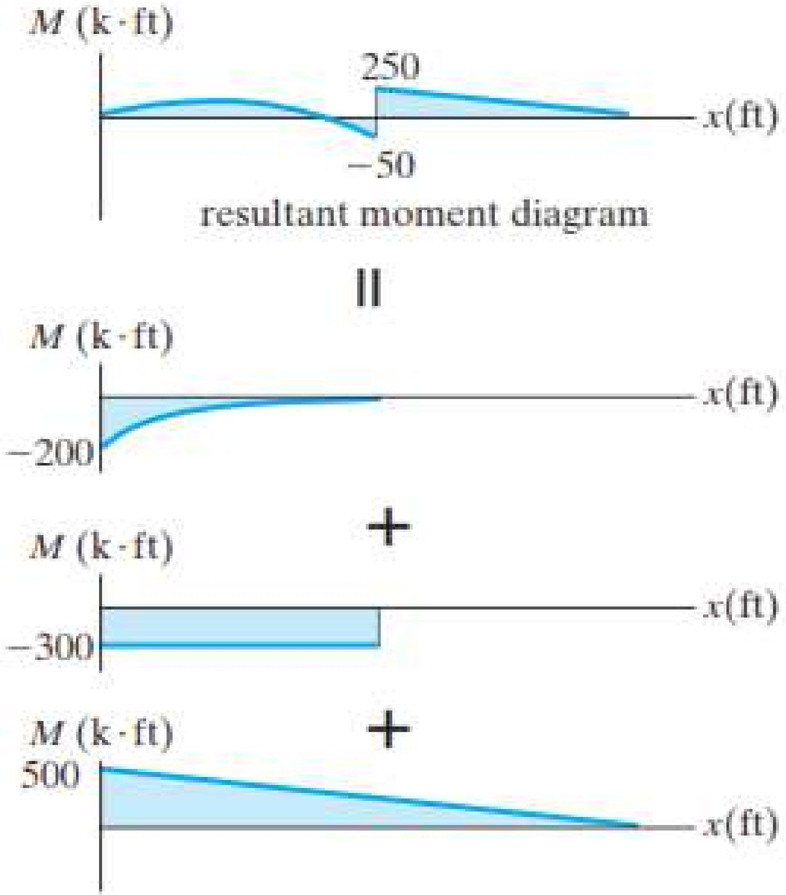
Most loadings on beams in structural analysis will be a combination of the loadings shown in Fig. 4–20.





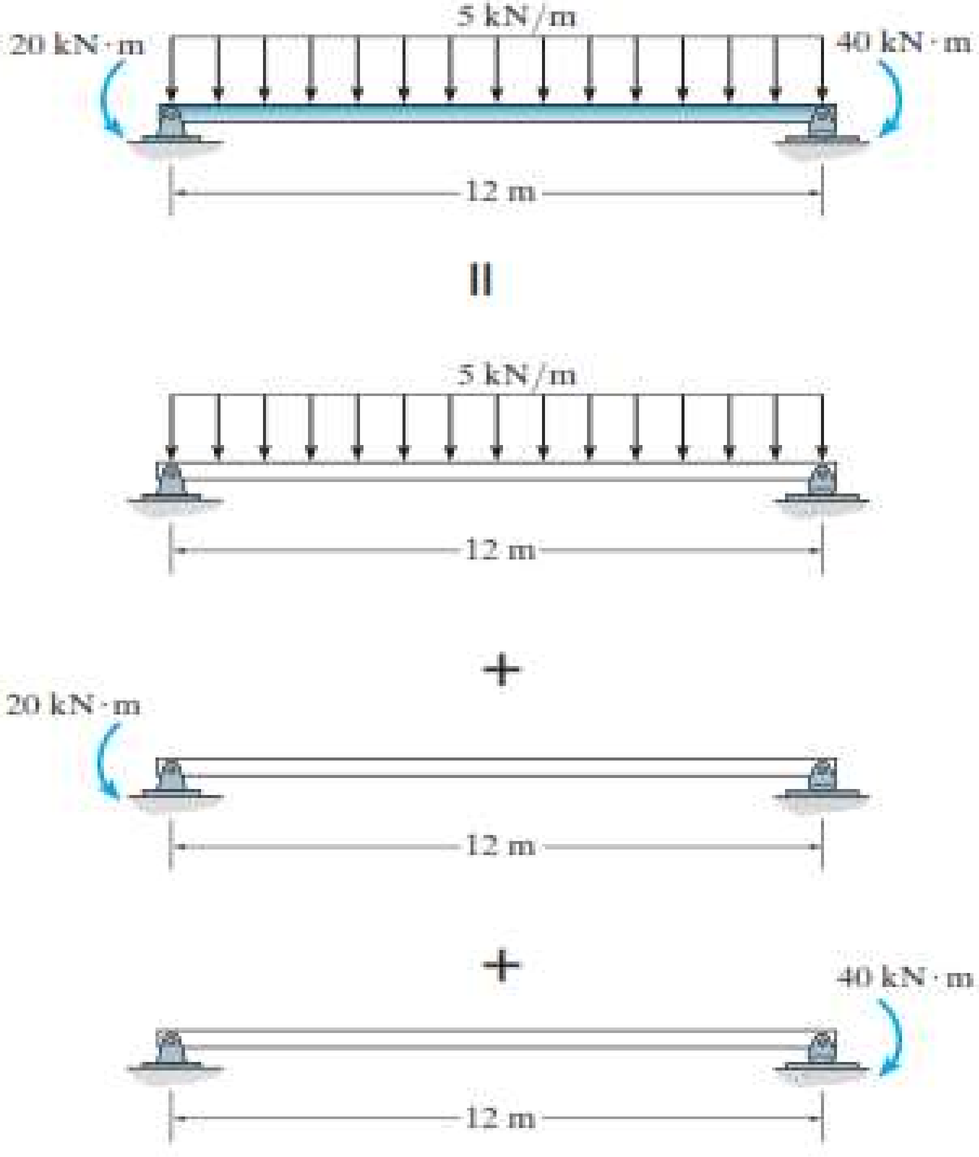
superposition of cantilevered beams

(a)



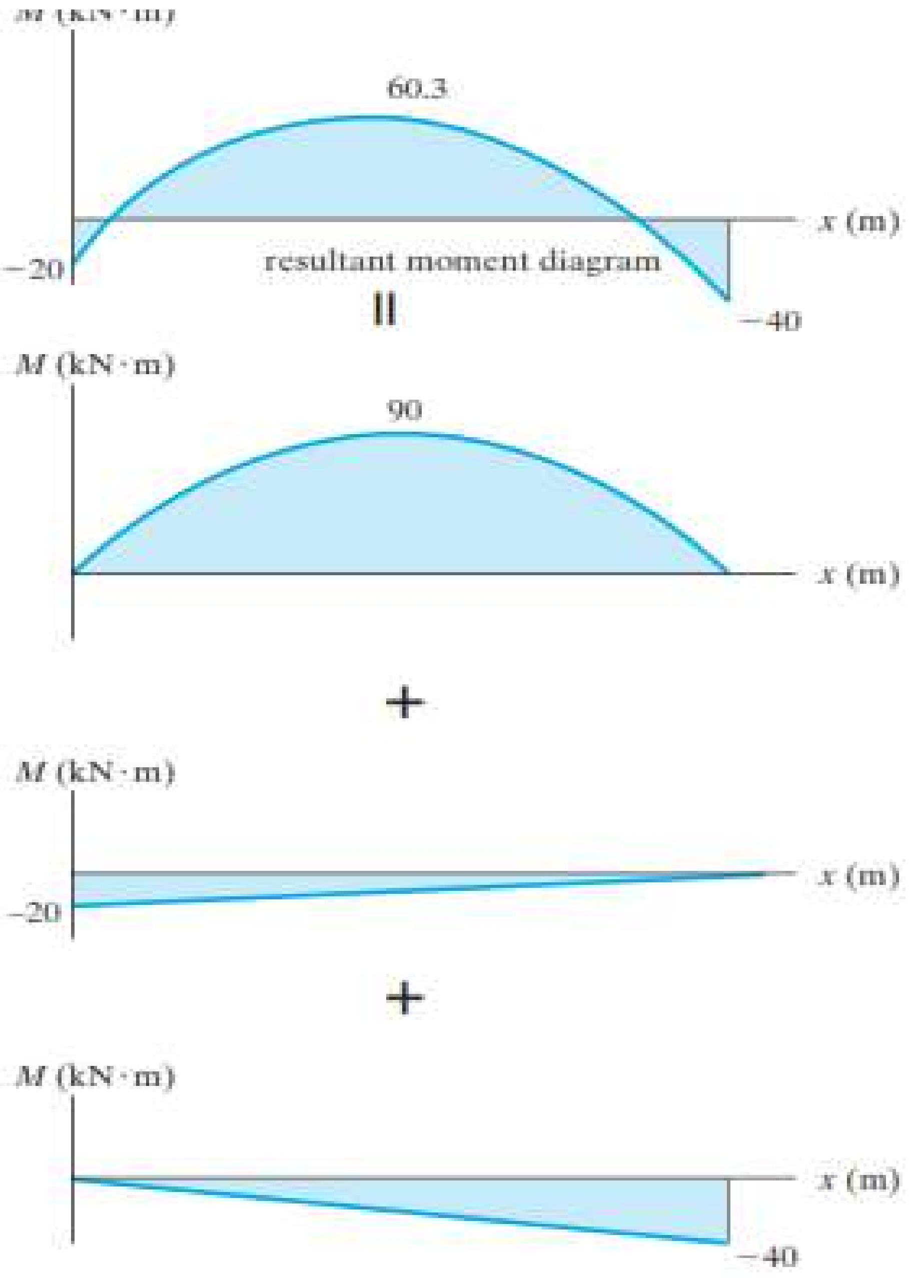
superposition of associated moment diagrams

(b)



superposition of simply supported beams

(a)



superposition of associated moment diagrams

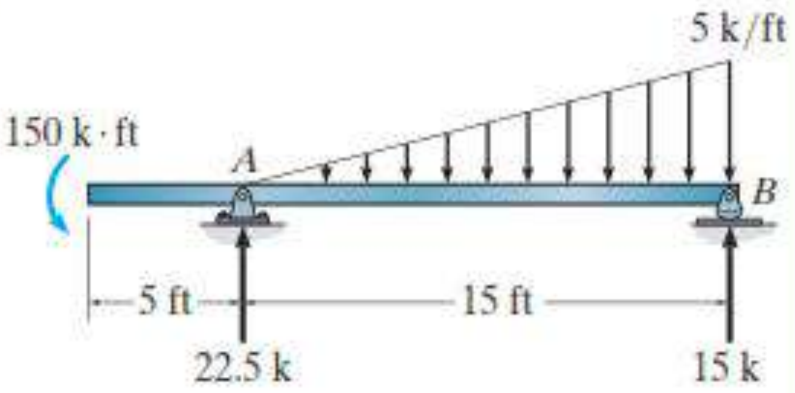
(b)

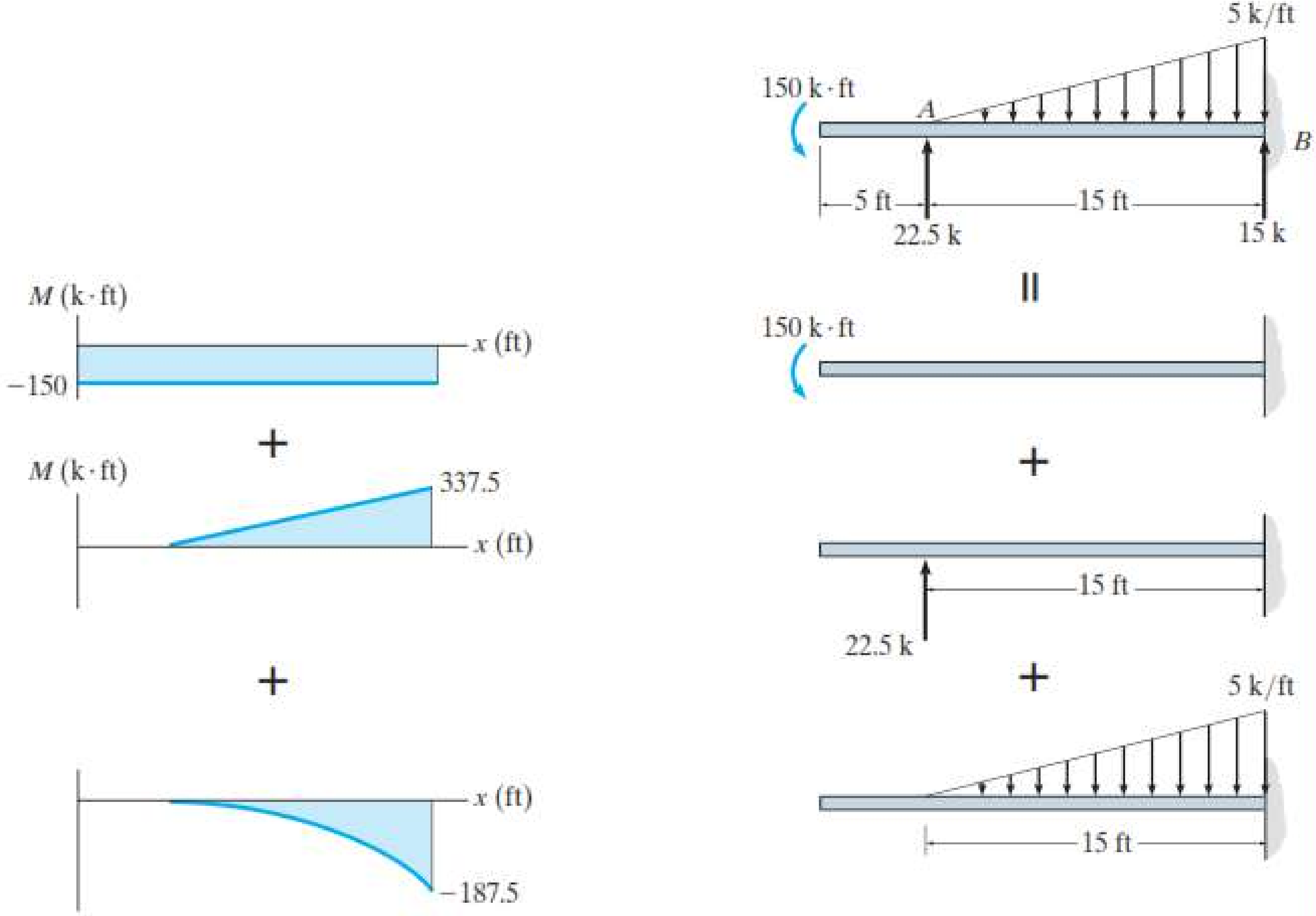
EXAMPLE 4.16

Draw the moment diagrams for the beam shown at the top of Fig. 4–23*a* using the method of superposition. Consider the beam to be cantilevered from the support at *B*.

SOLUTION

If the beam were supported as a cantilever from *B*, it would be subjected to the statically equivalent loadings shown in Fig. 4–23*a*. The superimposed three cantilevered beams are shown below it together with their associated moment diagrams in Fig. 4–23*b*. (As an aid to their construction, refer to Fig. 4–20.) Although *not needed here*, the sum of these diagrams will yield the resultant moment diagram for the beam. For practice, try drawing this diagram and check the results.





superposition of associated moment diagrams

(b)

superposition of cantilevered beams

(a)

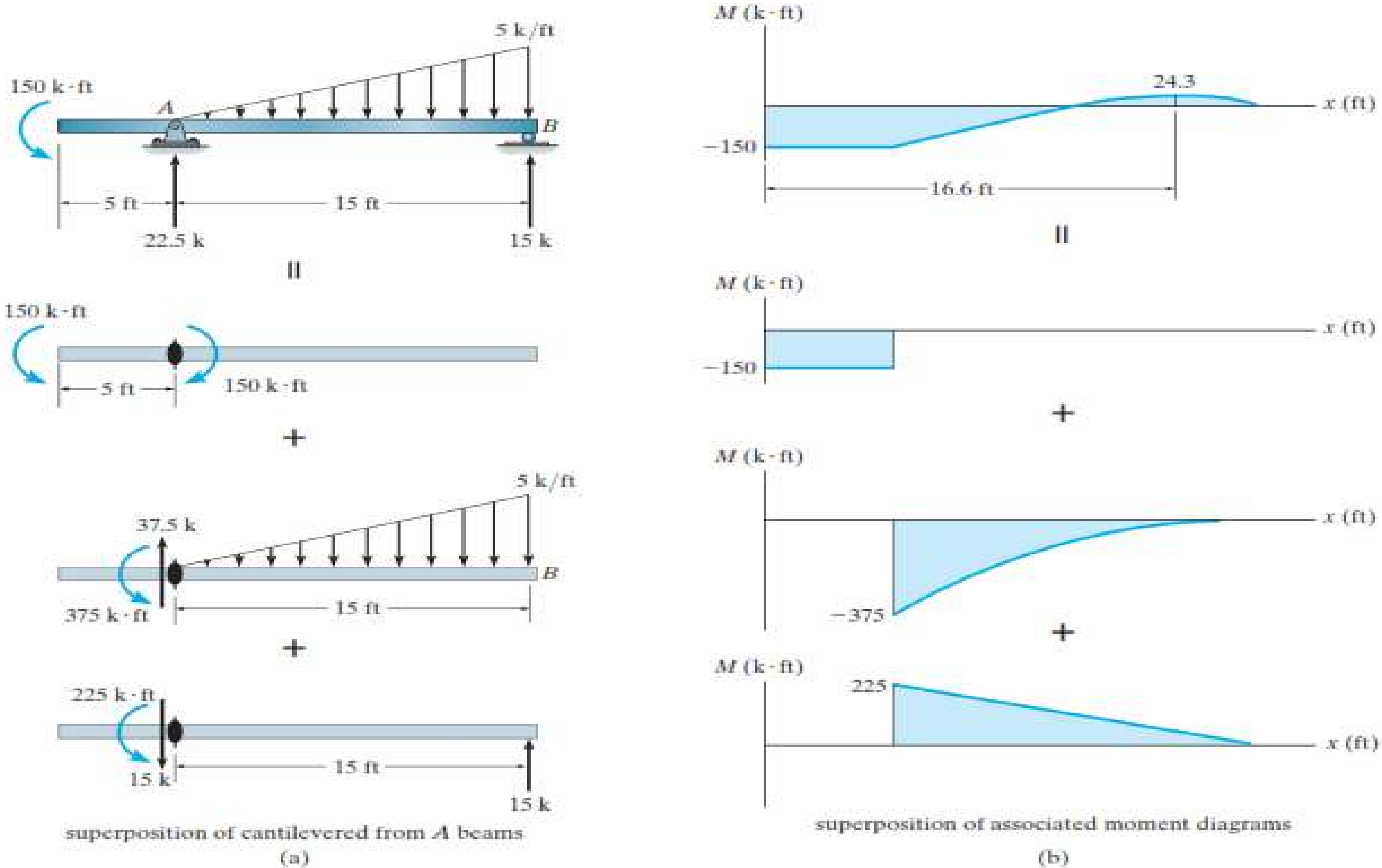
Fig. 4–23

EXAMPLE 4.17

Draw the moment diagrams for the beam shown at the top of Fig. 4–24*a* using the method of superposition. Consider the beam to be cantilevered from the pin at *A*.

SOLUTION

The superimposed cantilevered beams are shown in Fig. 4–24*a* together with their associated moment diagrams, Fig. 4–24*b*. Notice that the reaction at the pin (22.5 k) is not considered since it produces no moment diagram. As an exercise verify that the resultant moment diagram is given at the top of Fig. 4–24*b*.



5 k/ft
150 k · ft
A
B
5 ft
15 ft
22.5 k
15 k
150 k · ft
5 ft
150 k · ft
+
5 k/ft
37.5 k
B
375 k · ft
15 ft
+
225 k · ft
15 ft
15 k
15 k
superposition of cantilevered from A beams
(a)
M (k · ft)
24.3
x (ft)
−150
16.6 ft
M (k · ft)
x (ft)
−150
+
M (k · ft)
x (ft)
−375
+
M (k · ft)
225
x (ft)
superposition of associated moment diagrams
(b)

Fig. 4–24

5

Cables and Arches

5.1 Cables

Cables are often used in engineering structures for support and to transmit loads from one member to another. When used to support suspension roofs, bridges, and trolley wheels, cables form the main load-carrying element in the structure. In the force analysis of such

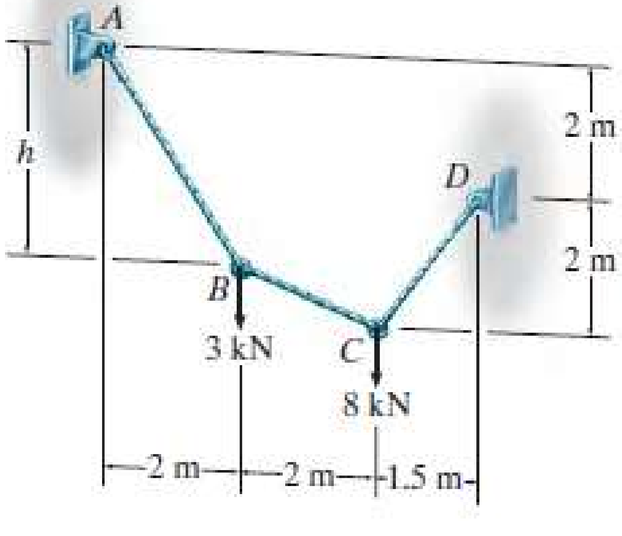
When deriving the necessary relations between the force in the cable and its slope, we will make the assumption that the cable is *perfectly flexible* and *inextensible*. Due to its flexibility, the cable offers no resistance to shear or bending and, therefore, the force acting in the cable is always tangent to the cable at points along its length.

5.2 Cable Subjected to Concentrated Loads

When a cable of negligible weight supports several concentrated loads, the cable takes the form of several straight-line segments, each of which is subjected to a constant tensile force.

EXAMPLE 5.1

Determine the tension in each segment of the cable shown in Fig. 5–2*a*. Also, what is the dimension *h*?

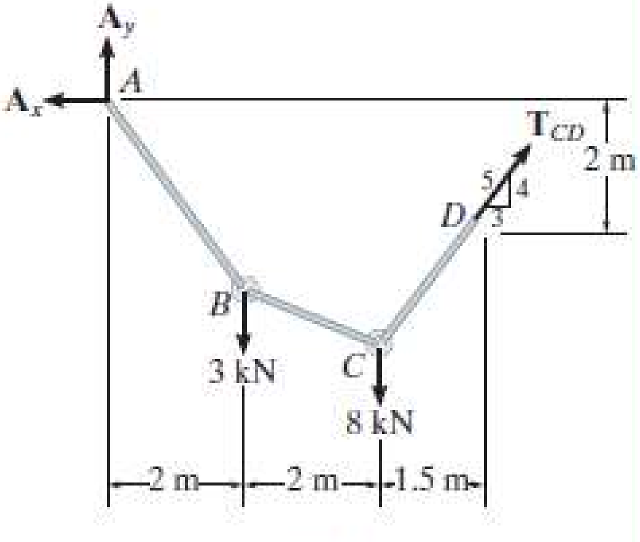


(a)

SOLUTION

By inspection, there are four unknown external reactions (A_x, A_y, D_x, and D_y) and three unknown cable tensions, one in each cable segment. These seven unknowns along with the sag *h* can be determined from the eight available equilibrium equations ($\Sigma F_x = 0$, $\Sigma F_y = 0$) applied to points *A* through *D*.

A more direct approach to the solution is to recognize that the slope of cable *CD* is specified, and so a free-body diagram of the entire cable is shown in Fig. 5–2*b*. We can obtain the tension in segment *CD* as follows:



(b)

$$\circlearrowright + \Sigma M_A = 0;$$

$$T_{CD}(3/5)(2\text{ m}) + T_{CD}(4/5)(5.5\text{ m}) - 3\text{ kN}(2\text{ m}) - 8\text{ kN}(4\text{ m}) = 0$$

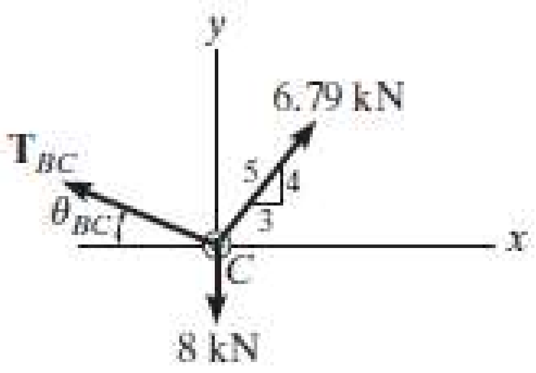
$$T_{CD} = 6.79\text{ kN} \qquad \textit{Ans.}$$

Now we can analyze the equilibrium of points *C* and *B* in sequence. Point *C* (Fig. 5–2*c*);

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad 6.79 \text{ kN}(3/5) - T_{BC} \cos \theta_{BC} = 0$

$+\uparrow \Sigma F_y = 0;$ $\quad 6.79 \text{ kN}(4/5) - 8 \text{ kN} + T_{BC} \sin \theta_{BC} = 0$

$$\theta_{BC} = 32.3^\circ \qquad T_{BC} = 4.82 \text{ kN}$$ *Ans.*



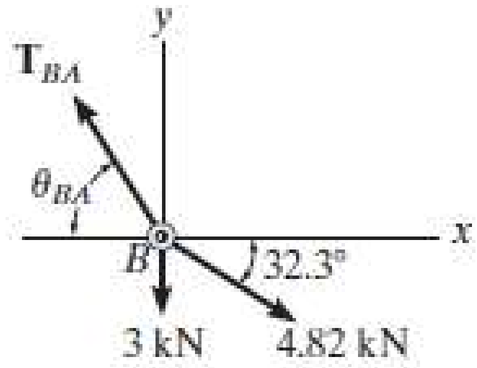
(c)

Point B (Fig. 5–2d);

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad -T_{BA} \cos \theta_{BA} + 4.82 \text{ kN} \cos 32.3^\circ = 0$

$+\uparrow \Sigma F_y = 0;$ $\quad T_{BA} \sin \theta_{BA} - 4.82 \text{ kN} \sin 32.3^\circ - 3 \text{ kN} = 0$

$$\theta_{BA} = 53.8^\circ \qquad T_{BA} = 6.90 \text{ kN}$$ *Ans.*



(d)

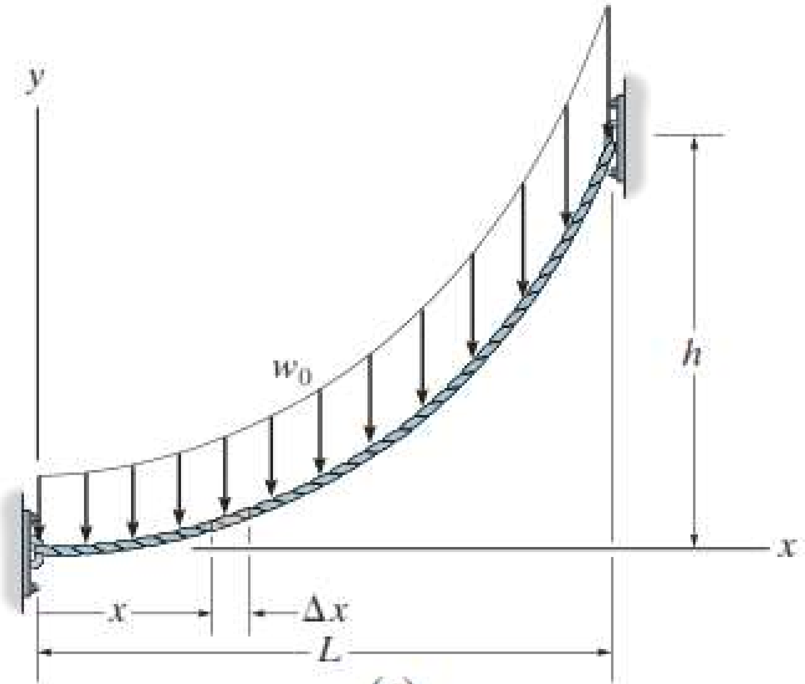
Hence, from Fig. 5–2a,

$$h = (2 \text{ m}) \tan 53.8^\circ = 2.74 \text{ m}$$ *Ans.*

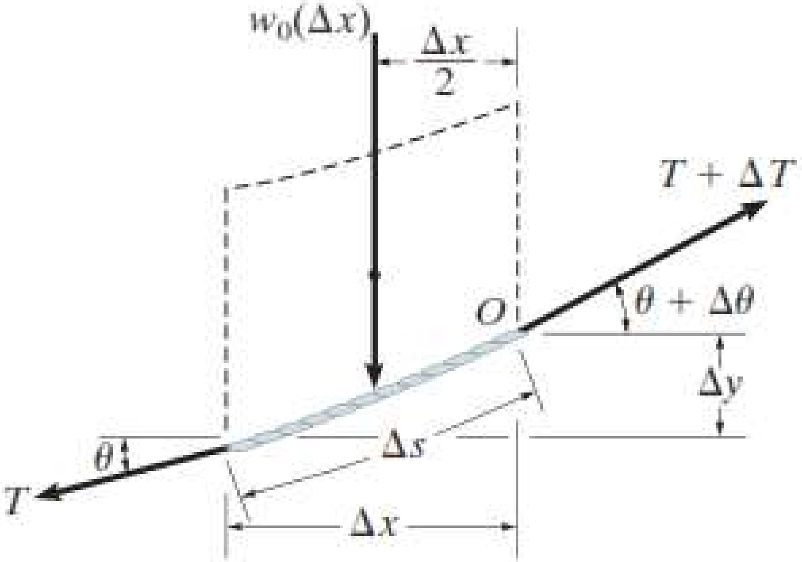
Fig. 5–2

5.3 Cable Subjected to a Uniform Distributed Load

Cables provide a very effective means of supporting the dead weight of girders or bridge decks having very long spans. A suspension bridge is a typical example, in which the deck is suspended from the cable using a series of close and equally spaced hangers.



(a)



(b)

$$T \cos \theta = F_H \tag{5–4}$$

$$\tan \theta = \frac{dy}{dx} = \frac{w_0 x}{F_H} \tag{5–6}$$

$$y = \frac{w_0}{2F_H} x^2 \tag{5–7}$$

$$F_H = \frac{w_0 L^2}{2h} \tag{5–8}$$

$$T_{\max} = \sqrt{F_H^2 + (w_0 L)^2} \tag{5–10}$$

Or, using Eq. 5–8, we can express $T_{\max}$ in terms of w_0, i.e.,

$$T_{\max} = w_0 L \sqrt{1 + (L/2h)^2} \tag{5–11}$$

EXAMPLE 5.2

The cable in Fig. 5–5*a* supports a girder which weighs 850 lb/ft. Determine the tension in the cable at points *A*, *B*, and *C*.

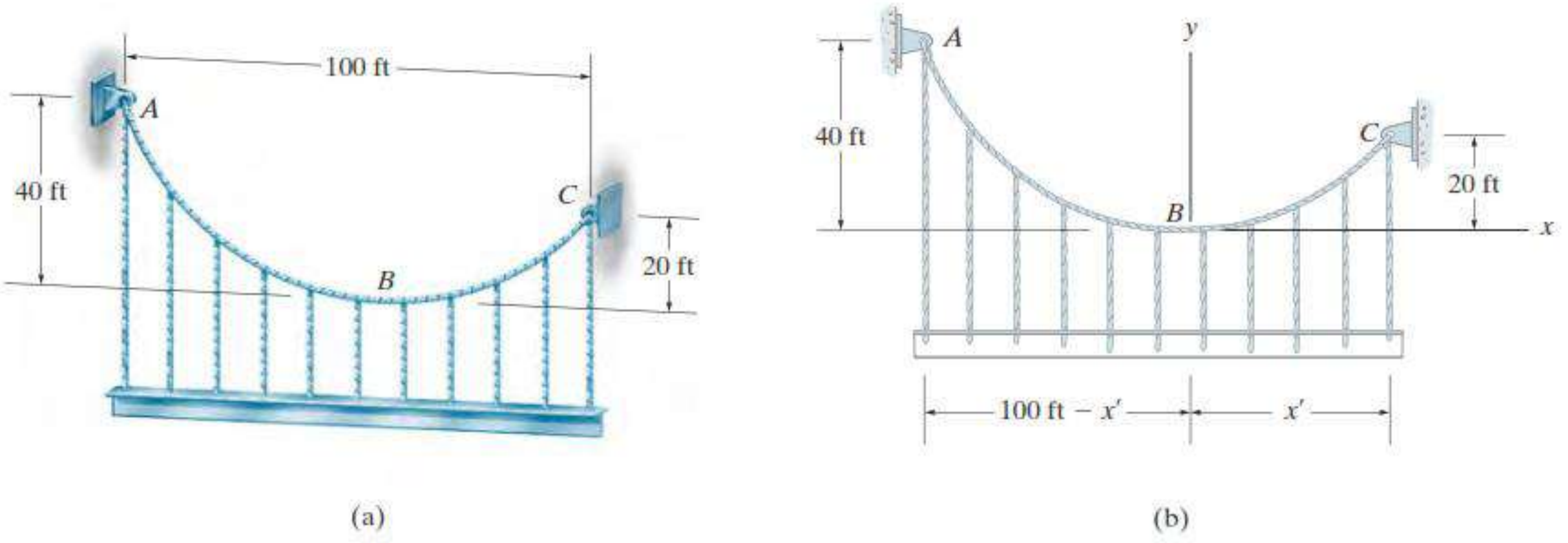


Fig. 5–5

SOLUTION

The origin of the coordinate axes is established at point B, the lowest point on the cable, where the slope is zero, Fig. 5–5b. From Eq. 5–7, the parabolic equation for the cable is:

$$y = \frac{w_0}{2F_H}x^2 = \frac{850 \text{ lb/ft}}{2F_H}x^2 = \frac{425}{F_H}x^2 \qquad (1)$$

Assuming point C is located x' from B, we have

$$20 = \frac{425}{F_H}x'^2$$

$$F_H = 21.25x'^2 \qquad (2)$$

Also, for point A,

$$40 = \frac{425}{F_H}[-(100 - x')]^2$$

$$40 = \frac{425}{21.25x'^2}[-(100 - x')]^2$$

$$x'^2 + 200x' - 10\,000 = 0$$

$$x' = 41.42 \text{ ft}$$

Thus, from Eqs. 2 and 1 (or Eq. 5–6) we have

$$F_H = 21.25(41.42)^2 = 36\,459.2\ \text{lb}$$

$$\frac{dy}{dx} = \frac{850}{36\,459.2}x = 0.02331x \tag{3}$$

At point A,

$$x = -(100 - 41.42) = -58.58\ \text{ft}$$

$$\tan\theta_A = \left.\frac{dy}{dx}\right|_{x=-58.58} = 0.02331(-58.58) = -1.366$$

$$\theta_A = -53.79^\circ$$

Using Eq. 5–4,

$$T_A = \frac{F_H}{\cos\theta_A} = \frac{36\,459.2}{\cos(-53.79^\circ)} = 61.7\ \text{k} \qquad \textit{Ans.}$$

At point B, $x = 0$,

$$\tan\theta_B = \frac{dy}{dx}\bigg|_{x=0} = 0, \qquad \theta_B = 0^\circ$$

$$T_B = \frac{F_H}{\cos\theta_B} = \frac{36\,459.2}{\cos 0^\circ} = 36.5\text{ k} \qquad \textit{Ans.}$$

At point C,

$$x = 41.42\text{ ft}$$

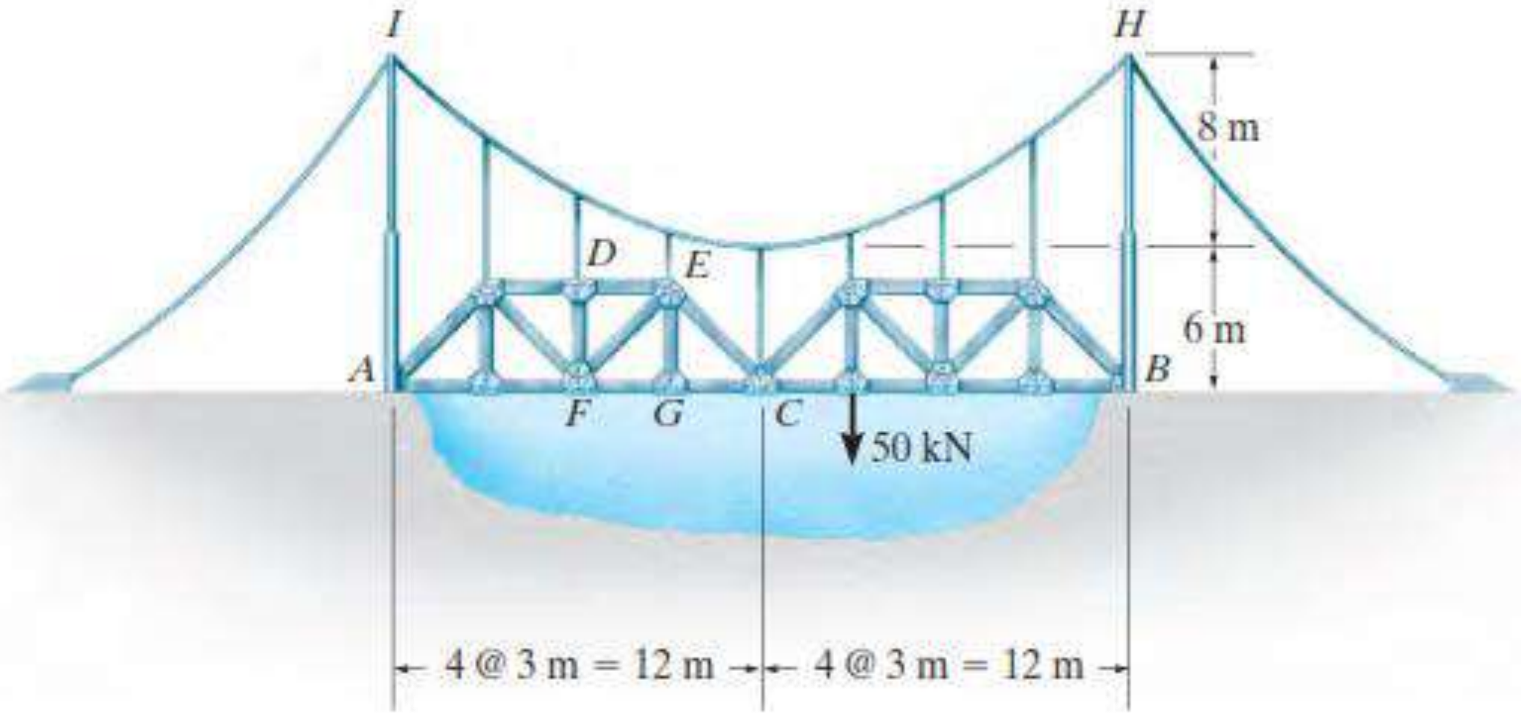
$$\tan\theta_C = \frac{dy}{dx}\bigg|_{x=41.42} = 0.02331(41.42) = 0.9657$$

$$\theta_C = 44.0^\circ$$

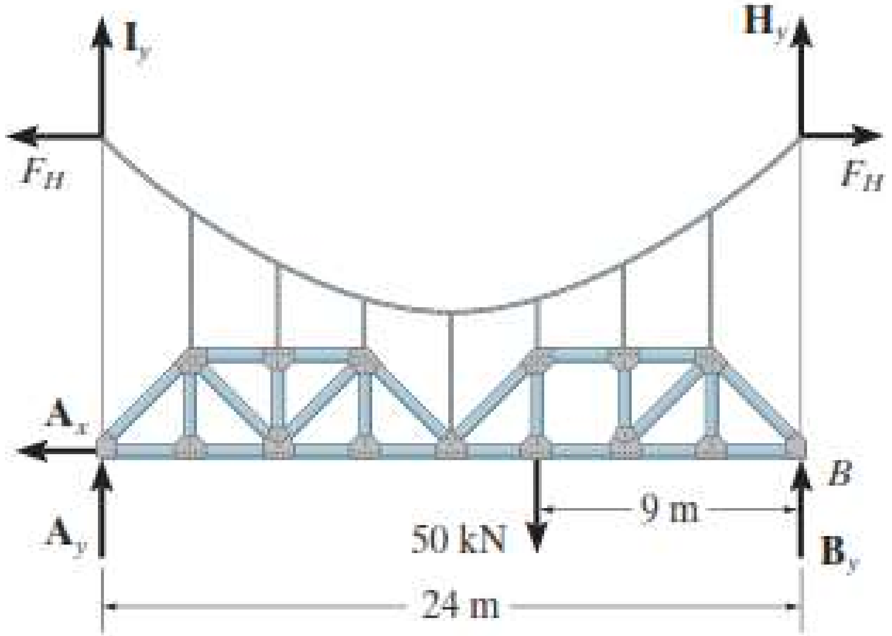
$$T_C = \frac{F_H}{\cos\theta_C} = \frac{36\,459.2}{\cos 44.0^\circ} = 50.7\text{ k} \qquad \textit{Ans.}$$

EXAMPLE 5.3

The suspension bridge in Fig. 5–6*a* is constructed using the two stiffening trusses that are pin connected at their ends *C* and supported by a pin at *A* and a rocker at *B*. Determine the maximum tension in the cable *IH*. The cable has a parabolic shape and the bridge is subjected to the single load of 50 kN.



(a)



(b)

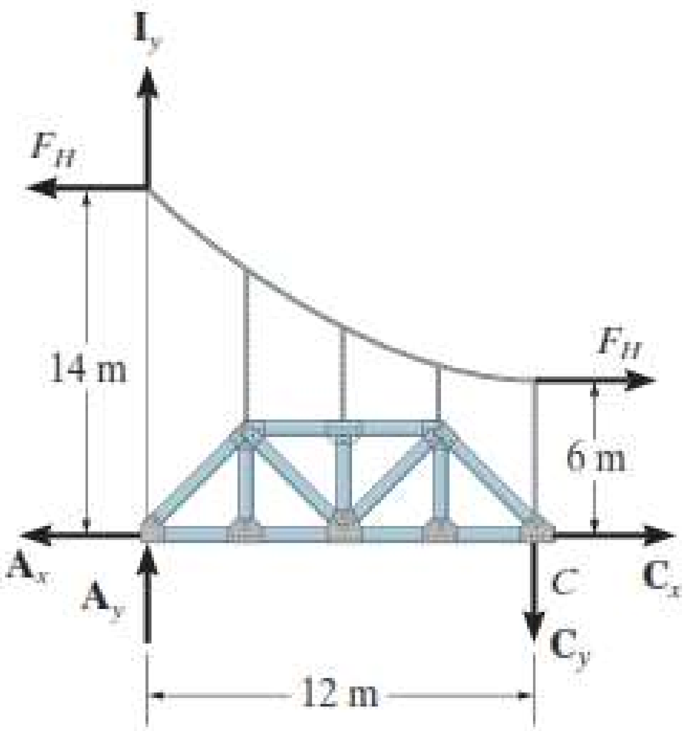
Fig. 5–6

SOLUTION

The free-body diagram of the cable-truss system is shown in Fig. 5–6*b*. According to Eq. 5–4 ($T \cos \theta = F_H$), the horizontal component of cable tension at I and H must be constant, F_H. Taking moments about B, we have

$$\circlearrowleft + \Sigma M_B = 0; \qquad -I_y(24 \text{ m}) - A_y(24 \text{ m}) + 50 \text{ kN}(9 \text{ m}) = 0$$

$$I_y + A_y = 18.75$$



(c)

If only half the suspended structure is considered, Fig. 5–6c, then summing moments about the pin at C, we have

$$\circlearrowright + \Sigma M_C = 0; \quad F_H(14\text{ m}) - F_H(6\text{ m}) - I_y(12\text{ m}) - A_y(12\text{ m}) = 0$$

$$I_y + A_y = 0.667F_H$$

From these two equations,

$$18.75 = 0.667F_H$$
$$F_H = 28.125 \text{ kN}$$

To obtain the maximum tension in the cable, we will use Eq. 5–11, but first it is necessary to determine the value of an assumed uniform distributed loading w_0 from Eq. 5–8:

$$w_0 = \frac{2F_H h}{L^2} = \frac{2(28.125 \text{ kN})(8 \text{ m})}{(12 \text{ m})^2} = 3.125 \text{ kN/m}$$

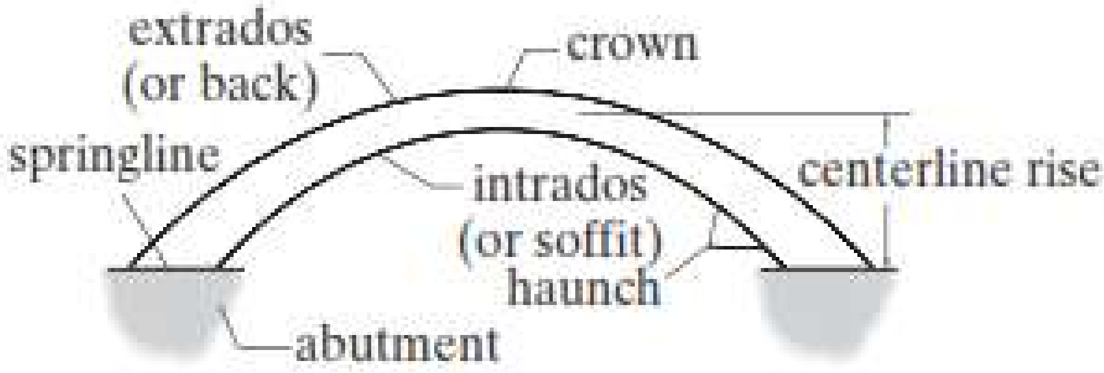
Thus, using Eq. 5–11, we have

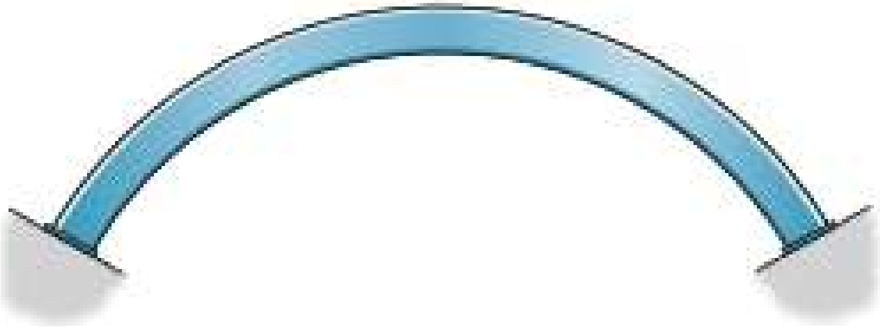
$$\begin{aligned} T_{\max} &= w_0 L\sqrt{1 + (L/2h)^2} \\ &= 3.125(12 \text{ m})\sqrt{1 + (12 \text{ m}/2(8 \text{ m}))^2} \\ &= 46.9 \text{ kN} \end{aligned}$$

Ans.

5.4 Arches

Like cables, arches can be used to reduce the bending moments in long-span structures. Essentially, an arch acts as an inverted cable, so it receives its load mainly in compression although, because of its rigidity, it must also resist some bending and shear depending upon how it is loaded and shaped. In particular, if the arch has a *parabolic shape* and it is subjected to a *uniform* horizontally distributed vertical load, then from the analysis of cables it follows that *only compressive forces* will be resisted by the arch. Under these conditions the arch shape is called a *funicular arch* because no bending or shear forces occur within the arch.

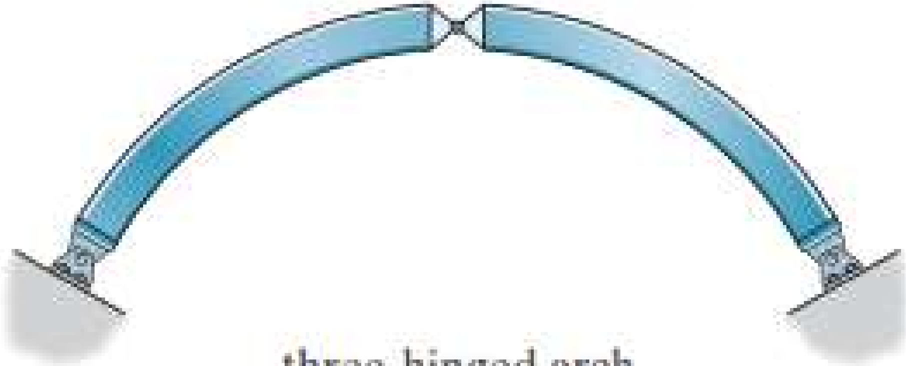




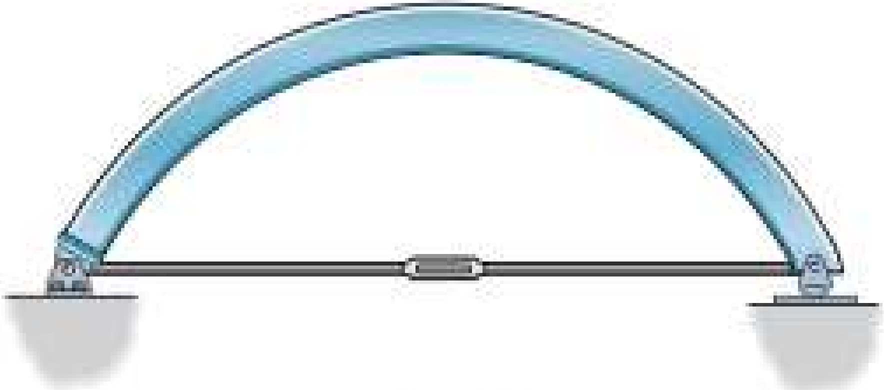
fixed arch
(a)

two-hinged arch

(b)



three-hinged arch
(c)



tied arch
(d)

5.5 Three-Hinged Arch

To provide some insight as to how arches transmit loads, we will now consider the analysis of a three-hinged arch such as the one shown in Fig. 5–9*a*. In this case, the third hinge is located at the crown and the supports are located at different elevations. In order to determine the reactions at the supports, the arch is disassembled and the free-body diagram of each member is shown in Fig. 5–9*b*. Here there are six unknowns for which six equations of equilibrium are available. One method of solving this problem is to apply the moment equilibrium equations about points A and B. Simultaneous solution will yield the reactions C_x and C_y. The support reactions are then determined from the force equations of equilibrium. Once obtained, the internal normal force, shear, and moment loadings at any point along the arch can be found using the method of sections. Here, of course, the section should be taken perpendicular to the axis of the arch at the point considered. For example, the free-body diagram for segment AD is shown in Fig. 5–9*c*.

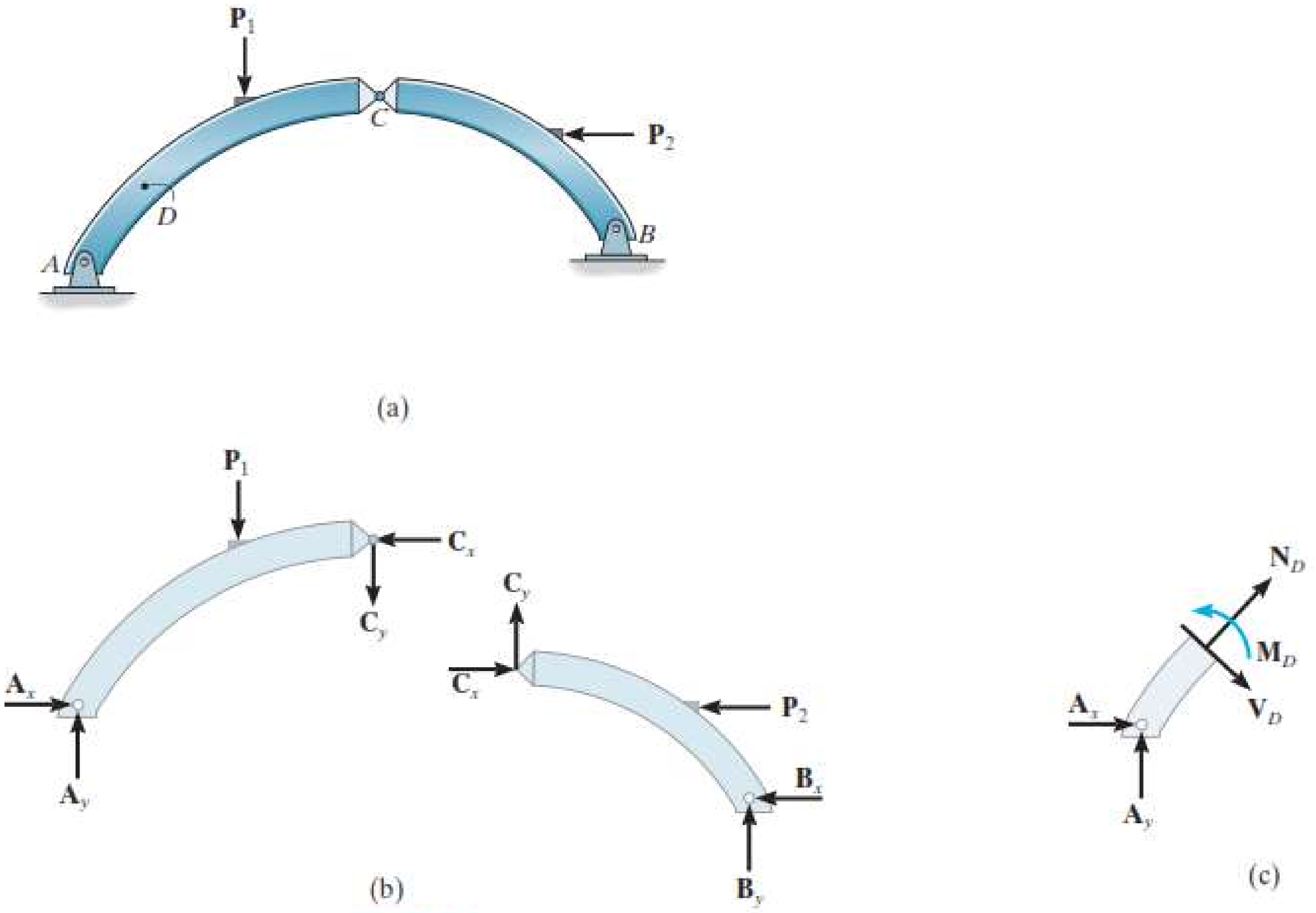

$\mathbf{P}_1$
C
$\mathbf{P}_2$
D
A
B
(a)
$\mathbf{P}_1$
$\mathbf{C}_x$
$\mathbf{C}_y$
$\mathbf{C}_y$
$\mathbf{C}_x$
$\mathbf{A}_x$
$\mathbf{P}_2$
$\mathbf{B}_x$
$\mathbf{A}_y$
$\mathbf{B}_y$
(b)
$\mathbf{N}_D$
$\mathbf{M}_D$
$\mathbf{A}_x$
$\mathbf{V}_D$
$\mathbf{A}_y$
(c)

Fig. 5–9

EXAMPLE 5.4

The three-hinged open-spandrel arch bridge like the one shown in the photo has a parabolic shape. If this arch were to support a uniform load and have the dimensions shown in Fig. 5–10*a*, show that the arch is subjected *only to axial compression* at any intermediate point such as point D. Assume the load is uniformly transmitted to the arch ribs.

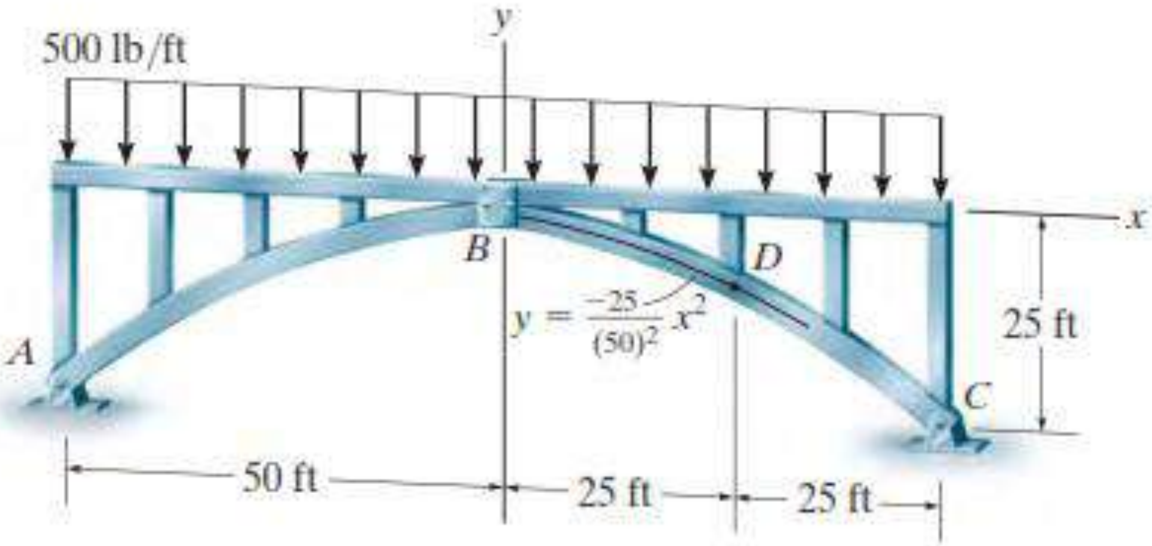
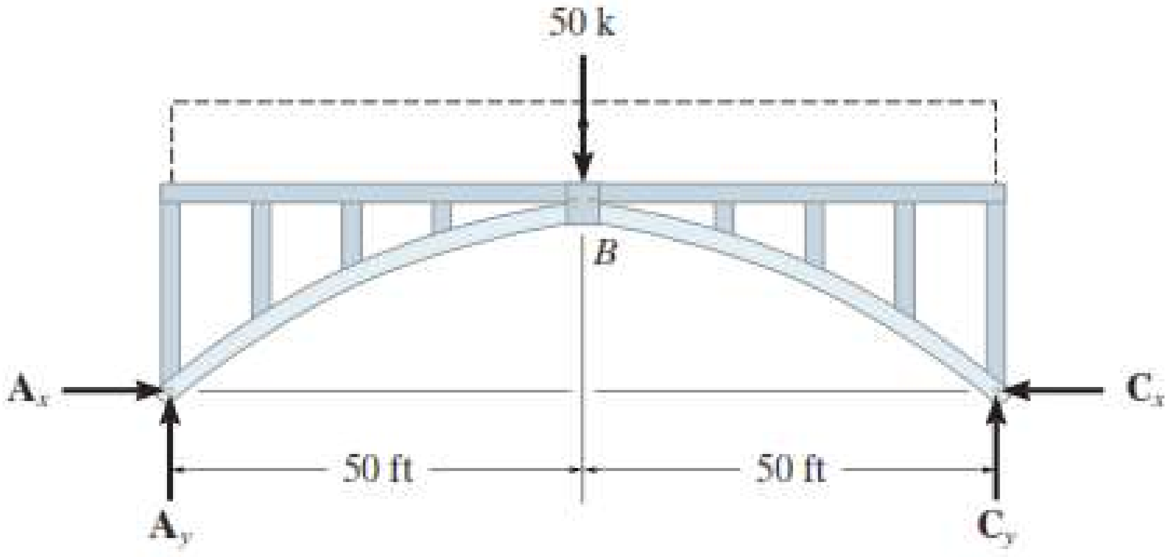


Fig. 5–10

SOLUTION

Here the supports are at the same elevation. The free-body diagrams of the entire arch and part BC are shown in Fig. 5–10b and Fig. 5–10c. Applying the equations of equilibrium, we have:

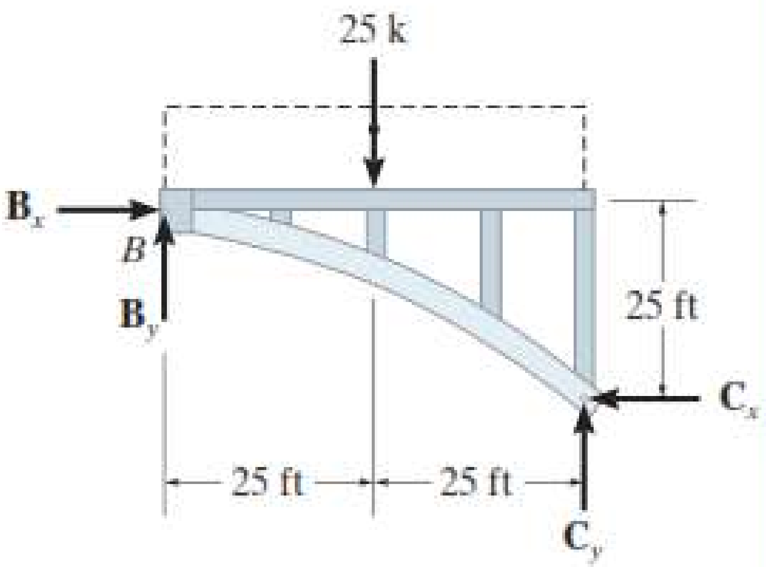


(b)

Entire arch:

$$\circlearrowright\!+\Sigma M_A = 0; \qquad C_y(100\text{ ft}) - 50\text{ k}(50\text{ ft}) = 0$$
$$C_y = 25\text{ k}$$

Arch segment BC:



(c)

$\circlearrowright + \Sigma M_B = 0; \qquad -25\text{ k}(25\text{ ft}) + 25\text{ k}(50\text{ ft}) - C_x(25\text{ ft}) = 0$

$$C_x = 25\text{ k}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad B_x = 25\text{ k}$

$+\uparrow \Sigma F_y = 0; \qquad B_y - 25\text{ k} + 25\text{ k} = 0$

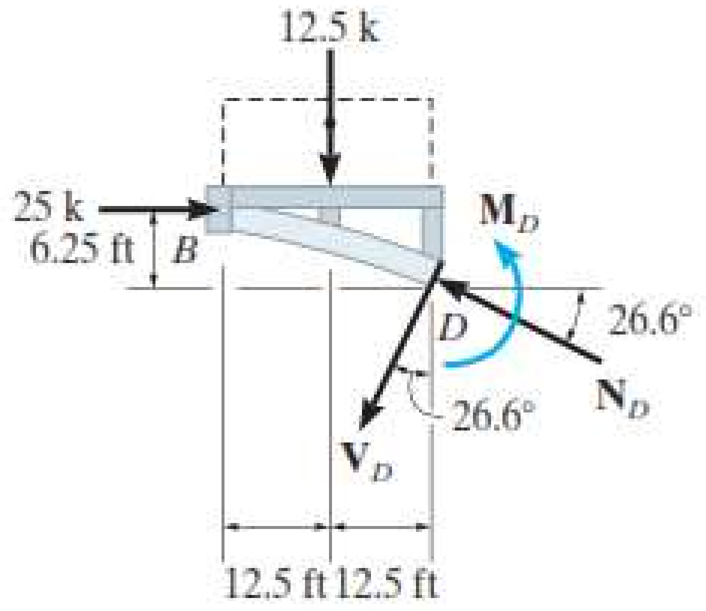
$$B_y = 0$$

A section of the arch taken through point D, $x = 25$ ft, $y = -25(25)^2/(50)^2 = -6.25$ ft, is shown in Fig. 5–10d. The slope of the segment at D is

$$\tan\theta = \frac{dy}{dx} = \frac{-50}{(50)^2}x\bigg|_{x=25\text{ ft}} = -0.5$$

$$\theta = -26.6^\circ$$

Applying the equations of equilibrium, Fig. 5–10*d* we have



(d)

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 25\text{ k} - N_D \cos 26.6^\circ - V_D \sin 26.6^\circ = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad -12.5\text{ k} + N_D \sin 26.6^\circ - V_D \cos 26.6^\circ = 0$$

$$\circlearrowright + \Sigma M_D = 0; \qquad M_D + 12.5\text{ k}(12.5\text{ ft}) - 25\text{ k}(6.25\text{ ft}) = 0$$

$$N_D = 28.0\text{ k} \qquad \textit{Ans.}$$

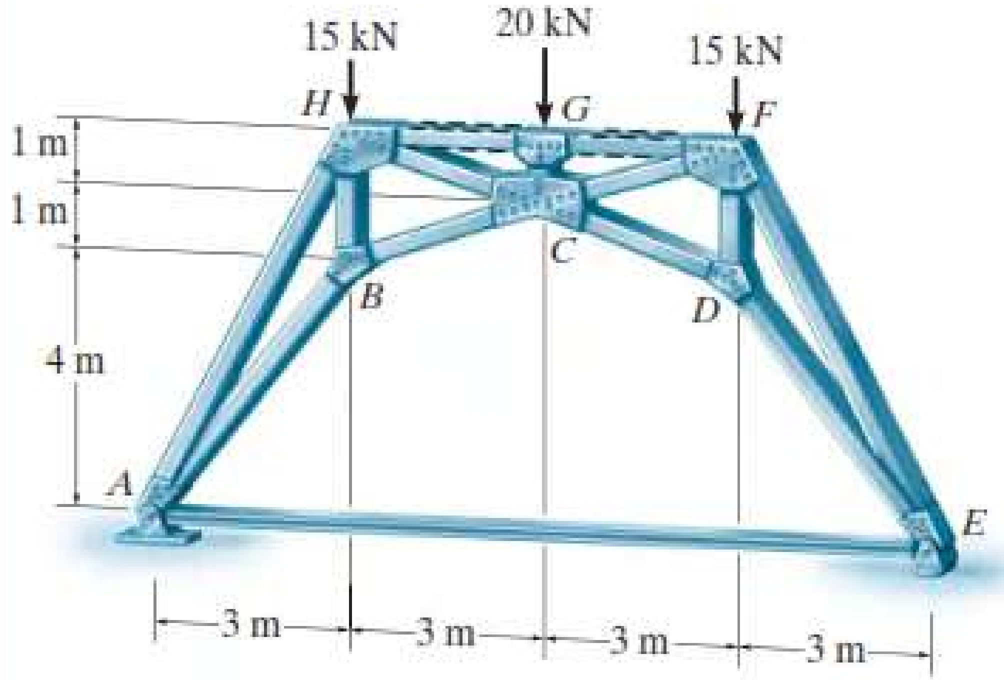
$$V_D = 0 \qquad \textit{Ans.}$$

$$M_D = 0 \qquad \textit{Ans.}$$

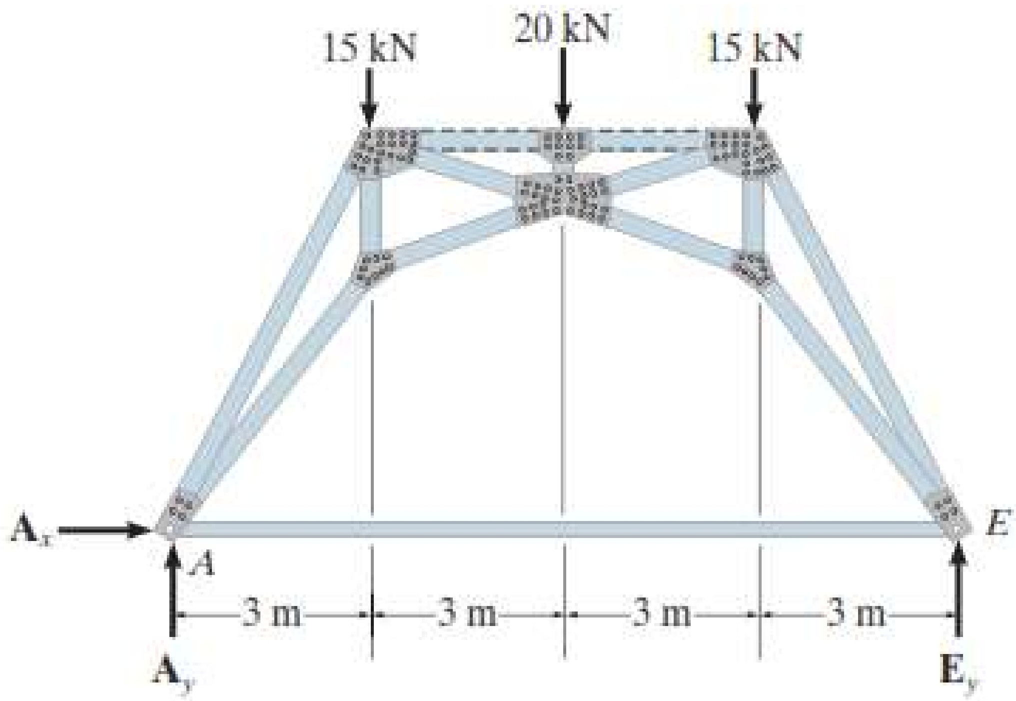
Note: If the arch had a different shape or if the load were nonuniform, then the internal shear and moment would be nonzero. Also, if a simply supported beam were used to support the distributed loading, it would have to resist a maximum bending moment of $M = 625\text{ k} \cdot \text{ft}$. By comparison, it is more efficient to structurally resist the load in direct compression (although one must consider the possibility of buckling) than to resist the load by a bending moment.

EXAMPLE 5.5

The three-hinged tied arch is subjected to the loading shown in Fig. 5–11*a*. Determine the force in members *CH* and *CB*. The dashed member *GF* of the truss is intended to carry no force.



(a)



(b)

Fig. 5–11

SOLUTION

The support reactions can be obtained from a free-body diagram of the entire arch, Fig. 5–11b:

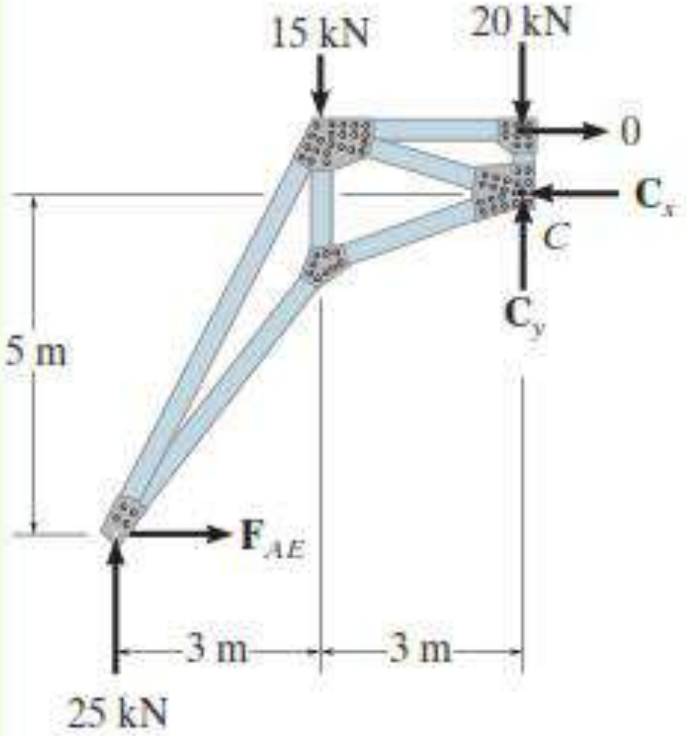
$$\circlearrowright + \Sigma M_A = 0; \quad E_y(12\text{ m}) - 15\text{ kN}(3\text{ m}) - 20\text{ kN}(6\text{ m}) - 15\text{ kN}(9\text{ m}) = 0$$

$$E_y = 25\text{ kN}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 15\text{ kN} - 20\text{ kN} - 15\text{ kN} + 25\text{ kN} = 0$$

$$A_y = 25\text{ kN}$$



(c)

The force components acting at joint C can be determined by considering the free-body diagram of the left part of the arch, Fig. 5–11c. First, we determine the force:

$$\circlearrowright + \Sigma M_C = 0; \quad F_{AE}(5\text{ m}) - 25\text{ kN}(6\text{ m}) + 15\text{ kN}(3\text{ m}) = 0$$

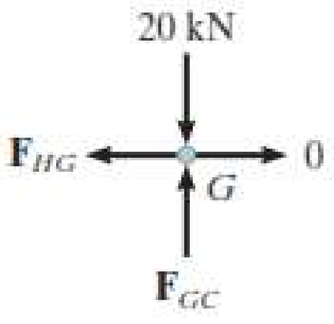
$$F_{AE} = 21.0\text{ kN}$$

Then,

$\xrightarrow{+} \Sigma F_x = 0; \qquad -C_x + 21.0\text{ kN} = 0, \quad C_x = 21.0\text{ kN}$

$+\uparrow \Sigma F_y = 0; \qquad 25\text{ kN} - 15\text{ kN} - 20\text{ kN} + C_y = 0, \quad C_y = 10\text{ kN}$

To obtain the forces in *CH* and *CB*, we can use the method of joints as follows:

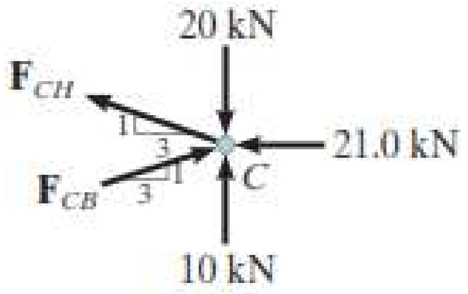


(d)

Joint *G*; Fig. 5–11*d*,

$$+\uparrow \Sigma F_y = 0; \qquad F_{GC} - 20\text{ kN} = 0$$

$$F_{GC} = 20\text{ kN (C)}$$



(e)

Joint *C*; Fig. 5–11*e*,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{CB}\left(\frac{3}{\sqrt{10}}\right) - 21.0\text{ kN} - F_{CH}\left(\frac{3}{\sqrt{10}}\right) = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{CB}\left(\frac{1}{\sqrt{10}}\right) + F_{CH}\left(\frac{1}{\sqrt{10}}\right) - 20\text{ kN} + 10\text{ kN} = 0$$

Thus,

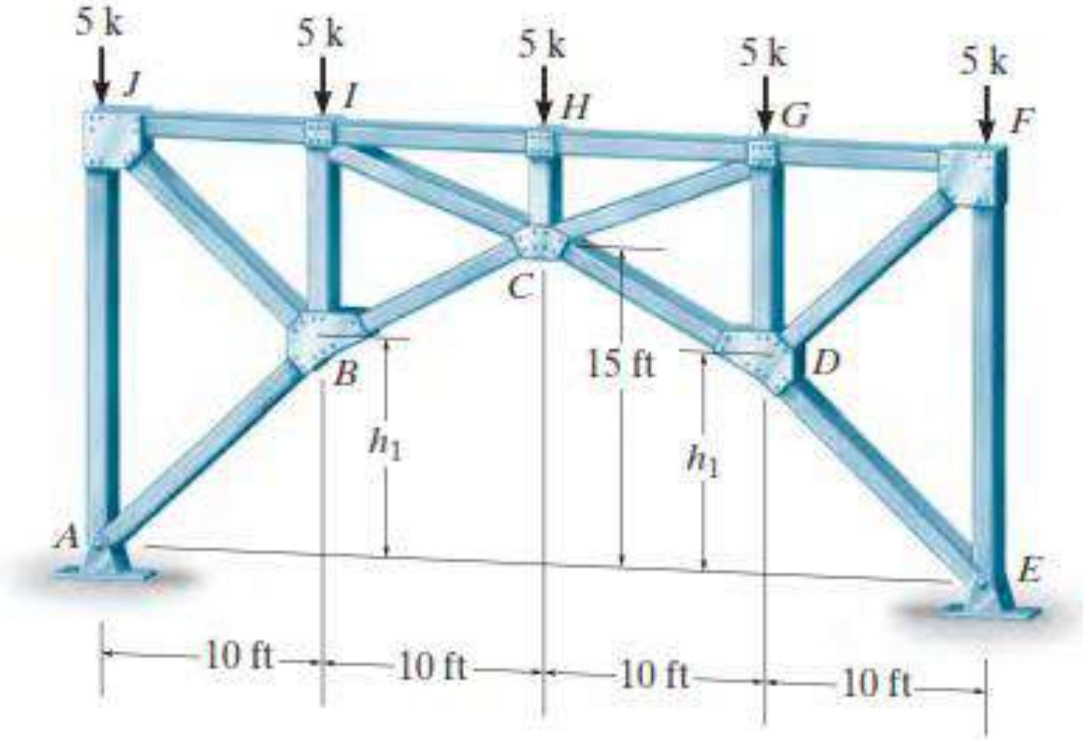
$$F_{CB} = 26.9 \text{ kN (C)} \qquad \textit{Ans.}$$

$$F_{CH} = 4.74 \text{ kN (T)} \qquad \textit{Ans.}$$

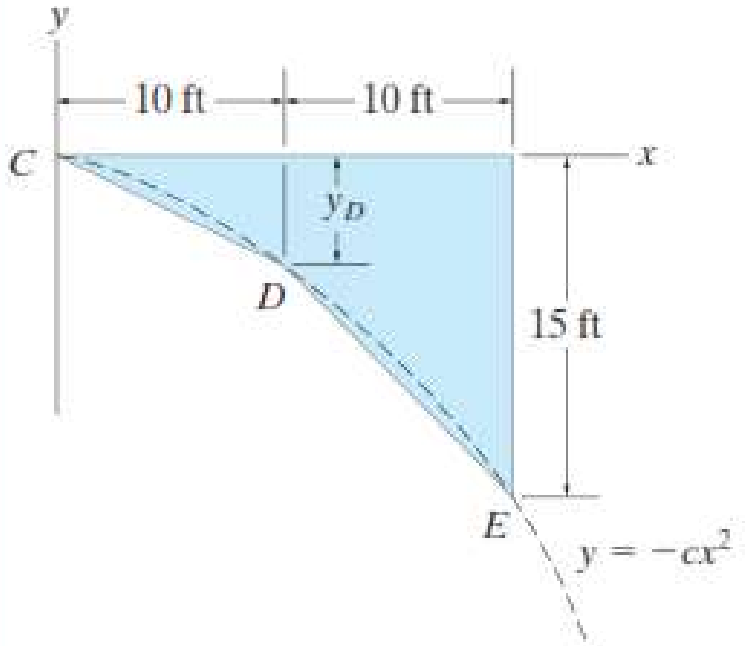
Note: Tied arches are sometimes used for bridges. Here the deck is supported by suspender bars that transmit their load to the arch. The deck is in tension so that it supports the actual thrust or horizontal force at the ends of the arch.

EXAMPLE 5.6

The three-hinged trussed arch shown in Fig. 5–12*a* supports the symmetric loading. Determine the required height h_1 of the joints *B* and *D*, so that the arch takes a funicular shape. Member *HG* is intended to carry no force.



(a)



(b)

Fig. 5–12

SOLUTION

For a symmetric loading, the funicular shape for the arch must be *parabolic* as indicated by the dashed line (Fig. 5–12*b*). Here we must find the equation which fits this shape. With the x, y axes having an origin at C, the equation is of the form $y = -cx^2$. To obtain the constant c, we require

$$-(15\text{ ft}) = -c(20\text{ ft})^2$$
$$c = 0.0375/\text{ft}$$

Therefore,

$$y_D = -(0.0375/\text{ft})(10\text{ ft})^2 = -3.75\text{ ft}$$

So that from Fig. 5–12*a*,

$$h_1 = 15\text{ ft} - 3.75\text{ ft} = 11.25\text{ ft} \qquad \textit{Ans.}$$

Using this value, if the method of joints is now applied to the truss, the results will show that the top cord and diagonal members will all be zero-force members, and the symmetric loading will be supported *only by the bottom cord* members AB, BC, CD, and DE of the truss.

6

Influence Lines for Statically Determinate Structures

6.1 Influence Lines

In the previous chapters we developed techniques for analyzing the forces in structural members due to *dead* or *fixed loads*. It was shown that the *shear* and *moment diagrams* represent the most descriptive methods for displaying the variation of these loads in a member. If a structure is subjected to a *live* or *moving load*, however, the variation of the shear and bending moment in the member is best described using the *influence line*. An influence line represents the variation of either the reaction, shear, moment, or deflection at a *specific point* in a member as a concentrated force moves over the member. Once this line is constructed, one can tell at a glance where the moving load should be placed on the structure so that it creates the greatest influence at the specified point. Furthermore, the magnitude of the associated reaction, shear, moment, or deflection at the point can then be calculated from the ordinates of the influence-line diagram. For these reasons, influence lines play an important part in the design of bridges, industrial crane rails, conveyors, and other structures where loads move across their span.

Procedure for Analysis

Either of the following two procedures can be used to construct the influence line at a specific point P in a member for any function (reaction, shear, or moment). For both of these procedures we will choose the moving force to have a *dimensionless magnitude of unity.**

Tabulate Values

- Place a unit load at various locations, x, along the member, and at *each* location use statics to determine the value of the function (reaction, shear, or moment) at the specified point.
- If the influence line for a vertical force *reaction* at a point on a beam is to be constructed, consider the reaction to be *positive* at the point when it acts *upward* on the beam.
- If a shear or moment influence line is to be drawn for a point, take the shear or moment at the point as positive according to the same sign convention used for drawing shear and moment diagrams. (See Fig. 4–1.)
- All statically determinate beams will have influence lines that consist of straight line segments. After some practice one should be able to minimize computations and locate the unit load *only* at points representing the *end points* of each line segment.

- To avoid errors, it is recommended that one first construct a table, listing "unit load at x" versus the corresponding value of the function calculated at the specific point; that is, "reaction R," "shear V," or "moment M." Once the load has been placed at various points along the span of the member, the tabulated values can be plotted and the influence-line segments constructed.

Influence-Line Equations

- The influence line can also be constructed by placing the unit load at a *variable* position x on the member and then computing the value of R, V, or M at the point as a function of x. In this manner, the equations of the various line segments composing the influence line can be determined and plotted.

*The reason for this choice will be explained in Sec. 6–2.

EXAMPLE 6.1

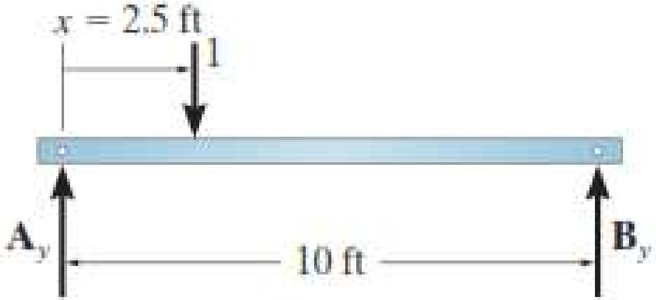
Construct the influence line for the vertical reaction at A of the beam in Fig. 6–1*a*.

(a)

Fig. 6–1

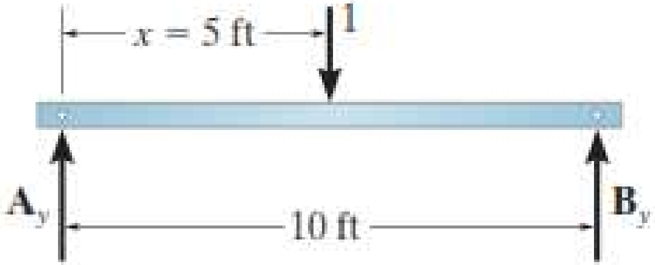
SOLUTION

Tabulate Values. A unit load is placed on the beam at each selected point x and the value of A_y is calculated by summing moments about B. For example, when $x = 2.5$ ft and $x = 5$ ft, see Figs. 6–1*b* and 6–1*c*, respectively. The results for A_y are entered in the table, Fig. 6–1*d*. A plot of these values yields the influence line for the reaction at A, Fig. 6–1*e*.



$$\circlearrowleft + \Sigma M_B = 0; \; -A_y\,(10) + 1\,(7.5) = 0$$
$$A_y = 0.75$$

(b)

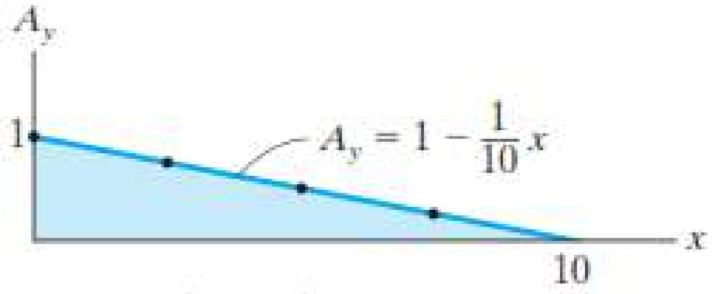


$$\circlearrowleft + \Sigma M_B = 0; \; -A_y\,(10) + 1\,(5) = 0$$
$$A_y = 0.5$$

(c)

x	A_y
0	1
2.5	0.75
5	0.5
7.5	0.25
10	0

(d)



influence line for A_y

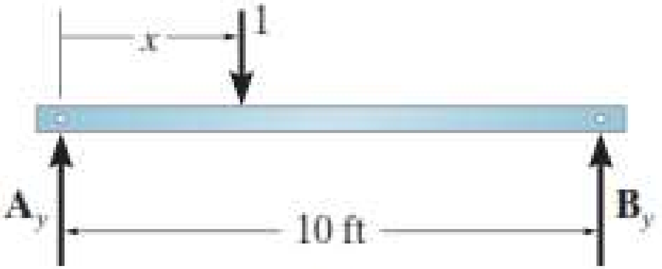
(e)

Influence-Line Equation. When the unit load is placed a variable distance x from A, Fig. 6–1f, the reaction A_y as a function of x can be determined from

$$\circlearrowleft + \Sigma M_B = 0; \qquad -A_y(10) + (10 - x)(1) = 0$$

$$A_y = 1 - \tfrac{1}{10}x$$

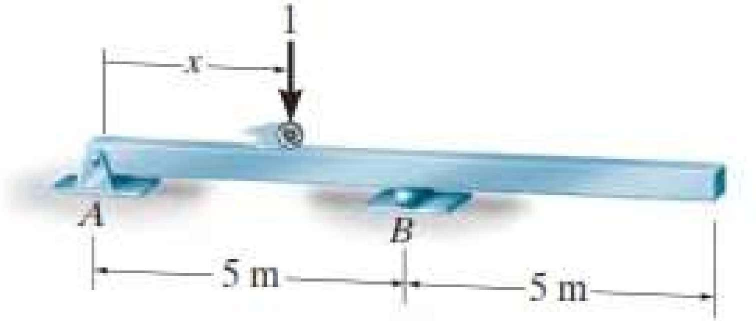
This line is plotted in Fig. 6–1e.



(f)

EXAMPLE 6.2

Construct the influence line for the vertical reaction at B of the beam in Fig. 6–2a.



(a)

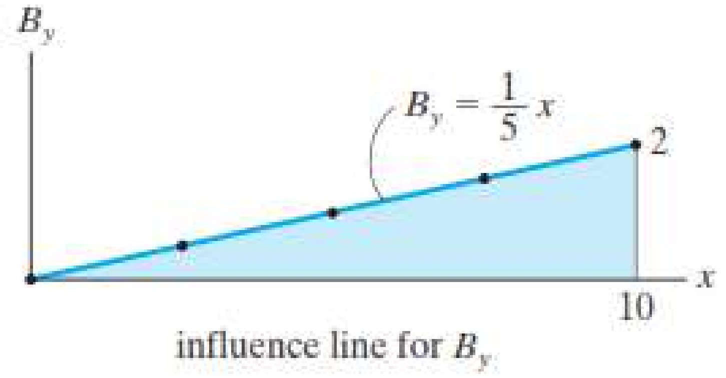
Fig. 6–2

SOLUTION

Tabulate Values. Using statics, verify that the values for the reaction B_y listed in the table, Fig. 6–2b, are correctly computed for each position x of the unit load. A plot of the values yields the influence line in Fig. 6–2c.

x	B_y
0	0
2.5	0.5
5	1
7.5	1.5
10	2

(b)



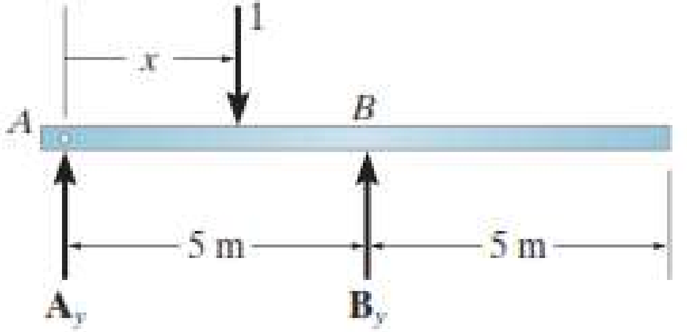
(c)

Influence-Line Equation. Applying the moment equation about A, in Fig. 6–2d,

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(5) - 1(x) = 0$$

$$B_y = \tfrac{1}{5}x$$

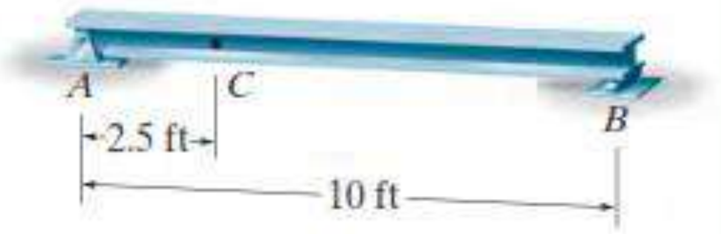
This is plotted in Fig. 6–2c.



(d)

EXAMPLE 6.3

Construct the influence line for the shear at point C of the beam in Fig. 6–3*a*.

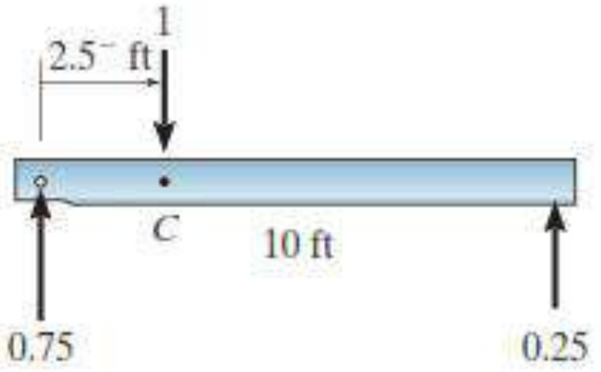


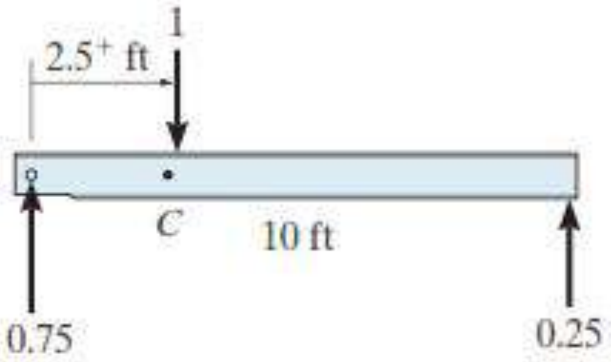
(a)

Fig. 6–3

SOLUTION

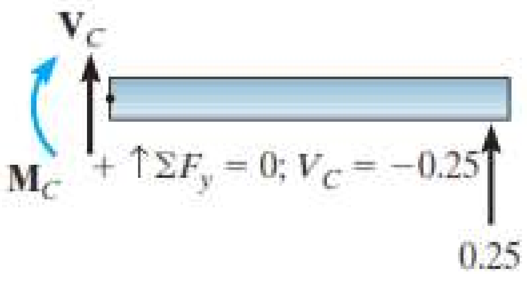
Tabulate Values. At each selected position x of the unit load, the method of sections is used to calculate the value of V_C. Note in particular that the unit load must be placed just to the left ($x = 2.5^-$) and just to the right ($x = 2.5^+$) of point C since the shear is discontinuous at C, Figs. 6–3*b* and 6–3*c*. A plot of the values in Fig. 6–3*d* yields the influence line for the shear at C, Fig. 6–3*e*.



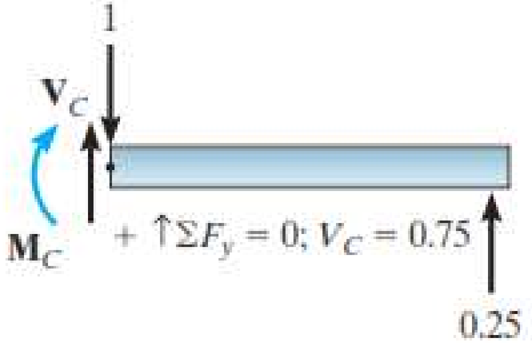


x	V_C
0	0
2.5^-	-0.25
2.5^+	0.75
5	0.5
7.5	0.25
10	0

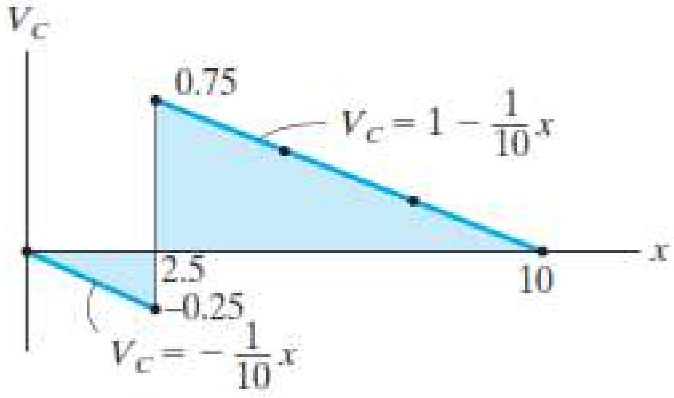
(d)



(b)



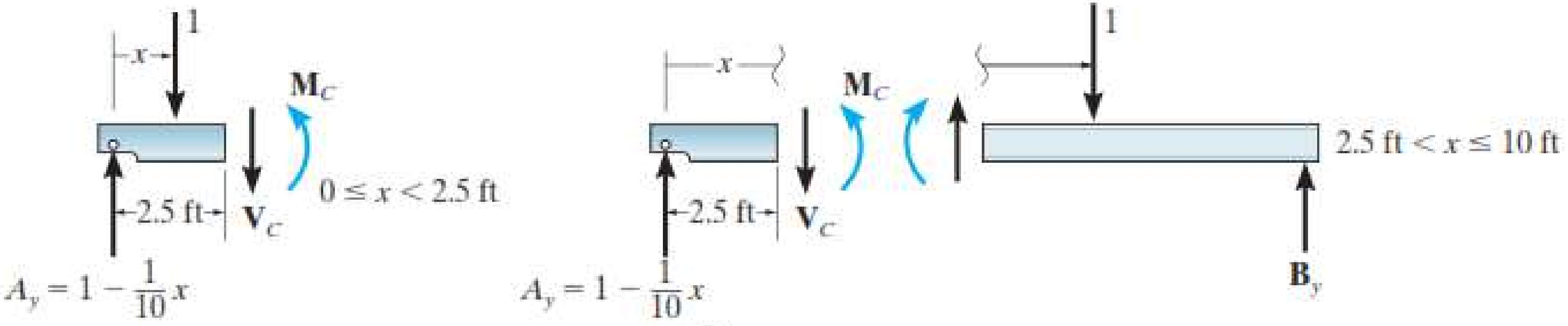
(c)



influence line for V_C

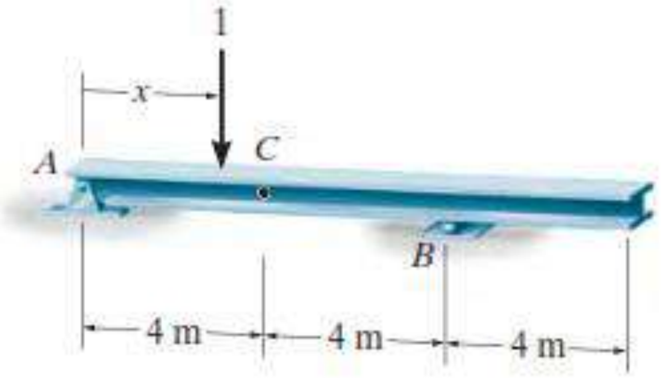
(e)

Influence-Line Equations. Here two equations have to be determined since there are two segments for the influence line due to the discontinuity of shear at C, Fig. 6–3*f*. These equations are plotted in Fig. 6–3*e*.



(f)

EXAMPLE 6.4



(a)

Fig. 6–4

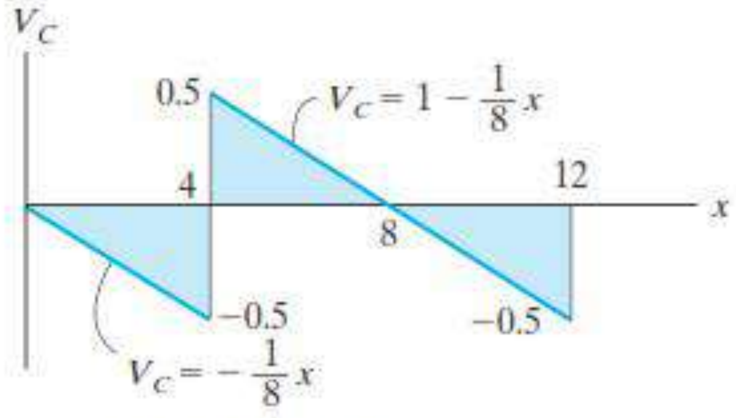
Construct the influence line for the shear at point C of the beam in Fig. 6–4a.

SOLUTION

Tabulate Values. Using statics and the method of sections, verify that the values of the shear V_C at point C in Fig. 6–4b correspond to each position x of the unit load on the beam. A plot of the values in Fig. 6–4b yields the influence line in Fig. 6–4c.

x	V_C
0	0
4^-	−0.5
4^+	0.5
8	0
12	−0.5

(b)



influence line for V_C

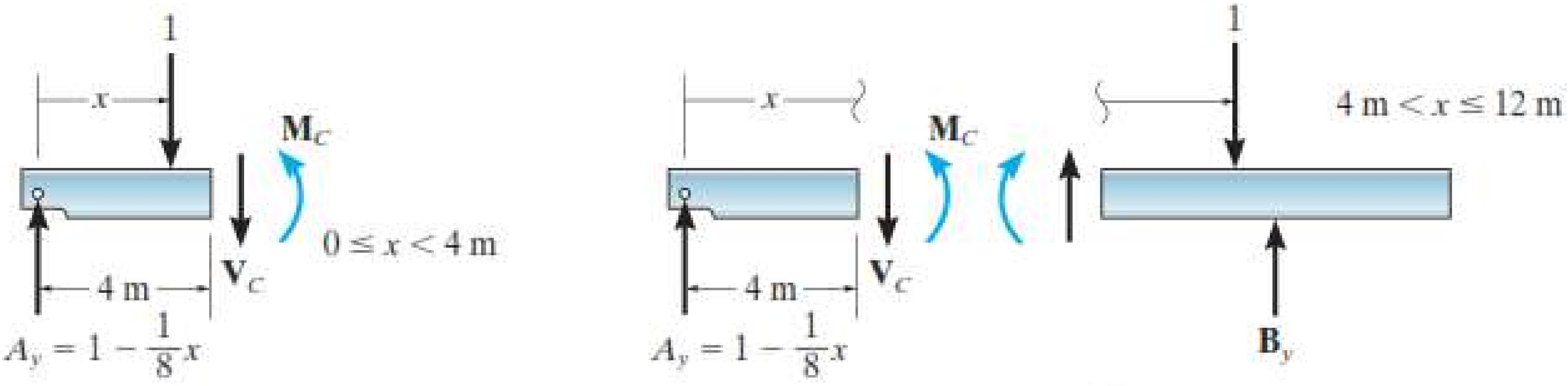
(c)

Influence-Line Equations. From Fig. 6–4*d*, verify that

$$V_C = -\tfrac{1}{8}x \qquad 0 \le x < 4\text{ m}$$

$$V_C = 1 - \tfrac{1}{8}x \qquad 4\text{ m} < x \le 12\text{ m}$$

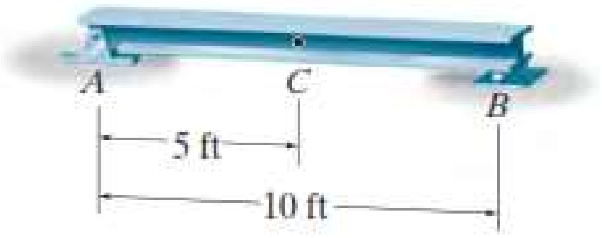
These equations are plotted in Fig. 6–4*c*.



(d)

EXAMPLE 6.5

Construct the influence line for the moment at point C of the beam in Fig. 6–5*a*.

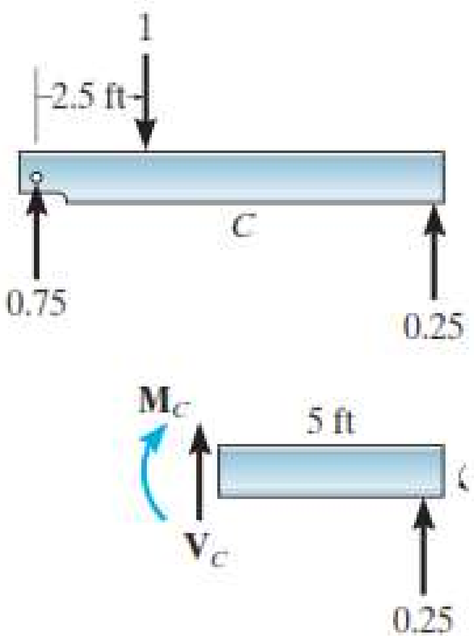


(a)

Fig. 6–5

SOLUTION

Tabulate Values. At each selected position of the unit load, the value of M_C is calculated using the method of sections. For example, see Fig. 6–5*b* for $x = 2.5$ ft. A plot of the values in Fig. 6–5*c* yields the influence line for the moment at C, Fig. 6–5*d*.

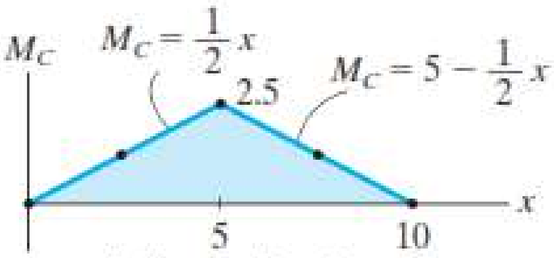


$\curvearrowleft + \Sigma M_C = 0;\ -M_C + 0.25\,(5) = 0$
$M_C = 1.25$

(b)

x	M_C
0	0
2.5	1.25
5	2.5
7.5	1.25
10	0

(c)



influence line for M_C

(d)

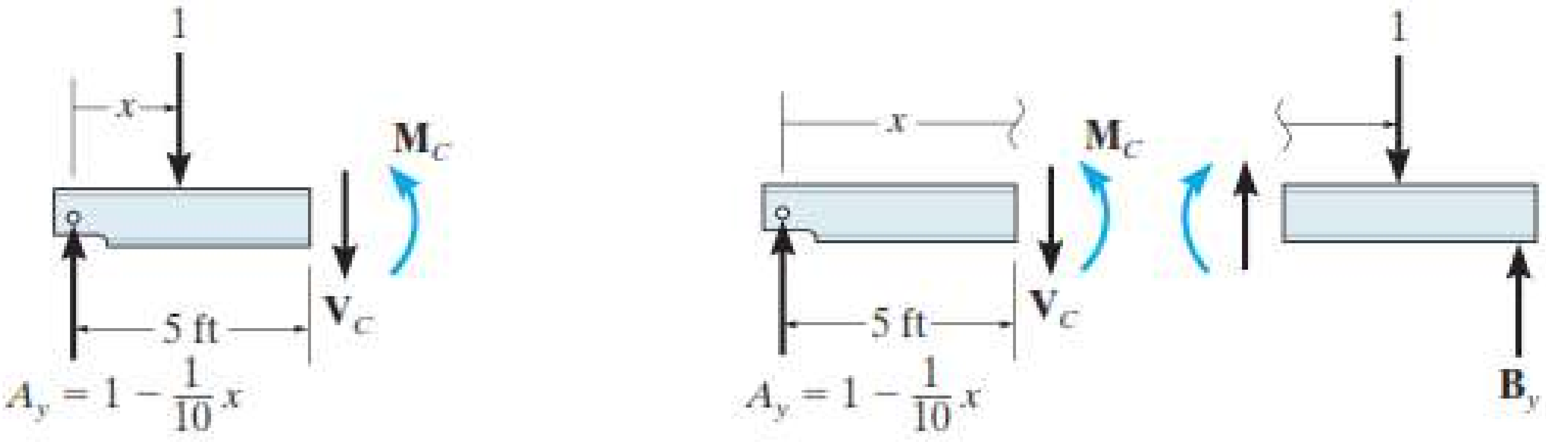
Influence-Line Equations. The two line segments for the influence line can be determined using $\Sigma M_C = 0$ along with the method of sections shown in Fig. 6–5*e*. These equations when plotted yield the influence line shown in Fig. 6–5*d*.

$$\circlearrowright + \Sigma M_C = 0; \qquad M_C + 1(5 - x) - \left(1 - \tfrac{1}{10}x\right)5 = 0$$

$$M_C = \tfrac{1}{2}x \qquad 0 \leq x < 5 \text{ ft}$$

$$\circlearrowright + \Sigma M_C = 0; \qquad M_C - \left(1 - \tfrac{1}{10}x\right)5 = 0$$

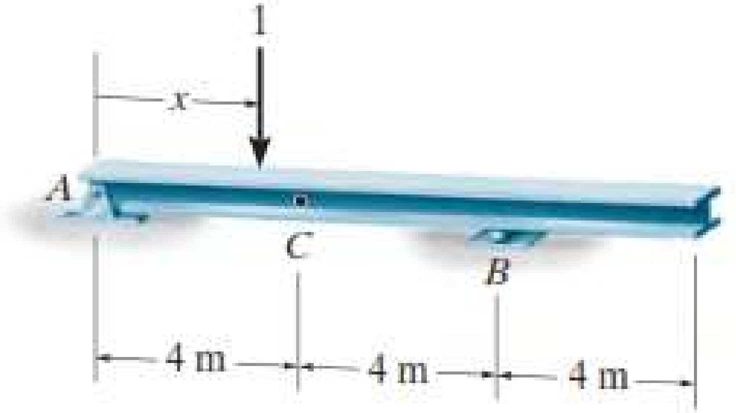
$$M_C = 5 - \tfrac{1}{2}x \qquad 5 \text{ ft} < x \leq 10 \text{ ft}$$



(e)

EXAMPLE 6.6

Construct the influence line for the moment at point C of the beam in Fig. 6–6a.



(a)

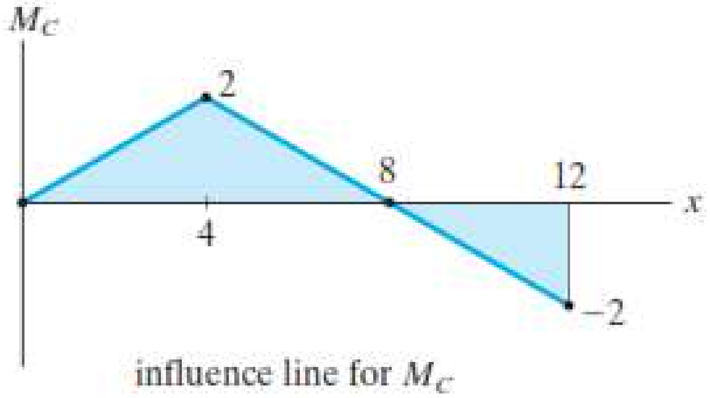
Fig. 6–6

SOLUTION

Tabulate Values. Using statics and the method of sections, verify that the values of the moment M_C at point C in Fig. 6–6b correspond to each position x of the unit load. A plot of the values in Fig. 6–6b yields the influence line in Fig. 6–6c.

x	M_C
0	0
4	2
8	0
12	−2

(b)



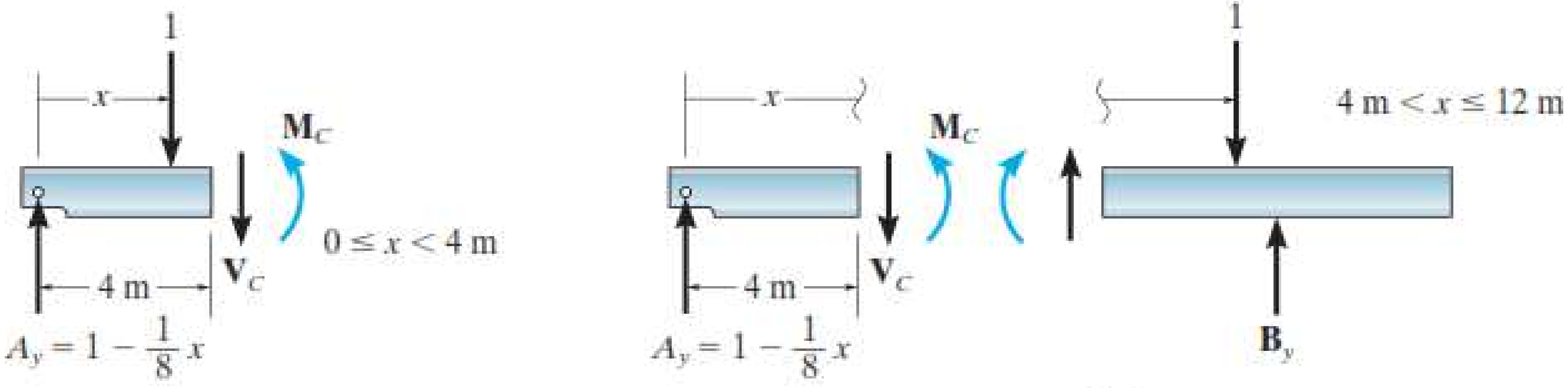
(c)

Influence-Line Equations. From Fig. 6–6*d* verify that

$$M_C = \tfrac{1}{2}x \qquad 0 \le x < 4 \text{ m}$$

$$M_C = 4 - \tfrac{1}{2}x \qquad 4 \text{ m} < x \le 12 \text{ m}$$

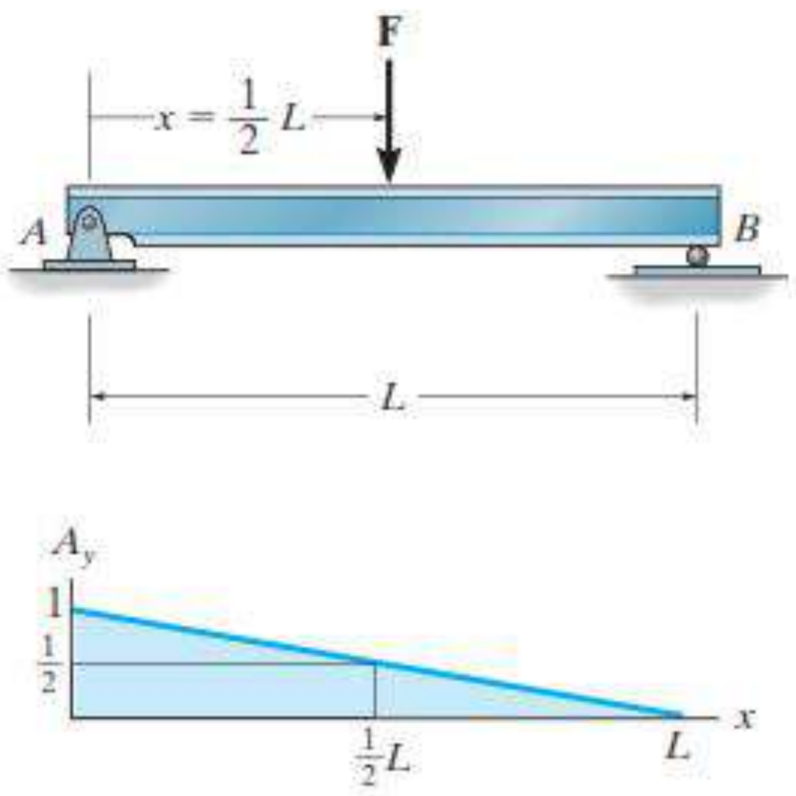
These equations are plotted in Fig. 6–6*c*.



(d)

6.2 Influence Lines for Beams

Since beams (or girders) often form the main load-carrying elements of a floor system or bridge deck, it is important to be able to construct the influence lines for the reactions, shear, or moment at any specified point in a beam.



influence line for A_y

Fig. 6–7

Loadings. Once the influence line for a function (reaction, shear, or moment) has been constructed, it will then be possible to position the live loads on the beam which will produce the maximum value of the function. Two types of loadings will now be considered.

Concentrated Force. Since the numerical values of a function for an influence line are determined using a dimensionless unit load, then for any concentrated force **F** acting on the beam at any position x, *the value of the function can be found by multiplying the ordinate of the influence line at the position x by the magnitude of* **F**.

Uniform Load. Consider a portion of a beam subjected to a uniform load w_0, Fig. 6–8. As shown, each dx segment of this load creates a concentrated force of $dF = w_0\,dx$ on the beam. If $d\mathbf{F}$ is located at x, where the beam's influence-line ordinate for some function (reaction, shear, moment) is y, then the value of the function is $(dF)(y) = (w_0\,dx)y$.

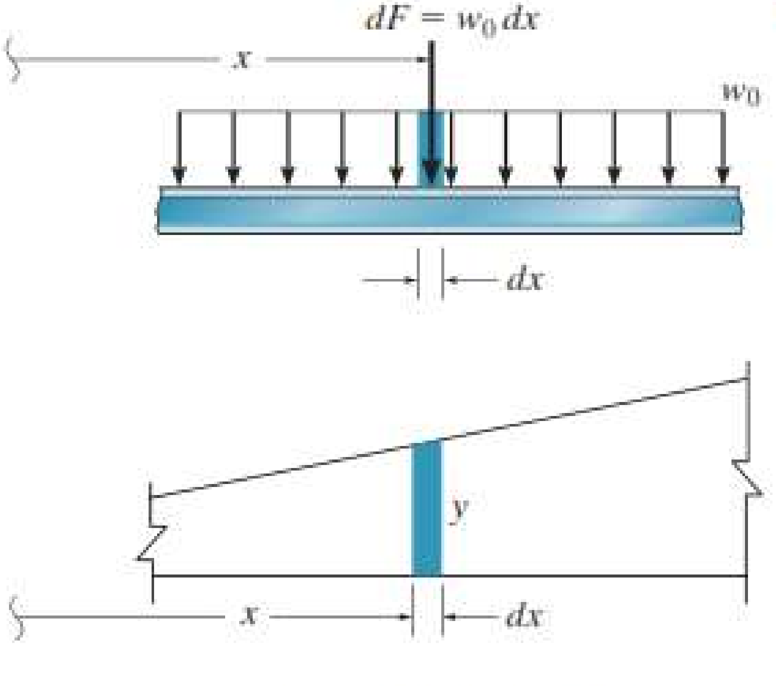


Fig. 6–8

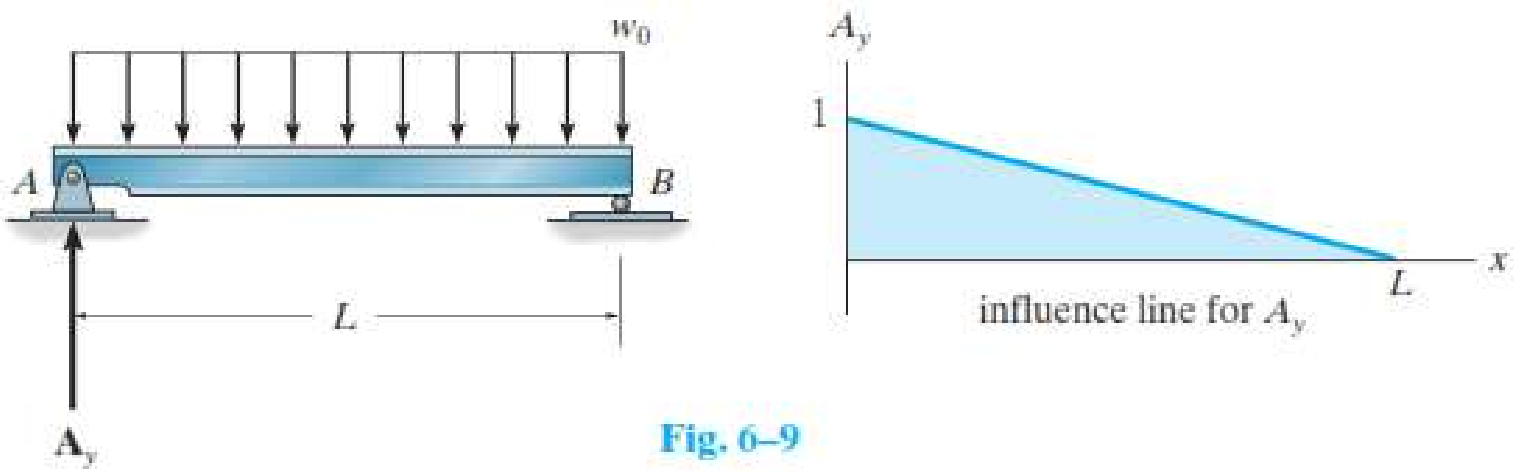
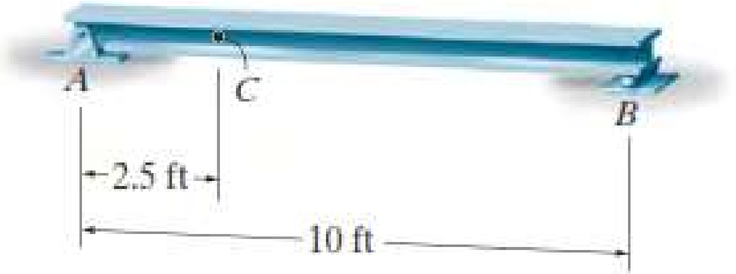


Fig. 6–9

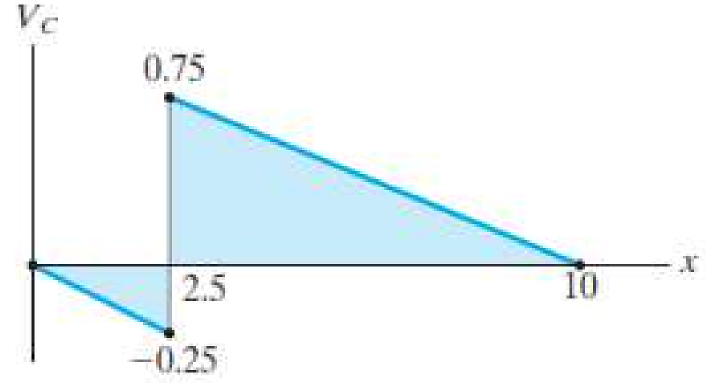
EXAMPLE 6.7

Determine the maximum *positive* shear that can be developed at point C in the beam shown in Fig. 6–10*a* due to a concentrated moving load of 4000 lb and a uniform moving load of 2000 lb/ft.



(a)

Fig. 6–10



influence line for V_C

(b)

SOLUTION

The influence line for the shear at C has been established in Example 6–3 and is shown in Fig. 6–10*b*.

Concentrated Force. The maximum positive shear at C will occur when the 4000-lb force is located at $x = 2.5^+$ ft, since this is the positive peak of the influence line. The ordinate of this peak is +0.75; so that

$$V_C = 0.75(4000 \text{ lb}) = 3000 \text{ lb}$$

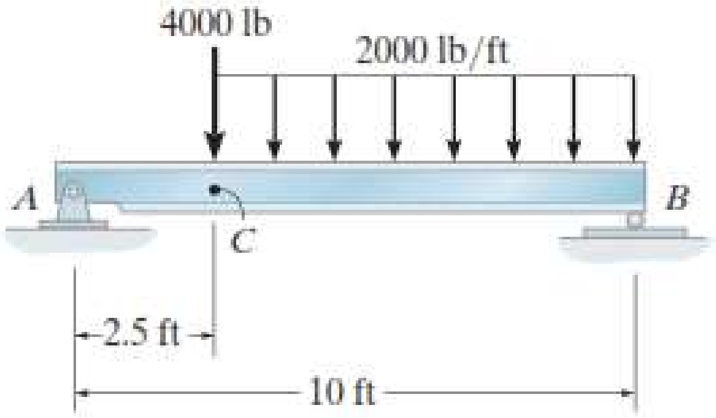
Uniform Load. The uniform moving load creates the maximum positive influence for V_C when the load acts on the beam between $x = 2.5^{+}$ ft and $x = 10$ ft, since within this region the influence line has a positive area. The magnitude of $\mathbf{V}_C$ due to this loading is

$$V_C = \left[\tfrac{1}{2}(10\text{ ft} - 2.5\text{ ft})(0.75)\right]2000\text{ lb/ft} = 5625\text{ lb}$$

Total Maximum Shear at C.

$$(V_C)_{\max} = 3000\text{ lb} + 5625\text{ lb} = 8625\text{ lb} \qquad \textit{Ans.}$$

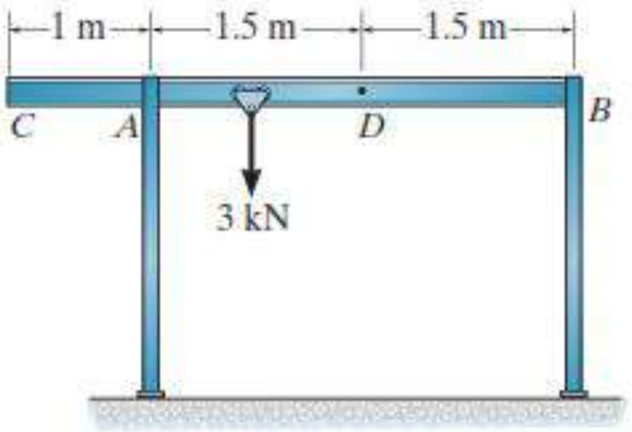
Notice that once the *positions* of the loads have been established using the influence line, Fig. 6–10*c*, this value of $(V_C)_{\max}$ can *also* be determined using statics and the method of sections. Show that this is the case.



(c)

EXAMPLE 6.8

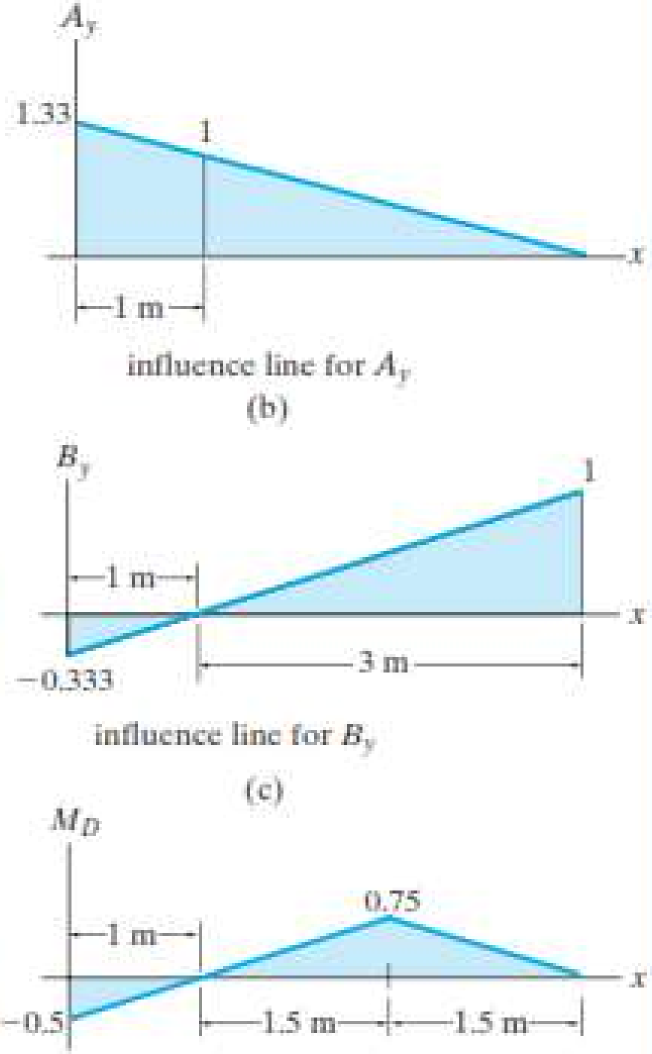
The frame structure shown in Fig. 6–11*a* is used to support a hoist for transferring loads for storage at points underneath it. It is anticipated that the load on the dolly is 3 kN and the beam *CB* has a mass of 24 kg/m. Assume the dolly has negligible size and can travel the entire length of the beam. Also, assume *A* is a pin and *B* is a roller. Determine the maximum vertical support reactions at *A* and *B* and the maximum moment in the beam at *D*.



(a)

A_y

(a)



influence line for A_y
(b)

influence line for B_y
(c)

influence line for M_D
(d)

Fig. 6–11

SOLUTION

Maximum Reaction at A. We first draw the influence line for A_y, Fig. 6–11*b*. Specifically, when a unit load is at A the reaction at A is 1 as shown. The ordinate at C, is 1.33. Here the maximum value for A_y occurs when the dolly is at C. Since the dead load (beam weight) must be placed over the entire length of the beam, we have,

$$(A_y)_{\max} = 3000(1.33) + 24(9.81)\left[\tfrac{1}{2}(4)(1.33)\right]$$
$$= 4.63 \text{ kN} \qquad \textit{Ans.}$$

Maximum Reaction at B. The influence line (or beam) takes the shape shown in Fig. 6–11*c*. The values at C and B are determined by statics. Here the dolly must be at B. Thus,

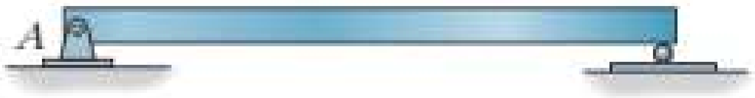
$$(B_y)_{\max} = 3000(1) + 24(9.81)\left[\tfrac{1}{2}(3)(1)\right] + 24(9.81)\left[\tfrac{1}{2}(1)(-0.333)\right]$$
$$= 3.31 \text{ kN} \qquad \textit{Ans.}$$

Maximum Moment at D. The influence line has the shape shown in Fig. 6–11*d*. The values at C and D are determined from statics. Here,

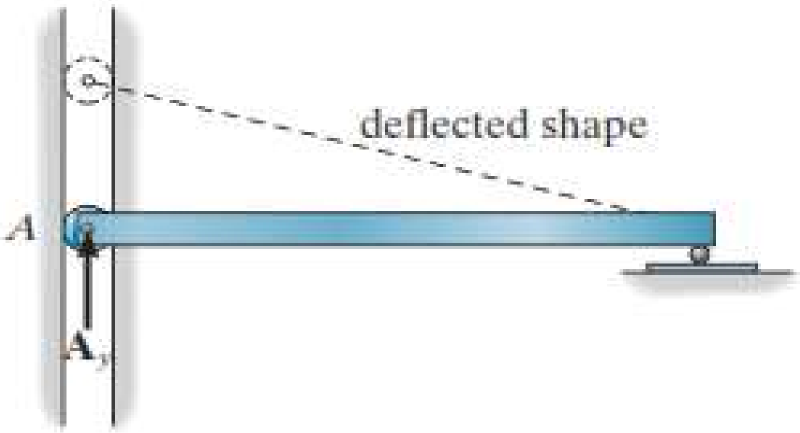
$$(M_D)_{\max} = 3000(0.75) + 24(9.81)\left[\tfrac{1}{2}(1)(-0.5)\right] + 24(9.81)\left[\tfrac{1}{2}(3)(0.75)\right]$$
$$= 2.46 \text{ kN}\cdot\text{m} \qquad \textit{Ans.}$$

6.3 Qualitative Influence Lines

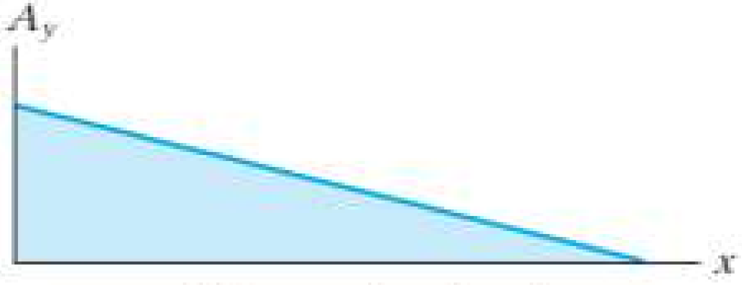
In 1886, Heinrich Müller-Breslau developed a technique for rapidly constructing the shape of an influence line. Referred to as the *Müller-Breslau principle, it states that the influence line for a function (reaction, shear, or moment) is to the same scale as the deflected shape of the beam when the beam is acted upon by the function.* In order to draw the deflected shape properly, the capacity of the beam to resist the applied function must be *removed* so the beam can deflect when the function is applied.



(a)



(b)



influence line for A_y

(c)

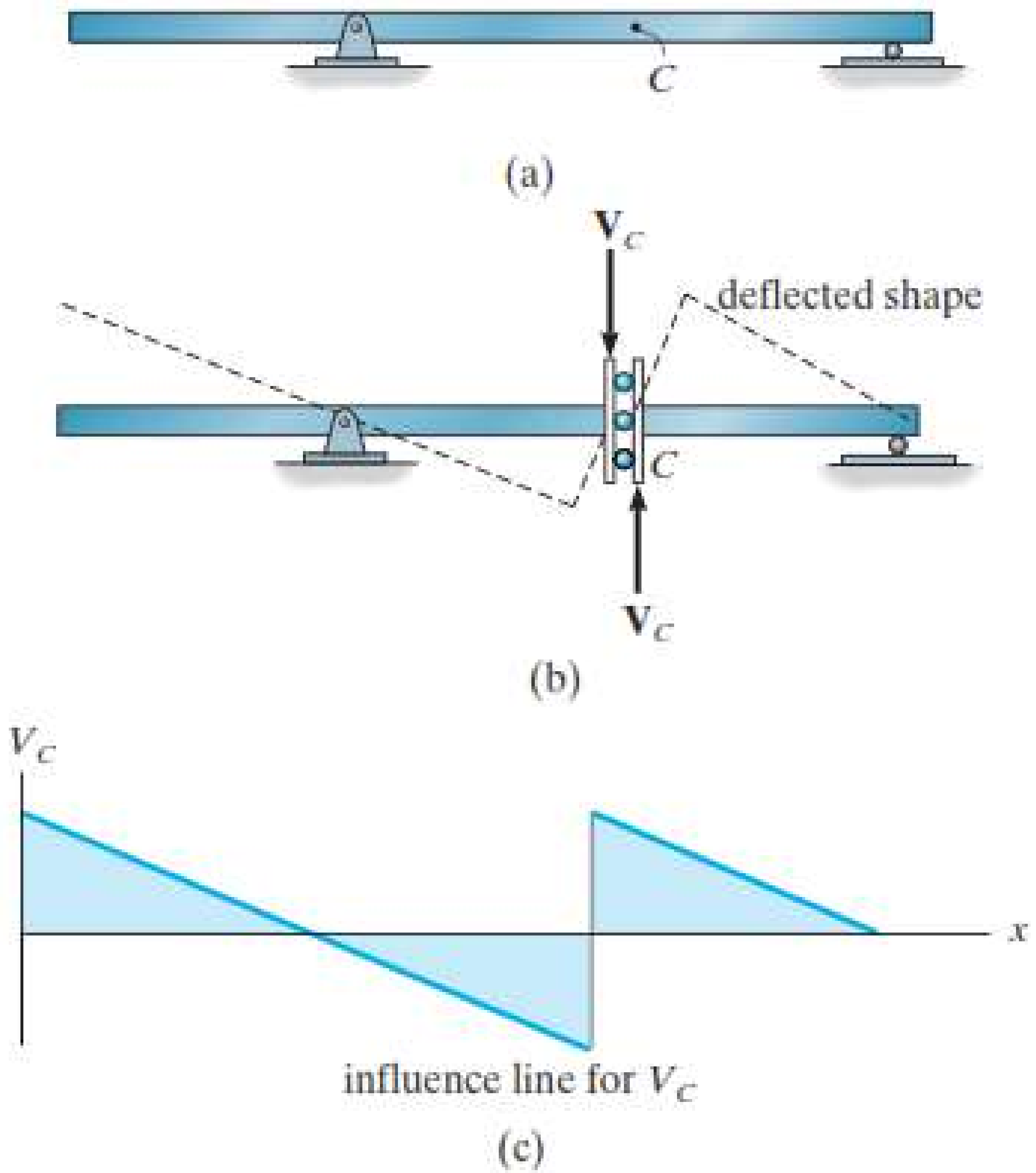


Fig. 6–13

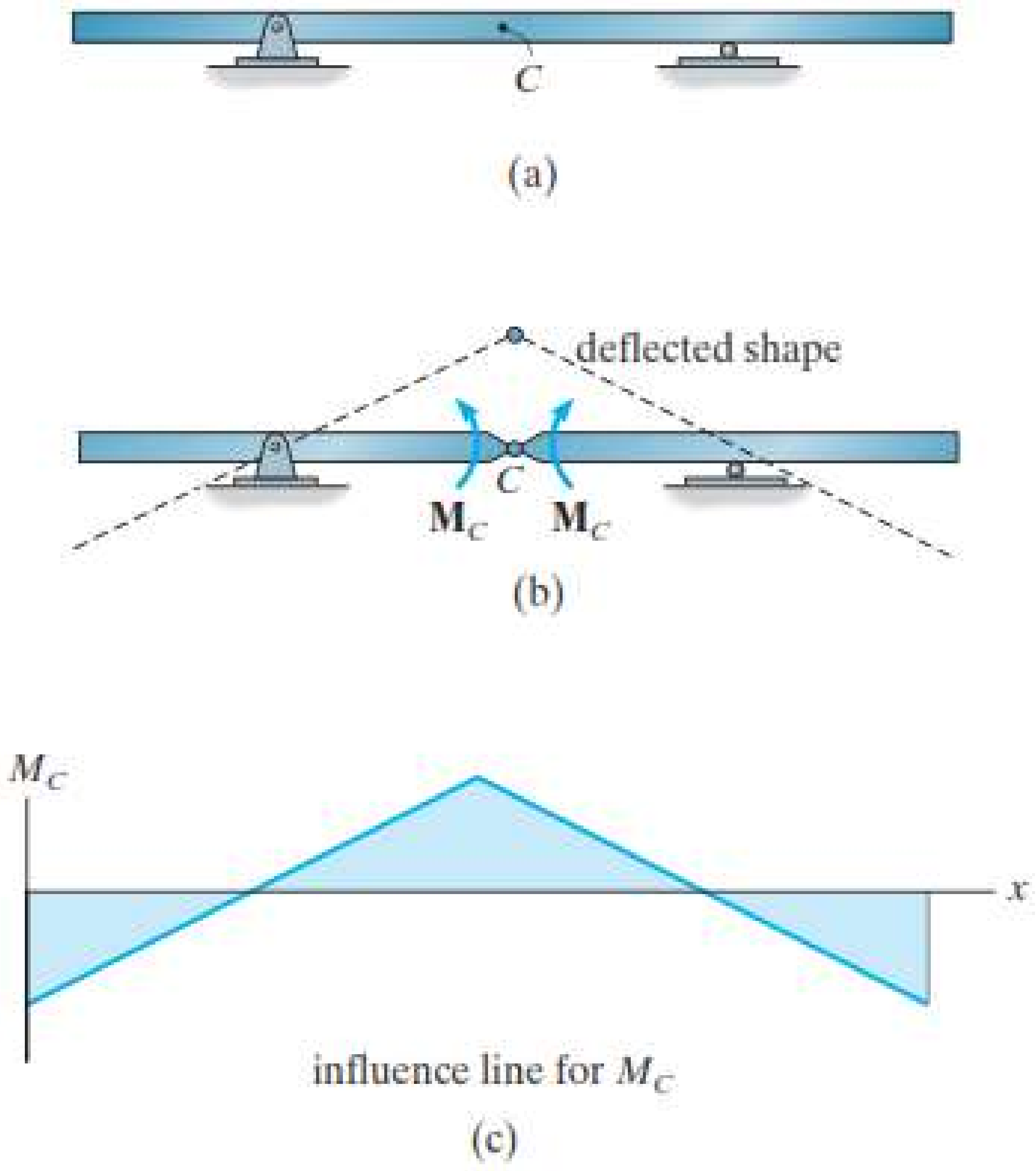


Fig. 6–14

EXAMPLE 6.9

For each beam in Fig. 6–16*a* through 6–16*c*, sketch the influence line for the vertical reaction at A.

SOLUTION

The support is replaced by a roller guide at A since it will resist $\mathbf{A}_x$, but not $\mathbf{A}_y$. The force $\mathbf{A}_y$ is then applied.

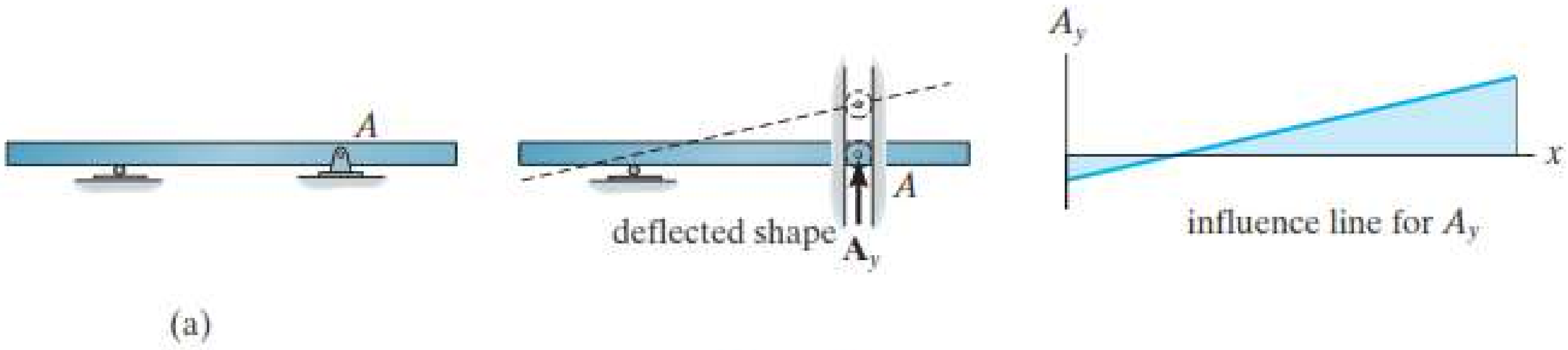
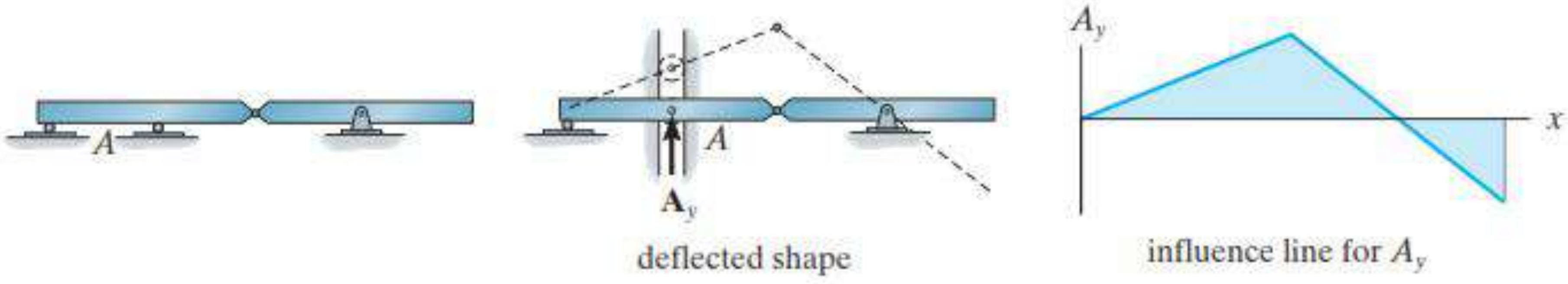


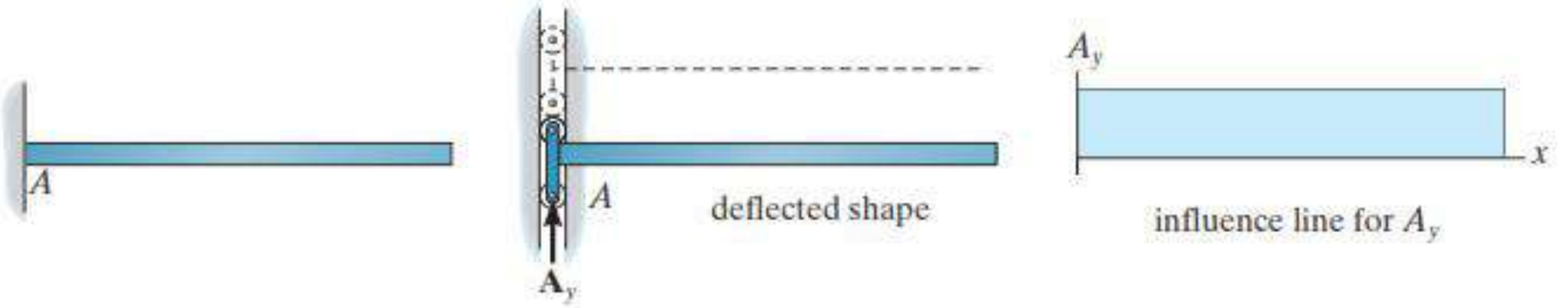
Fig. 6–16

Again, a roller guide is placed at A and the force $\mathbf{A}_y$ is applied.



(b)

A *double-roller guide* must be used at A in this case, since this type of support will resist both a moment $\mathbf{M}_A$ at the fixed support and axial load $\mathbf{A}_x$, but will not resist $\mathbf{A}_y$.



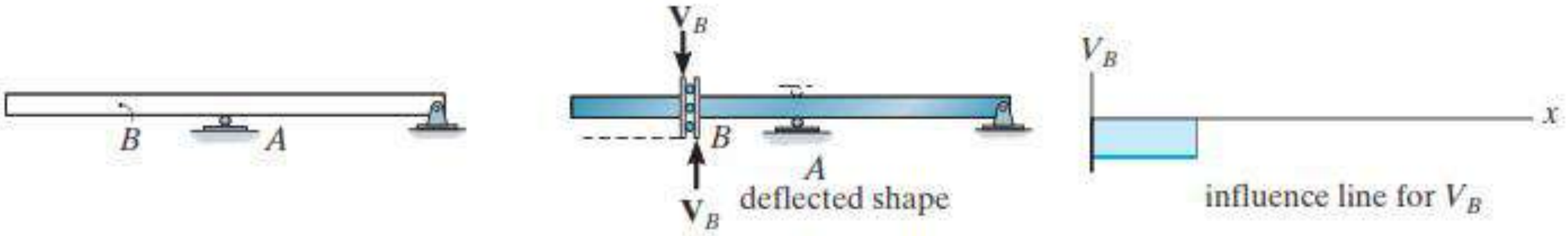
(c)

EXAMPLE 6.10

For each beam in Figs. 6–17*a* through 6–17*c*, sketch the influence line for the shear at *B*.

SOLUTION

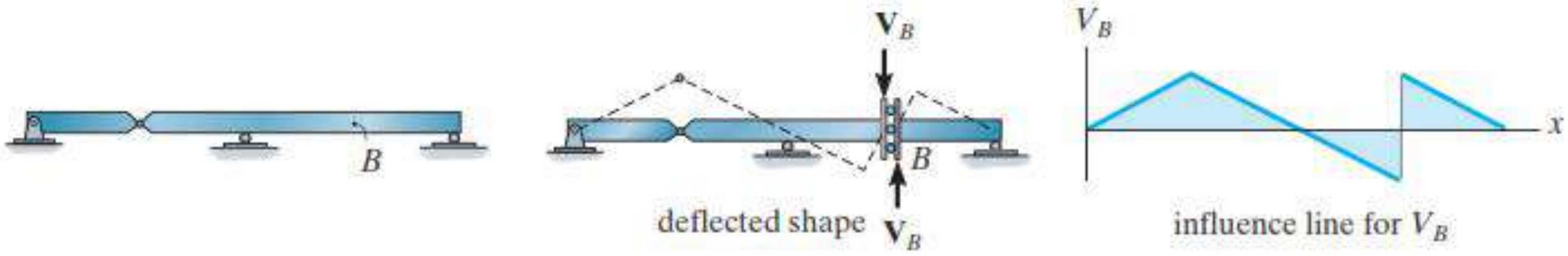
The roller guide is introduced at *B* and the positive shear $\mathbf{V}_B$ is applied. Notice that the right segment of the beam will *not deflect* since the roller at *A* actually constrains the beam from moving vertically, either up or down. [See support (2) in Table 2–1.]



(a)

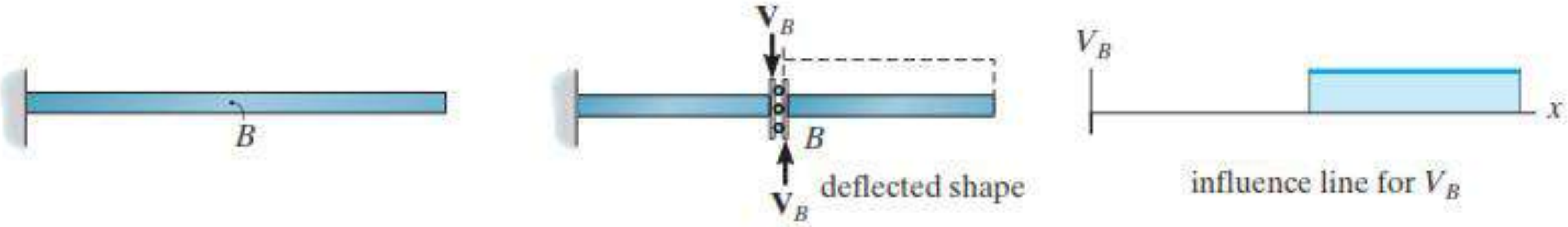
Fig. 6–17

Placing the roller guide at *B* and applying the positive shear at *B* yields the deflected shape and corresponding influence line.



(b)

Again, the roller guide is placed at B, the positive shear is applied, and the deflected shape and corresponding influence line are shown. Note that the left segment of the beam does not deflect, due to the fixed support.



(c)

EXAMPLE 6.11

For each beam in Figs. 6–18*a* through 6–18*c*, sketch the influence line for the moment at *B*.

SOLUTION

A hinge is introduced at *B* and positive moments $\mathbf{M}_B$ are applied to the beam. The deflected shape and corresponding influence line are shown.

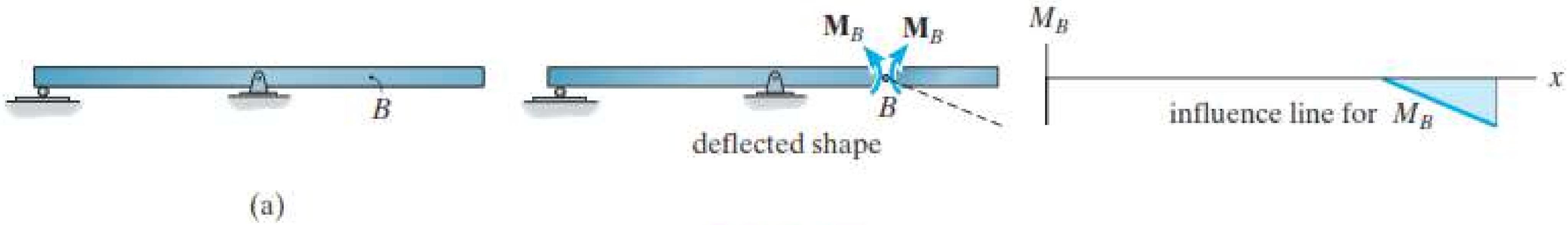
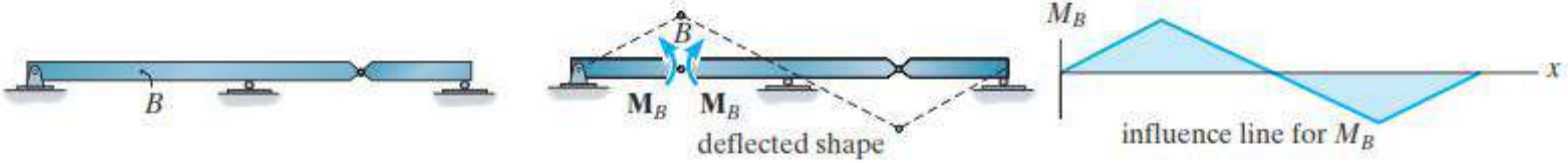


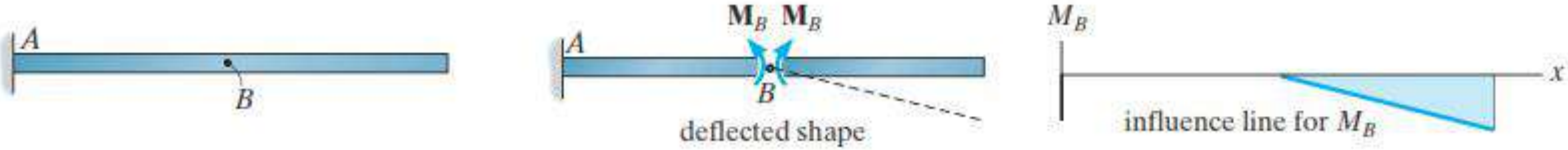
Fig. 6–18

Placing a hinge at *B* and applying positive moments $\mathbf{M}_B$ to the beam yields the deflected shape and influence line.



(b)

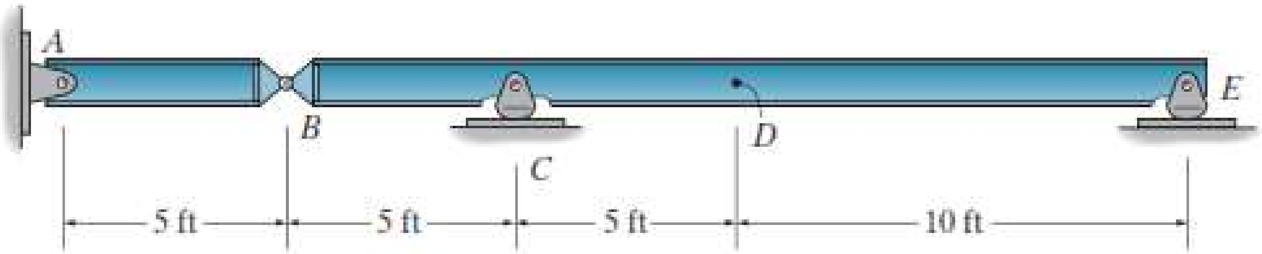
With the hinge and positive moment at B the deflected shape and influence line are shown. The left segment of the beam is constrained from moving due to the fixed wall at A.



(c)

EXAMPLE 6.12

Determine the maximum positive moment that can be developed at point D in the beam shown in Fig. 6–19a due to a concentrated moving load of 4000 lb, a uniform moving load of 300 lb/ft, and a beam weight of 200 lb/ft.



(a)

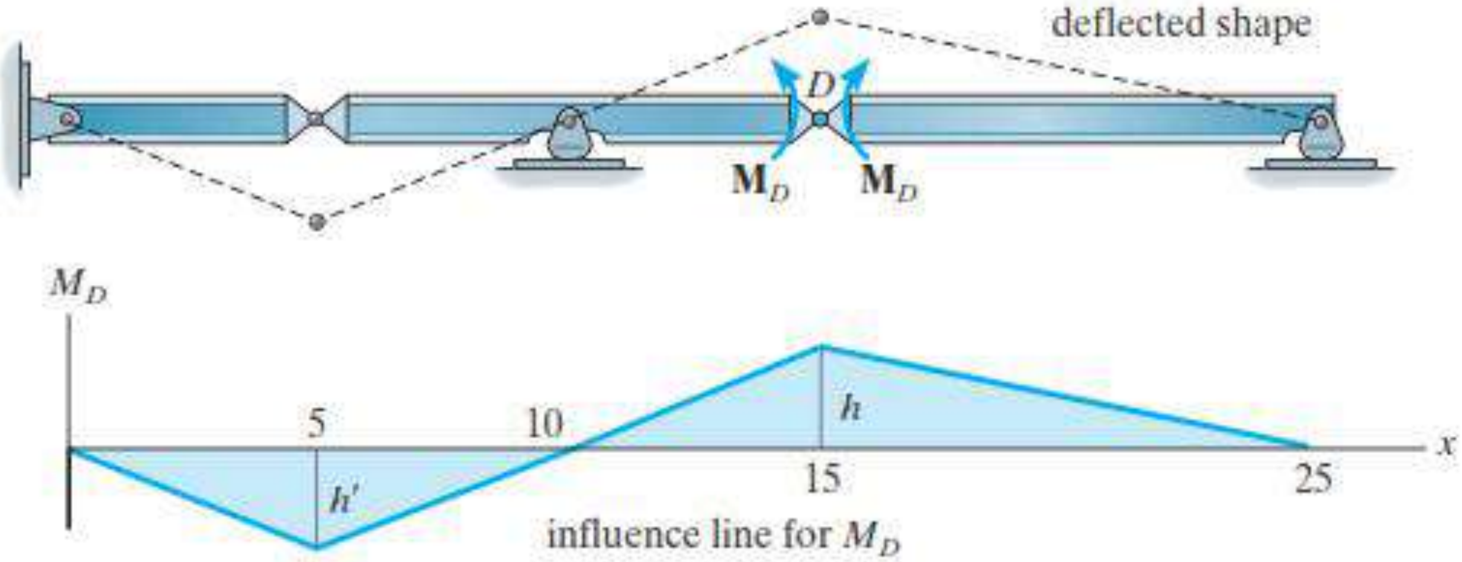
Fig. 6–19

SOLUTION

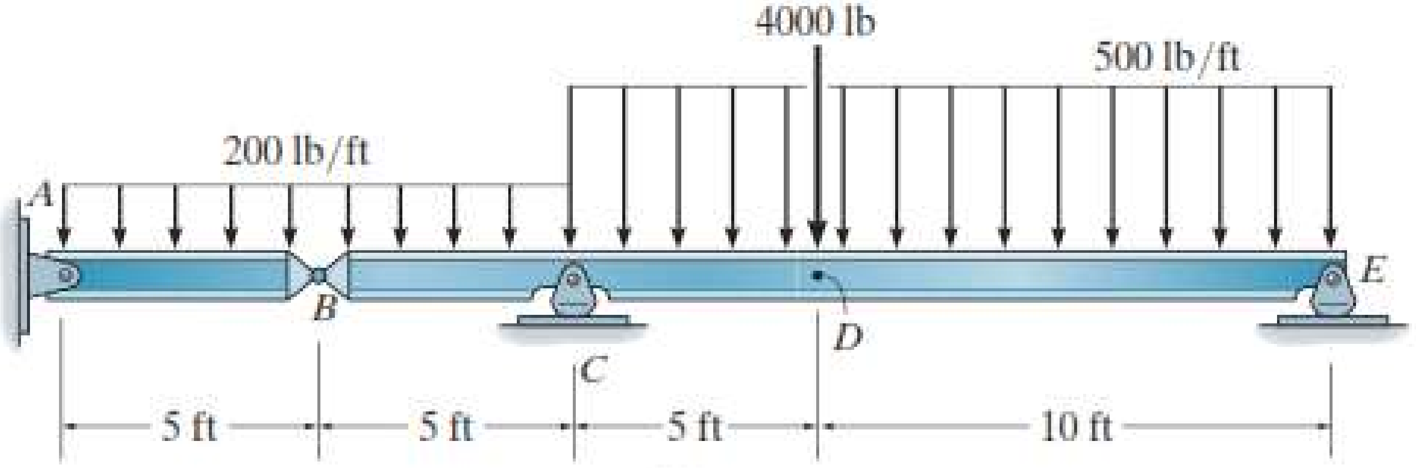
A hinge is placed at D and positive moments $\mathbf{M}_D$ are applied to the beam. The deflected shape and corresponding influence line are shown in Fig. 6–19b. Immediately one recognizes that the concentrated moving load of 4000 lb creates a maximum *positive* moment at D when it is placed at D, i.e., the peak of the influence line. Also, the uniform moving load of 300 lb/ft must extend from C to E in order to cover the region where the area of the influence line is positive. Finally, the uniform weight of 200 lb/ft acts over the *entire length* of the beam. The loading is shown on the beam in Fig. 6–19*c*. Knowing the position of the loads, we can now determine the maximum moment at D using statics. In Fig. 6–19*d* the reactions on BE have been computed. Sectioning the beam at D and using segment DE, Fig. 6–19*e*, we have

$$\circlearrowleft + \Sigma M_D = 0; \qquad -M_D - 5000(5) + 4750(10) = 0$$

$$M_D = 22\,500 \text{ lb} \cdot \text{ft} = 22.5 \text{ k} \cdot \text{ft} \qquad \textit{Ans.}$$

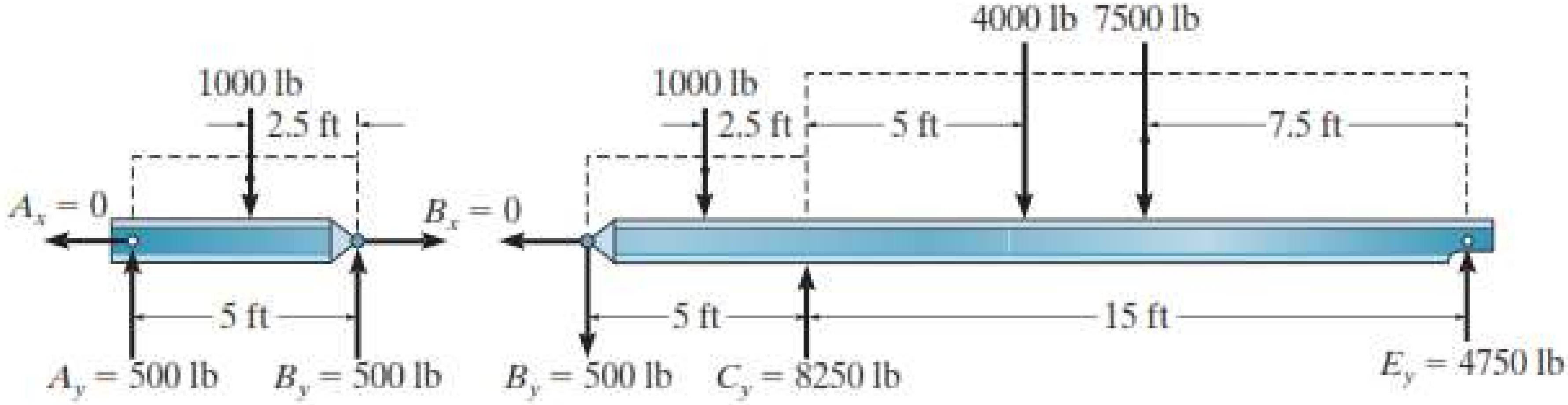


(b)



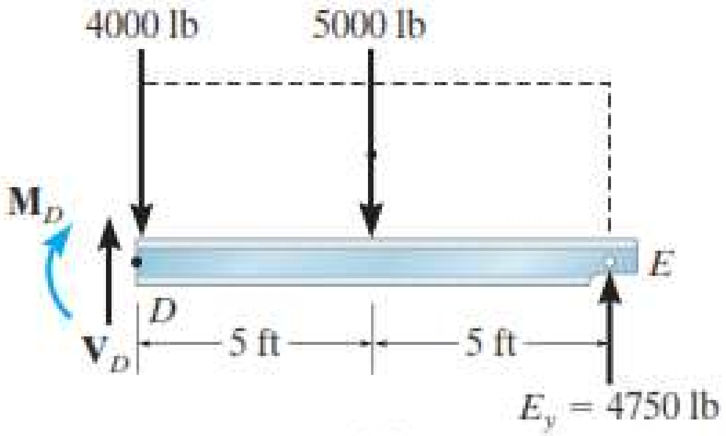
4000 lb
500 lb/ft
200 lb/ft
A
B
C
D
E
5 ft
5 ft
5 ft
10 ft

(c)



1000 lb
2.5 ft
$A_x = 0$
$B_x = 0$
5 ft
$A_y = 500$ lb
$B_y = 500$ lb
1000 lb
2.5 ft
5 ft
4000 lb
7500 lb
7.5 ft
5 ft
15 ft
$B_y = 500$ lb
$C_y = 8250$ lb
$E_y = 4750$ lb

(d)

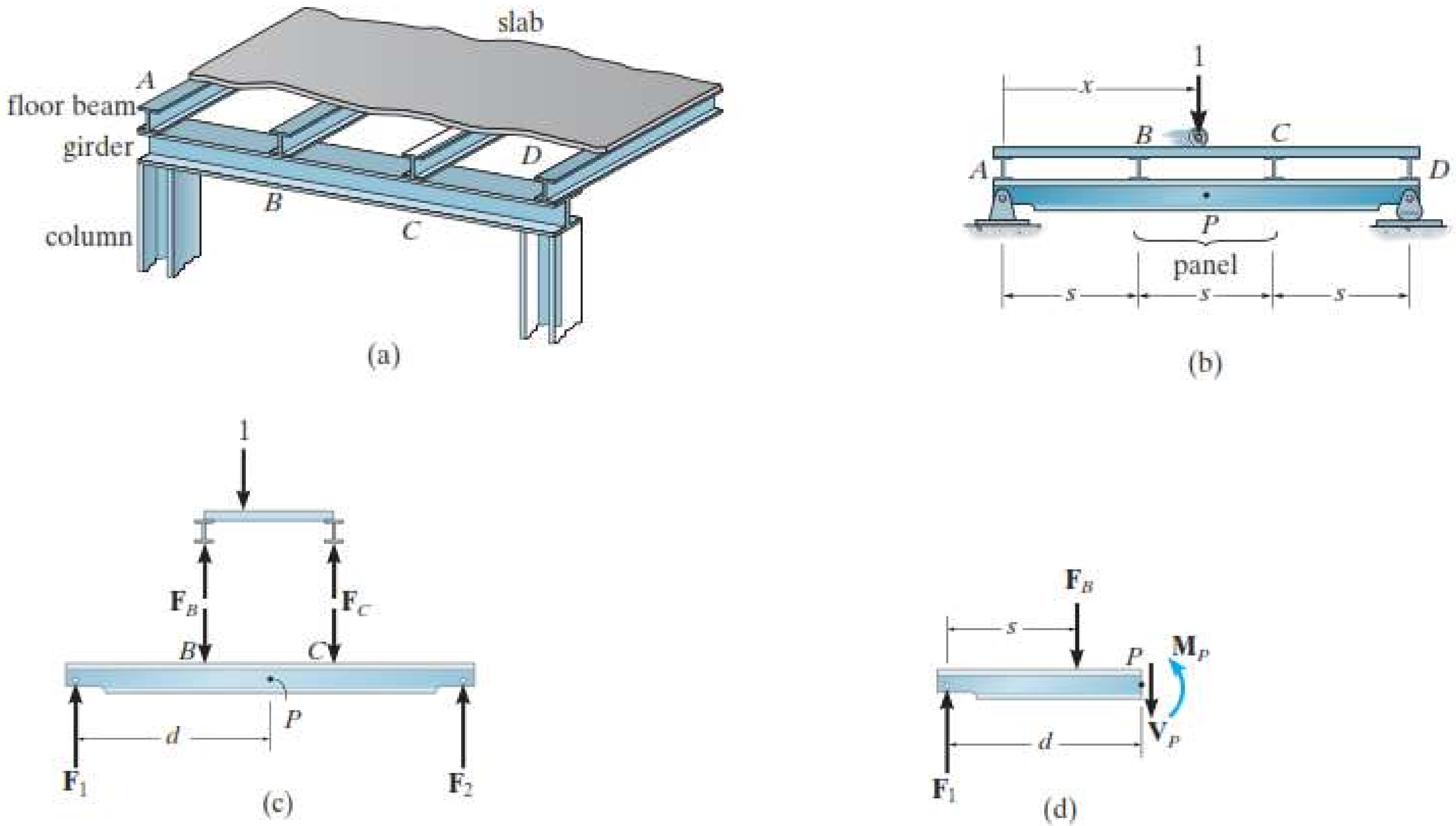


(e)

This problem can also be worked by using *numerical values* for the influence line as in Sec. 6–1. Actually, by inspection of Fig. 6–19*b*, only the peak value h at D must be determined. This requires placing a unit load on the beam at D in Fig. 6–19*a* and then solving for the internal moment in the beam at D. Show that the value obtained is $h = 3.33$. By proportional triangles, $h'/(10 - 5) = 3.33/(15 - 10)$ or $h' = 3.33$. Hence, with the loading on the beam as in Fig. 6–19*c*, using the areas and peak values of the influence line, Fig. 6–19*b*, we have

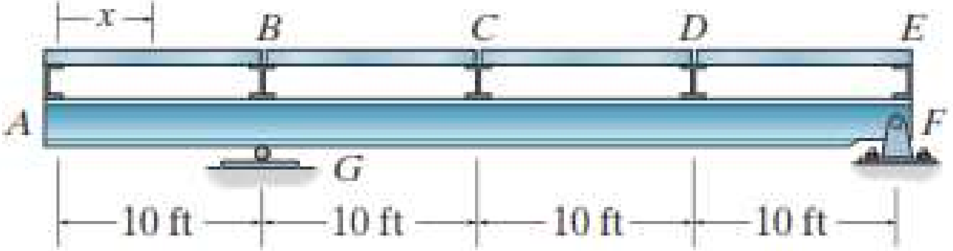
$$M_D = 500\left[\tfrac{1}{2}(25 - 10)(3.33)\right] + 4000(3.33) - 200\left[\tfrac{1}{2}(10)(3.33)\right]$$
$$= 22\,500 \text{ lb} \cdot \text{ft} = 22.5 \text{ k} \cdot \text{ft} \qquad \textit{Ans.}$$

6.4 Influence Lines for Floor Girders



EXAMPLE 6.13

Draw the influence line for the shear in panel CD of the floor girder in Fig. 6–21a.



(a)

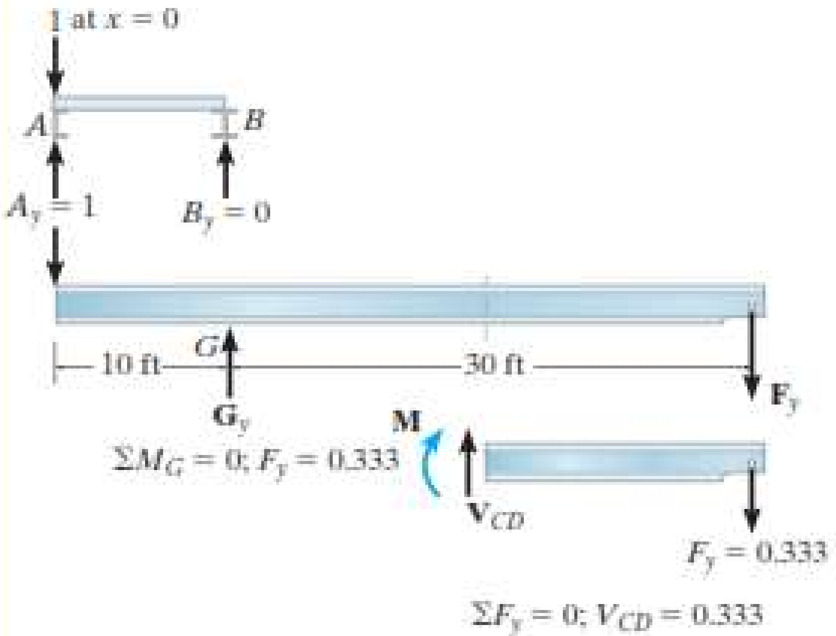
Fig. 6–21

x	V_{CD}
0	0.333
10	0
20	−0.333
30	0.333
40	0

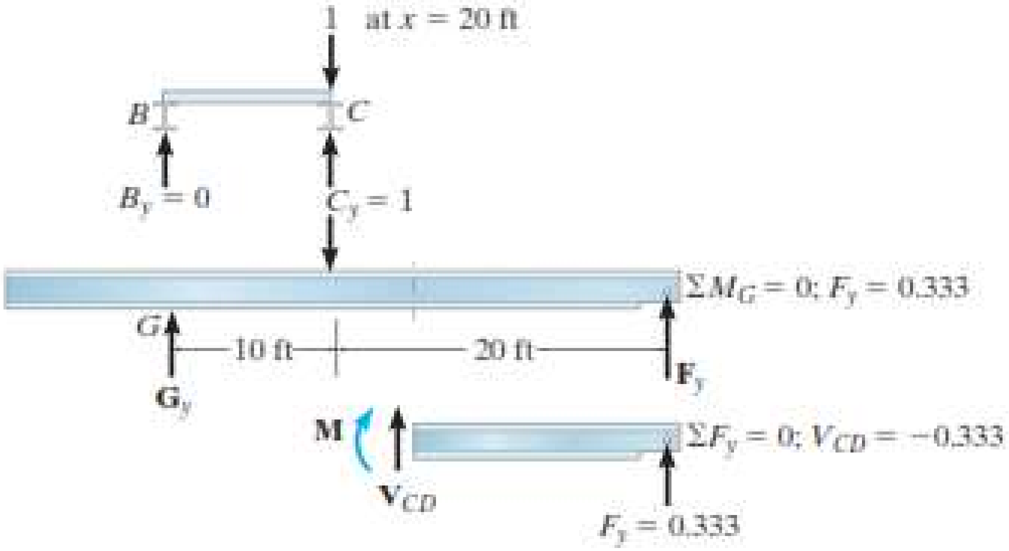
(b)

SOLUTION

Tabulate Values. The unit load is placed at each floor beam location and the shear in panel CD is calculated. A table of the results is shown in Fig. 6–21b. The details for the calculations when $x = 0$ and $x = 20$ ft are given in Figs. 6–21c and 6–21d, respectively. Notice how in each case the reactions of the floor beams on the girder are calculated first, followed by a determination of the girder support reaction at F ($\mathbf{G}_y$ is not needed), and finally, a segment of the girder is considered and the internal panel shear V_{CD} is calculated. As an exercise, verify the values for V_{CD} when $x = 10$ ft, 30 ft, and 40 ft.

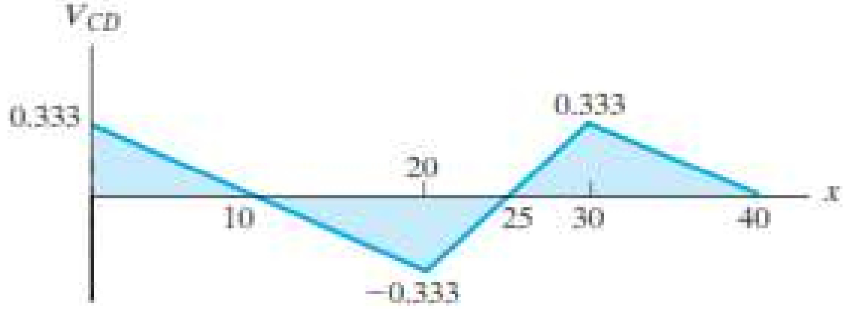


(c)



(d)

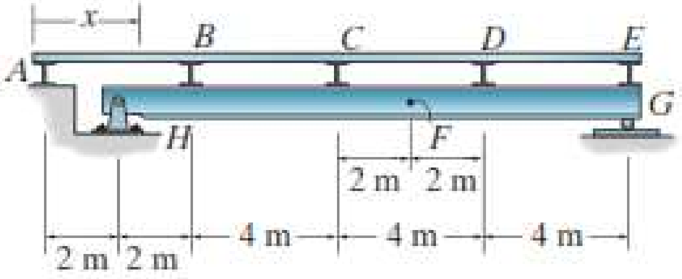
Influence Line. When the tabular values are plotted and the points connected with straight line segments, the resulting influence line for V_{CD} is as shown in Fig. 6–21*e*.



influence line for V_{CD}
(e)

EXAMPLE 6.14

Draw the influence line for the moment at point F for the floor girder in Fig. 6–22*a*.



(a)

x	M_F
0	0
2	0.429
4	0.857
8	2.571
10	2.429
12	2.286
16	0

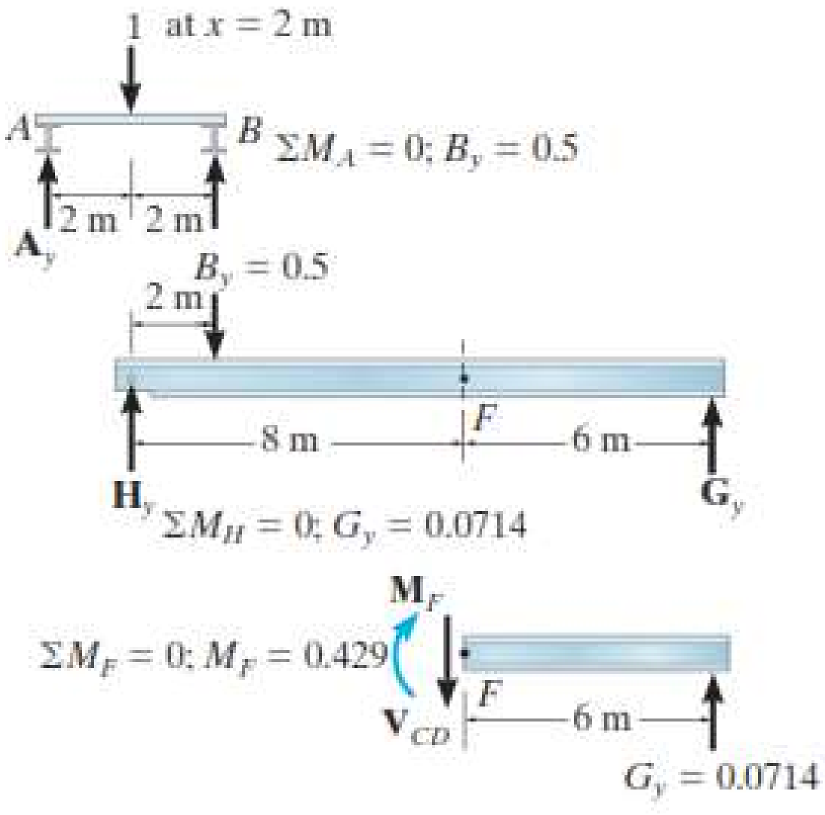
(b)

Fig. 6–22

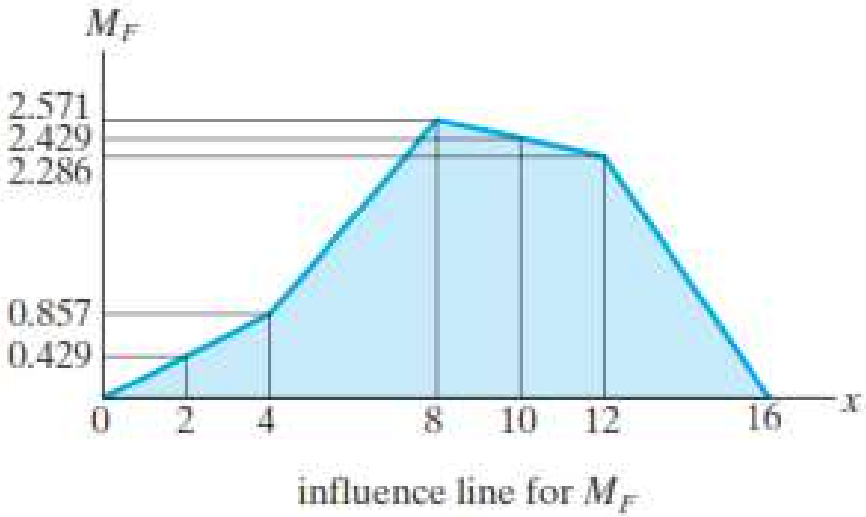
SOLUTION

Tabulate Values. The unit load is placed at $x = 0$ and each panel point thereafter. The corresponding values for M_F are calculated and shown in the table, Fig. 6–22*b*. Details of the calculations for $x = 2$ m are shown in Fig. 6–22*c*. As in the previous example, it is first necessary to determine the reactions of the floor beams on the girder, followed by a determination of the girder support reaction $\mathbf{G}_y$ ($\mathbf{H}_y$ is not needed), and finally segment GF of the girder is considered and the internal moment $\mathbf{M}_F$ is calculated. As an exercise, determine the other values of M_F listed in Fig. 6–22*b*.

Influence Line. A plot of the tabular values yields the influence line for M_F, Fig. 6–22d.



(c)



(d)

6.5 Influence Lines for Trusses

Trusses are often used as primary load-carrying elements for bridges. Hence, for design it is important to be able to construct the influence lines for each of its members. As shown in Fig. 6–23, the loading on the bridge deck is transmitted to stringers, which in turn transmit the loading to floor beams and then to the *joints* along the bottom cord of the truss. Since the truss members are affected only by the joint loading, we can therefore obtain the ordinate values of the influence line for a member by loading each joint along the deck with a unit load and then use the method of joints or the method of sections to calculate the force in the member.

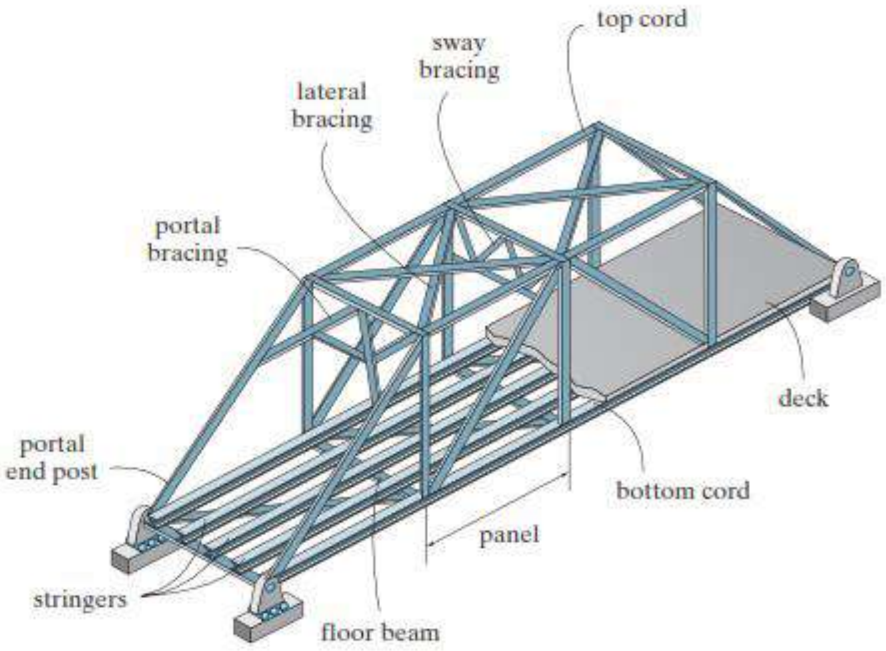
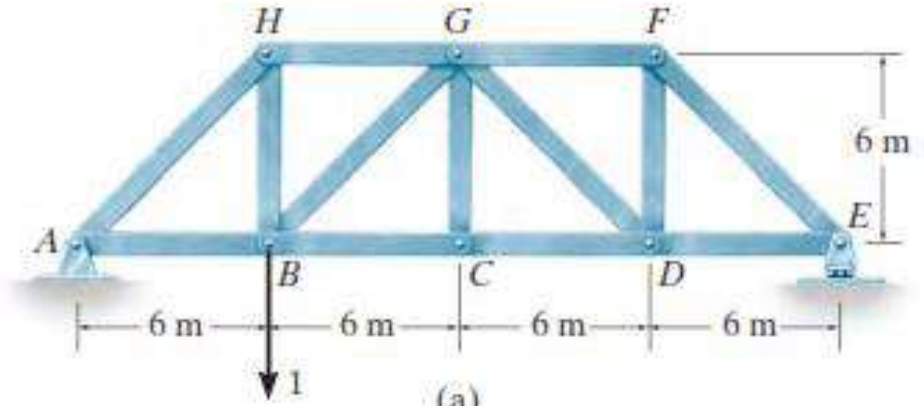


Fig. 6–23

EXAMPLE 6.15

Draw the influence line for the force in member GB of the bridge truss shown in Fig. 6–24a.



(a)

Fig. 6–24

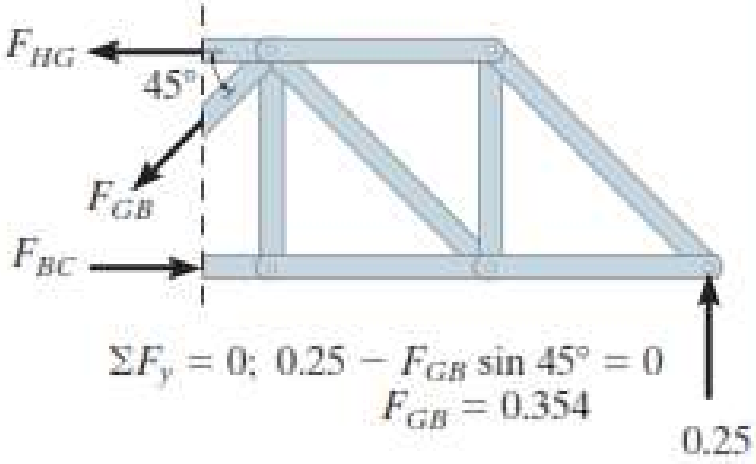
SOLUTION

Tabulate Values. Here each successive joint at the bottom cord is loaded with a unit load and the force in member GB is calculated using the method of sections, Fig. 6–24b. For example, placing the unit load at $x = 6$ m (joint B), the support reaction at E is calculated first, Fig. 6–24a, then passing a section through HG, GB, BC and isolating the right segment, the force in GB is determined, Fig. 6–24c. In the same manner, determine the other values listed in the table.

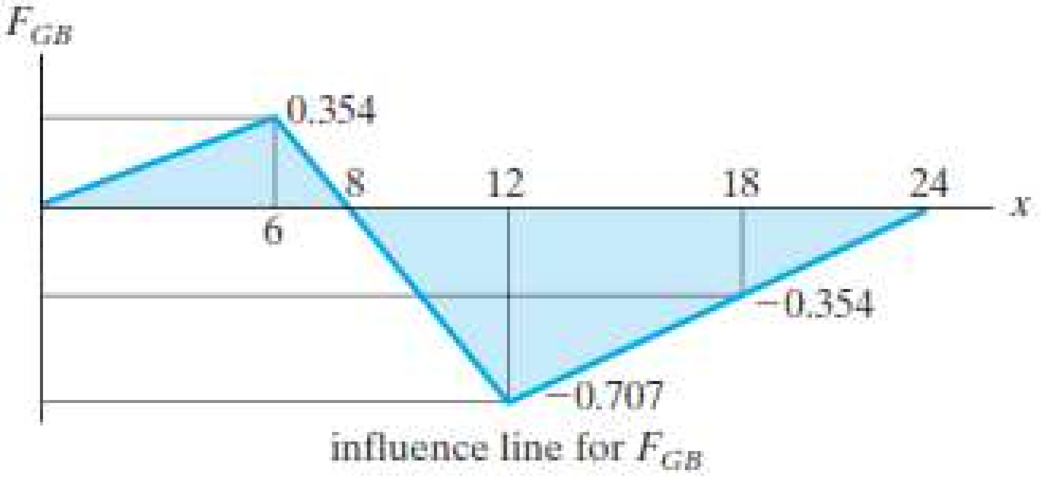
x	F_{GB}
0	0
6	0.354
12	−0.707
18	−0.354
24	0

(b)

Influence Line. Plotting the tabular data and connecting the points yields the influence line for member GB, Fig. 6–24d. Since the influence line extends over the entire span of the truss, member GB is referred to as a *primary member*. This means GB is subjected to a force regardless of where the bridge deck (roadway) is loaded, except, of course, at $x = 8$ m. The point of zero force, $x = 8$ m, is determined by similar triangles between $x = 6$ m and $x = 12$ m, that is, $(0.354 + 0.707)/(12 - 6) = 0.354/x'$, $x' = 2$ m, so $x = 6 + 2 = 8$ m.



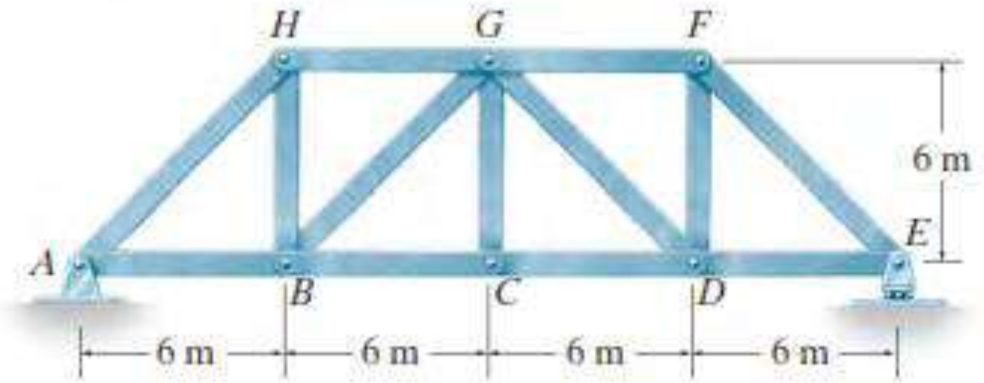
(c)



(d)

EXAMPLE 6.16

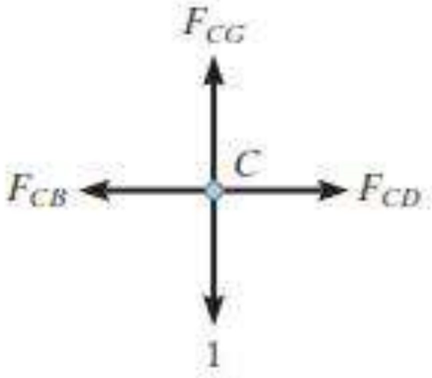
Draw the influence line for the force in member CG of the bridge truss shown in Fig. 6–25a.



(a)

x	F_{GC}
0	0
6	0
12	1
18	0
24	0

(b)



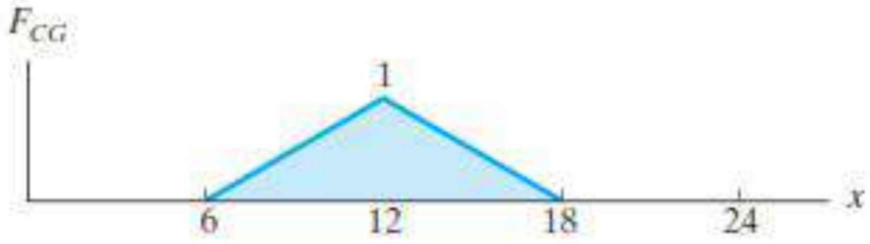
(c)

Fig. 6–25

SOLUTION

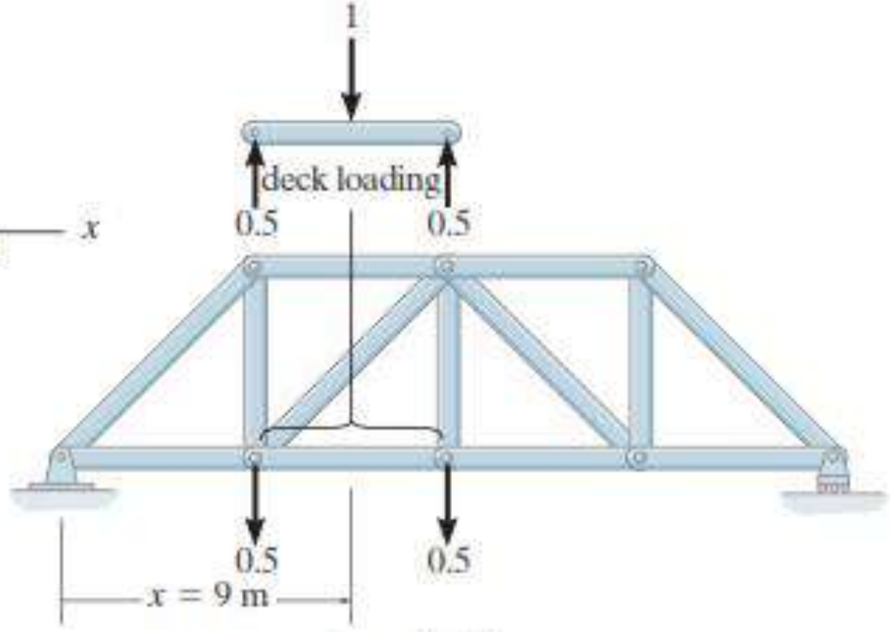
Tabulate Values. A table of unit-load position at the joints of the bottom cord versus the force in member CG is shown in Fig. 6–25b. These values are easily obtained by isolating joint C, Fig. 6–25c. Here it is seen that CG is a zero-force member unless the unit load is applied at joint C, in which case $F_{CG} = 1$ (T).

Influence Line. Plotting the tabular data and connecting the points yields the influence line for member *CG* as shown in Fig. 6–25*d*. In particular, notice that when the unit load is at $x = 9$ m, the force in member *CG* is $F_{CG} = 0.5$. This situation requires the unit load to be placed on the bridge deck *between* the joints. The transference of this load from the deck to the truss is shown in Fig. 6–25*e*. From this one can see that indeed $F_{CG} = 0.5$ by analyzing the equilibrium of joint *C*, Fig. 6–25*f*. Since the influence line for *CG* does *not* extend over the entire span of the truss, Fig. 6–25*d*, member *CG* is referred to as a *secondary member*.



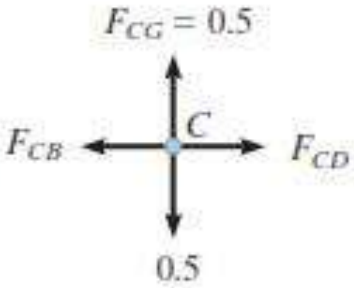
influence line for F_{CG}

(d)



truss loading

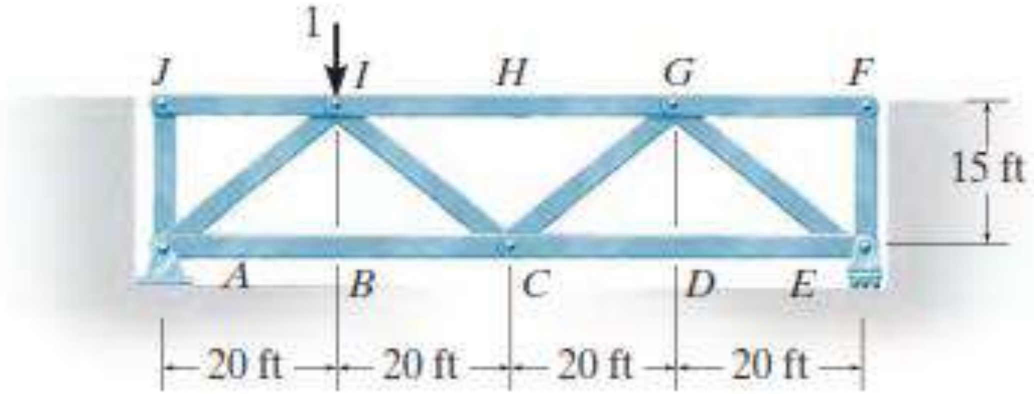
(e)



(f)

EXAMPLE 6.17

In order to determine the maximum force in each member of the Warren truss, shown in the photo, we must first draw the influence lines for each of its members. If we consider a similar truss as shown in Fig. 6–26*a*, determine the largest force that can be developed in member *BC* due to a moving force of 25 k and a moving distributed load of 0.6 k/ft. The loading is applied at the top cord.



(a)

x	F_{BC}
0	0
20	1
40	0.667
60	0.333
80	0

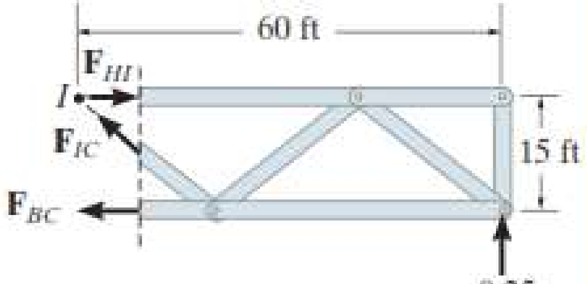
(b)

Fig. 6–26

SOLUTION

SOLUTION

Tabulate Values. A table of unit-load position x at the joints along the top cord versus the force in member BC is shown in Fig. 6–26b. The method of sections can be used for the calculations. For example, when the unit load is at joint I ($x = 20$ ft), Fig. 6–26a, the reaction $\mathbf{E}_y$ is determined first ($E_y = 0.25$). Then the truss is sectioned through BC, IC, and HI, and the right segment is isolated, Fig. 6–26c. One obtains $\mathbf{F}_{BC}$ by summing moments about point I, to eliminate $\mathbf{F}_{HI}$ and $\mathbf{F}_{IC}$. In a similar manner determine the other values in Fig. 6–26b.



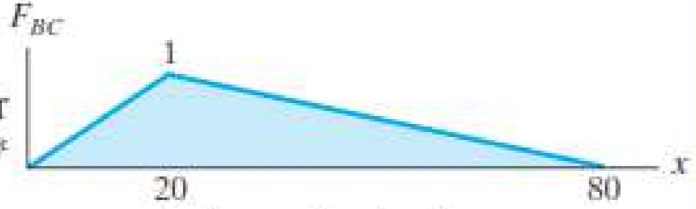
$$\circlearrowleft + \Sigma M_I = 0;\ -F_{BC}(15) + 0.25(60) = 0$$
$$F_{BC} = 1.00\ (\mathrm{T})$$

(c)

Influence Line. A plot of the tabular values yields the influence line, Fig. 6–26d. By inspection, BC is a primary member. Why?

Concentrated Live Force. The largest force in member BC occurs when the moving force of 25 k is placed at $x = 20$ ft. Thus,

$$F_{BC} = (1.00)(25) = 25.0\ \mathrm{k}$$



influence line for F_{BC}

(d)

Distributed Live Load. The uniform live load must be placed over the entire deck of the truss to create the largest tensile force in BC.* Thus,

$$F_{BC} = \left[\tfrac{1}{2}(80)(1.00)\right]0.6 = 24.0\ \mathrm{k}$$

Total Maximum Force.

$$(F_{BC})_{max} = 25.0\ \mathrm{k} + 24.0\ \mathrm{k} = 49.0\ \mathrm{k} \qquad \textit{Ans.}$$

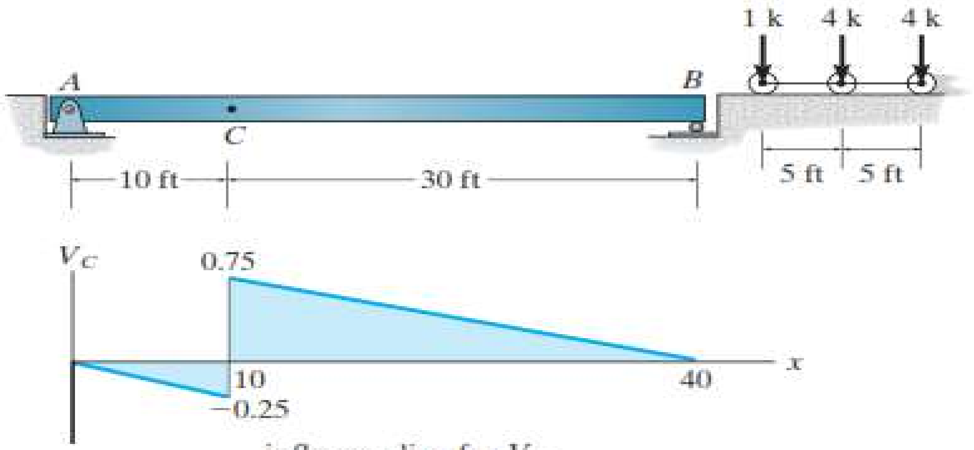
*The largest *tensile* force in member GB of Example 6–15 is created when the distributed load acts on the deck of the truss from $x = 0$ to $x = 8$ m, Fig. 6–24d.

6.6 Maximum Influence at a Point due to a Series of Concentrated Loads

Once the influence line of a function has been established for a point in a structure, the maximum effect caused by a live concentrated force is determined by multiplying the peak ordinate of the influence line by the magnitude of the force. In some cases, however, *several* concentrated forces must be placed on the structure. An example would be the wheel loadings of a truck or train. In order to determine the maximum effect in this case, either a trial-and-error procedure can be used or a method that is based on the change in the function that takes place as the load is moved. Each of these methods will now be explained specifically as it applies to shear and moment.

Shear. Consider the simply supported beam with the associated influence line for the shear at point C in Fig. 6–27*a*. The maximum *positive shear* at point C is to be determined due to the series of concentrated (wheel) loads which move from right to left over the beam. The critical loading will occur when one of the loads is placed *just to the right* of point C, which is coincident with the positive peak of the influence line. By trial and error each of three possible cases can therefore be investigated, Fig. 6–27*b*. We have

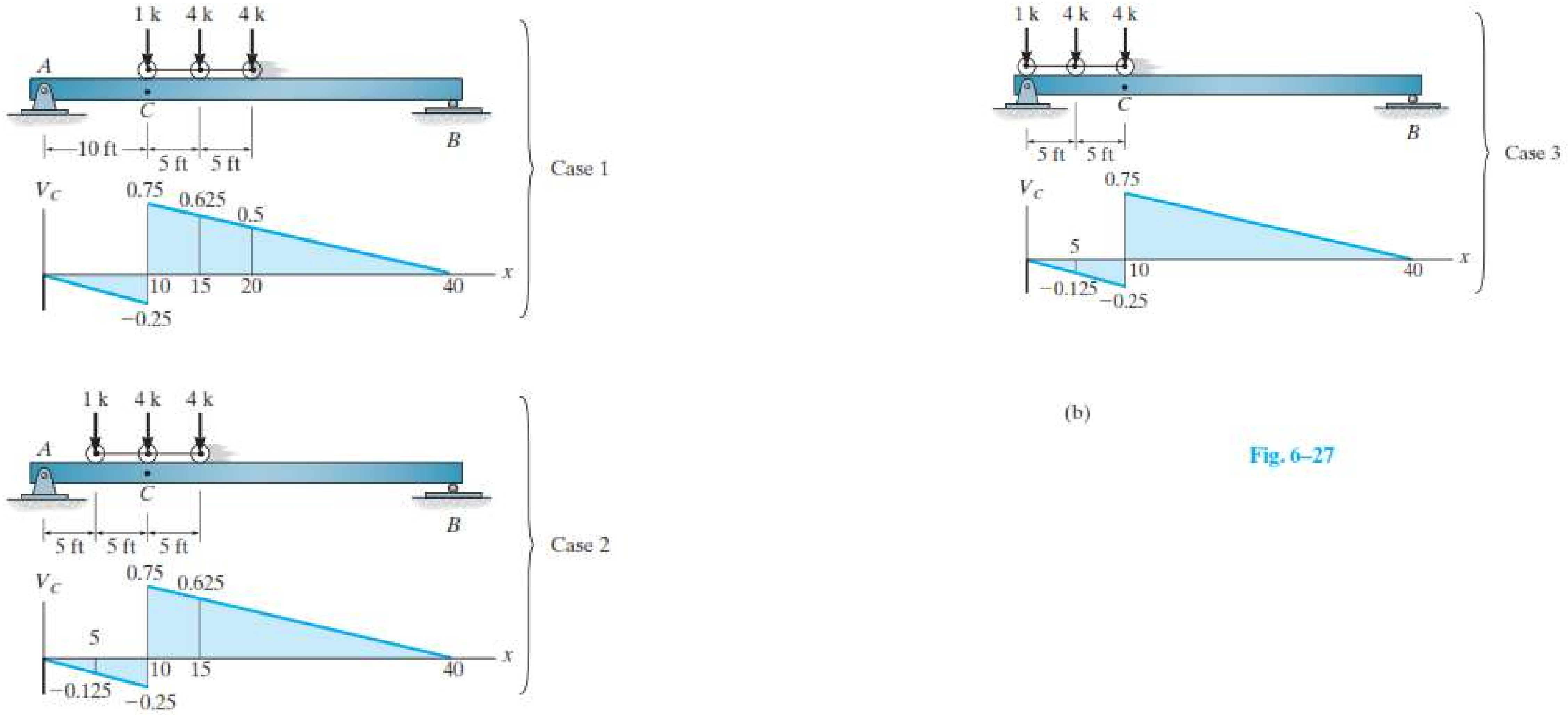
$$\text{Case 1:}\quad (V_C)_1 = 1(0.75) + 4(0.625) + 4(0.5) = 5.25\text{ k}$$
$$\text{Case 2:}\quad (V_C)_2 = 1(-0.125) + 4(0.75) + 4(0.625) = 5.375\text{ k}$$
$$\text{Case 3:}\quad (V_C)_3 = 1(0) + 4(-0.125) + 4(0.75) = 2.5\text{ k}$$



influence line for V_C
(a)

Fig. 6–27

Case 2, with the 1-k force located 5^{+} ft from the left support, yields the largest value for V_C and therefore represents the critical loading. Actually investigation of Case 3 is unnecessary, since by inspection such an arrangement of loads would yield a value of $(V_C)_3$ that would be less than $(V_C)_2$.



(b)

Fig. 6–27

When many concentrated loads act on the span, as in the case of the E-72 load of Fig. 1–11, the trial-and-error computations used above can be tedious. Instead, the critical position of the loads can be determined in a more direct manner by finding the change in shear, ΔV, which occurs when the loads are moved from Case 1 to Case 2, then from Case 2 to Case 3, and so on. As long as each computed ΔV is *positive*, the new position will yield a larger shear in the beam at C than the previous position. Each movement is investigated until a negative change in shear is computed. When this occurs, the previous position of the loads will give the critical value. The change in shear ΔV for a load P that moves from position x_1 to x_2 over a beam can be determined by multiplying P by the change in the ordinate of the influence line, that is, $(y_2 - y_1)$. If the slope of the influence line is s, then $(y_2 - y_1) = s(x_2 - x_1)$, and therefore

$$\boxed{\begin{array}{c}\Delta V = Ps(x_2 - x_1)\\ \text{Sloping Line}\end{array}} \tag{6–1}$$

If the load moves past a point where there is a discontinuity or "jump" in the influence line, as point C in Fig. 6–27a, then the change in shear is simply

$$\boxed{\begin{array}{c}\Delta V = P(y_2 - y_1)\\ \text{Jump}\end{array}} \tag{6–2}$$

The girders of this bridge must resist the maximum moment caused by the weight of this jet plane as it passes over it.

Moment. We can also use the foregoing methods to determine the critical position of a series of concentrated forces so that they create the largest internal moment at a specific point in a structure. Of course, it is first necessary to draw the influence line for the moment at the point and determine the slopes s of its line segments. For a horizontal movement $(x_2 - x_1)$ of a concentrated force P, the change in moment, ΔM, is equivalent to the magnitude of the force times the change in the influence-line ordinate under the load, that is,

$$\boxed{\begin{array}{c}\Delta M = Ps(x_2 - x_1)\\ \text{Sloping Line}\end{array}} \qquad (6\text{–}3)$$

As an example, consider the beam, loading, and influence line for the moment at point C in Fig. 6–29*a*. If each of the three concentrated forces is placed on the beam, coincident with the peak of the influence line, we will obtain the greatest influence from each force. The three cases of loading are shown in Fig. 6–29*b*. When the loads of Case 1 are moved 4 ft to the left to Case 2, it is observed that the 2-k load *decreases* ΔM_{1-2}, since the *slope* (7.5/10) is *downward*, Fig. 6–29*a*. Likewise, the 4-k and 3-k forces cause an *increase* of ΔM_{1-2}, since the *slope* [7.5/(40 − 10)] is *upward*. We have

$$\Delta M_{1-2} = -2\left(\frac{7.5}{10}\right)(4) + (4 + 3)\left(\frac{7.5}{40 - 10}\right)(4) = 1.0\ \text{k}\cdot\text{ft}$$

Since ΔM_{1-2} is positive, we must further investigate moving the loads 6 ft from Case 2 to Case 3.

$$\Delta M_{2-3} = -(2 + 4)\left(\frac{7.5}{10}\right)(6) + 3\left(\frac{7.5}{40 - 10}\right)(6) = -22.5\ \text{k}\cdot\text{ft}$$

Here the change is negative, so the greatest moment at C will occur when the beam is loaded as shown in Case 2, Fig. 6–29*c*. The maximum moment at C is therefore

$$(M_C)_{\max} = 2(4.5) + 4(7.5) + 3(6.0) = 57.0\ \text{k}\cdot\text{ft}$$

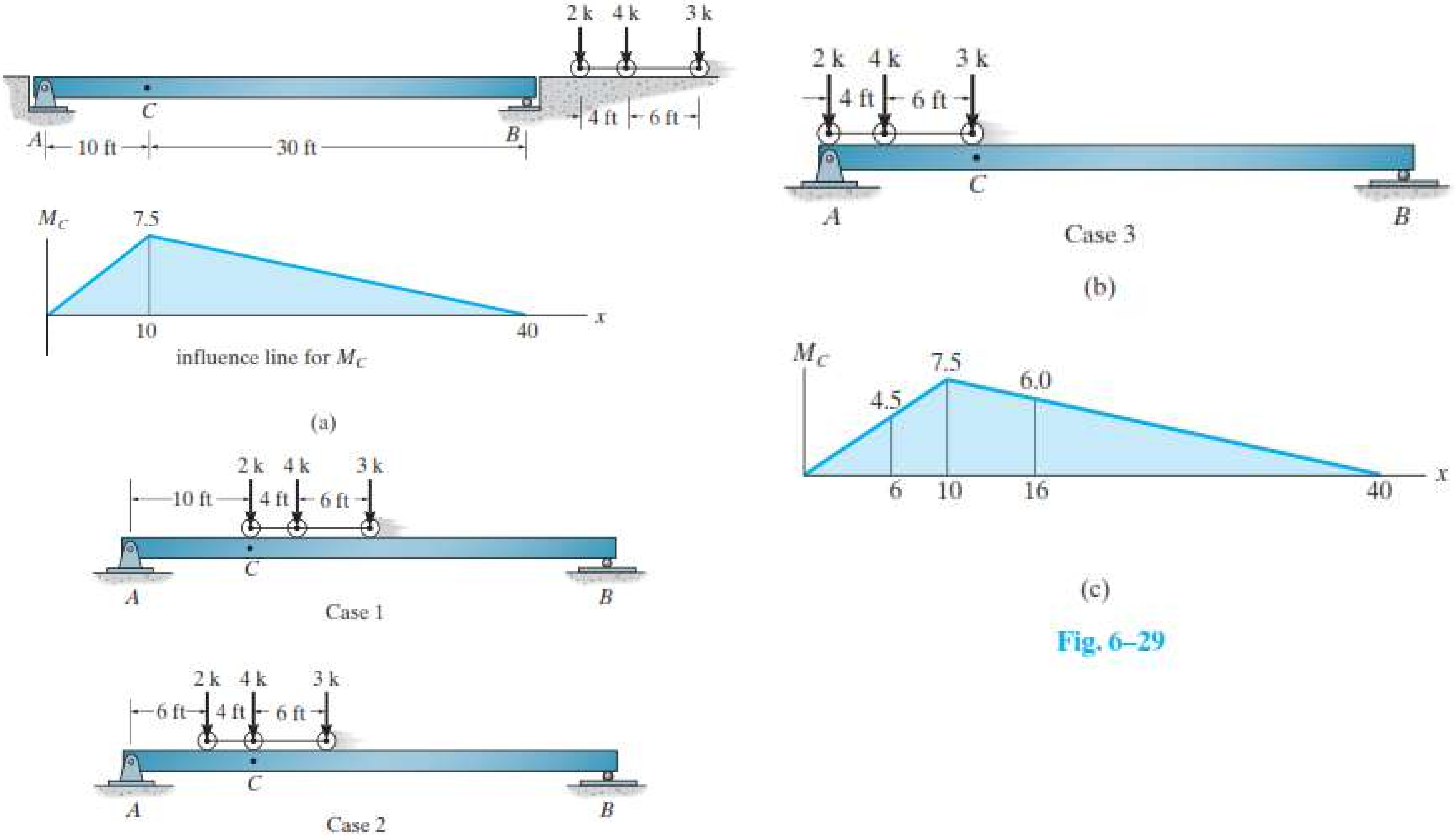
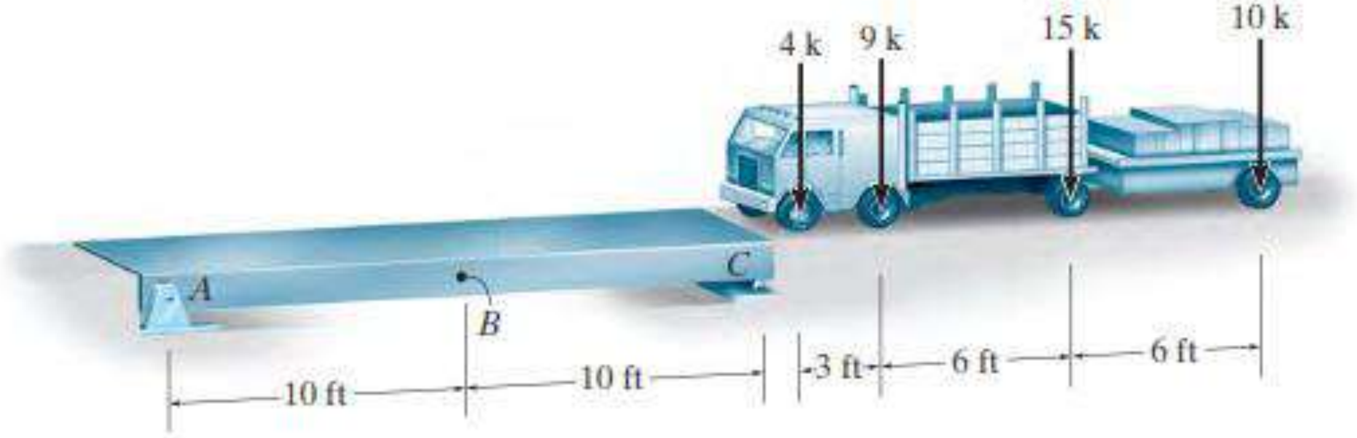

2 k
4 k
3 k
C
4 ft
6 ft
A
10 ft
30 ft
B
M_C
7.5
10
40
x
influence line for M_C
(a)
2 k
4 k
3 k
10 ft
4 ft
6 ft
C
A
B
Case 1
2 k
4 k
3 k
6 ft
4 ft
6 ft
C
A
B
Case 2
2 k
4 k
3 k
4 ft
6 ft
C
A
B
Case 3
(b)
M_C
7.5
6.0
4.5
6
10
16
40
x
(c)

Fig. 6–29

EXAMPLE 6.18

Determine the maximum positive shear created at point B in the beam shown in Fig. 6–30*a* due to the wheel loads of the moving truck.

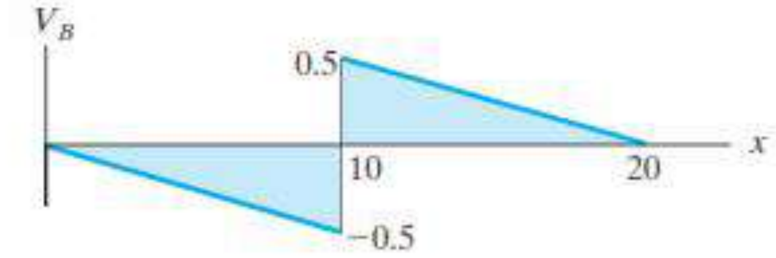


(a)

Fig. 6–30

SOLUTION

The influence line for the shear at B is shown in Fig. 6–30*b*.



influence line for V_B
(b)

3-ft Movement of 4-k Load. Imagine that the 4-k load acts just to the right of point B so that we obtain its maximum positive influence. Since the beam segment BC is 10 ft long, the 10-k load is not as yet on the beam. When the truck moves 3 ft to the left, the 4-k load jumps *downward* on the influence line 1 unit and the 4-k, 9-k, and 15-k loads create a positive increase in ΔV_B, since the slope is upward to the left. Although the 10-k load also moves forward 3 ft, it is still not on the beam. Thus,

$$\Delta V_B = 4(-1) + (4 + 9 + 15)\left(\frac{0.5}{10}\right)3 = +0.2 \text{ k}$$

6-ft Movement of 9-k Load. When the 9-k load acts just to the right of B, and then the truck moves 6 ft to the left, we have

$$\Delta V_B = 9(-1) + (4 + 9 + 15)\left(\frac{0.5}{10}\right)(6) + 10\left(\frac{0.5}{10}\right)(4) = +1.4 \text{ k}$$

Note in the calculation that the 10-k load only moves 4 ft on the beam.

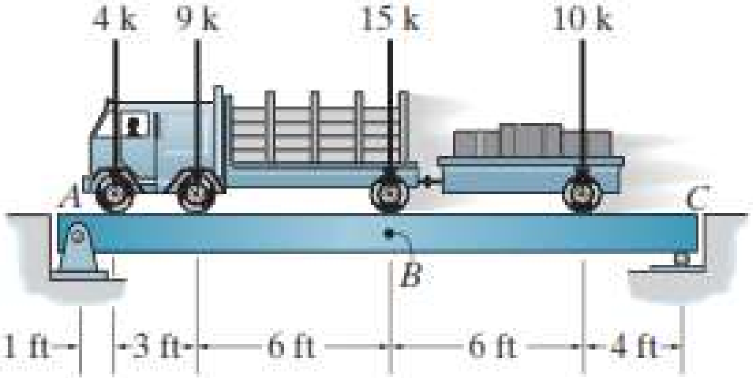
6-ft Movement of 15-k Load. If the 15-k load is positioned just to the right of B and then the truck moves 6 ft to the left, the 4-k load moves only 1 ft until it is off the beam, and likewise the 9-k load moves only 4 ft until it is off the beam. Hence,

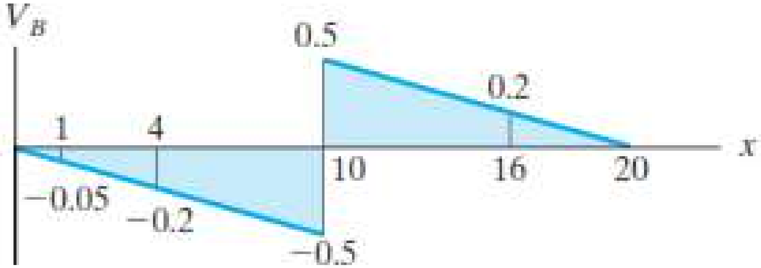
$$\Delta V_B = 15(-1) + 4\left(\frac{0.5}{10}\right)(1) + 9\left(\frac{0.5}{10}\right)(4) + (15 + 10)\left(\frac{0.5}{10}\right)(6)$$
$$= -5.5 \text{ k}$$

Since ΔV_B is now negative, the correct position of the loads occurs when the 15-k load is just to the right of point B, Fig. 6–30*c*. Consequently,

$$(V_B)_{\max} = 4(-0.05) + 9(-0.2) + 15(0.5) + 10(0.2)$$
$$= 7.5 \text{ k} \qquad \textit{Ans.}$$

In practice one also has to consider motion of the truck from left to right and then choose the maximum value between these two situations.





EXAMPLE 6.19

Determine the maximum positive moment created at point B in the beam shown in Fig. 6–31a due to the wheel loads of the crane.

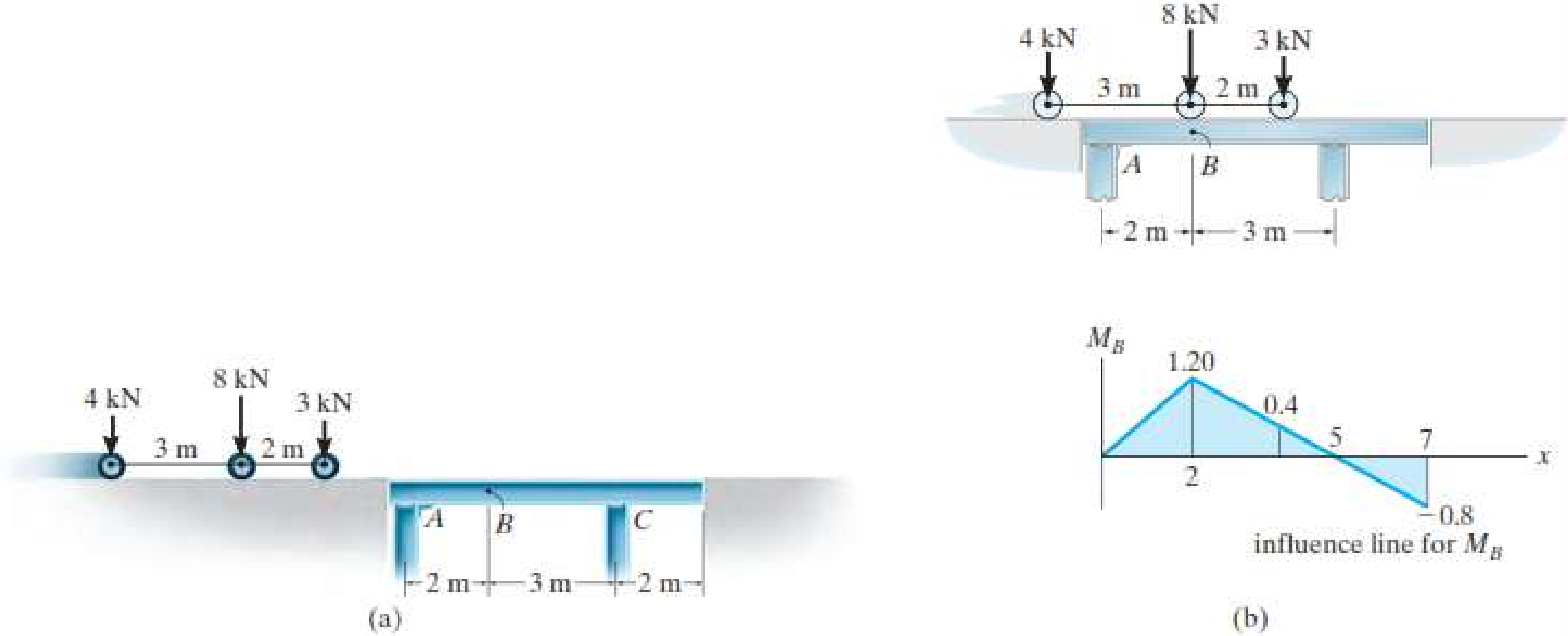


Fig. 6–31

SOLUTION

The influence line for the moment at B is shown in Fig. 6–31b.

2-m Movement of 3-kN Load. If the 3-kN load is assumed to act at B and then moves 2 m to the right, Fig. 6–31b, the change in moment is

$$\Delta M_B = -3\left(\frac{1.20}{3}\right)(2) + 8\left(\frac{1.20}{3}\right)(2) = 7.20\ \text{kN}\cdot\text{m}$$

Why is the 4-kN load not included in the calculations?

3-m Movement of 8-kN Load. If the 8-kN load is assumed to act at B and then moves 3 m to the right, the change in moment is

$$\begin{aligned}\Delta M_B &= -3\left(\frac{1.20}{3}\right)(3) - 8\left(\frac{1.20}{3}\right)(3) + 4\left(\frac{1.20}{2}\right)(2)\\ &= -8.40\ \text{kN}\cdot\text{m}\end{aligned}$$

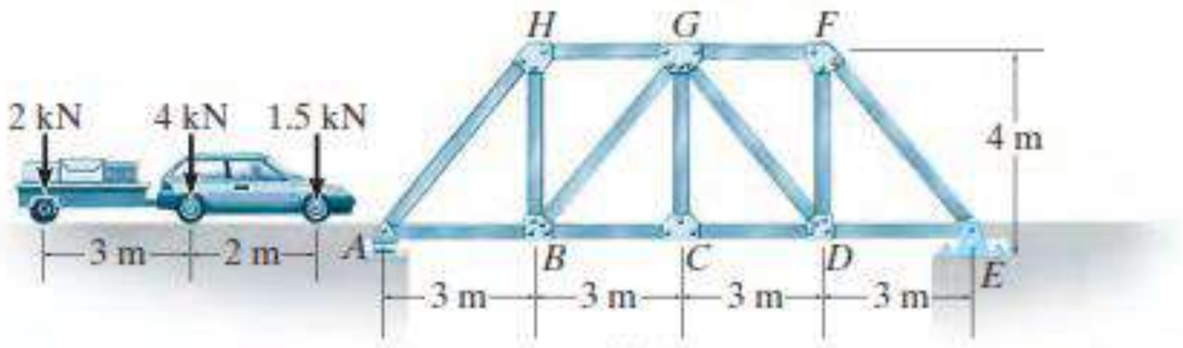
Notice here that the 4-kN load was initially 1 m off the beam, and therefore moves only 2 m on the beam.

Since there is a sign change in ΔM_B, the correct position of the loads for maximum positive moment at B occurs when the 8-kN force is at B, Fig. 6–31b. Therefore,

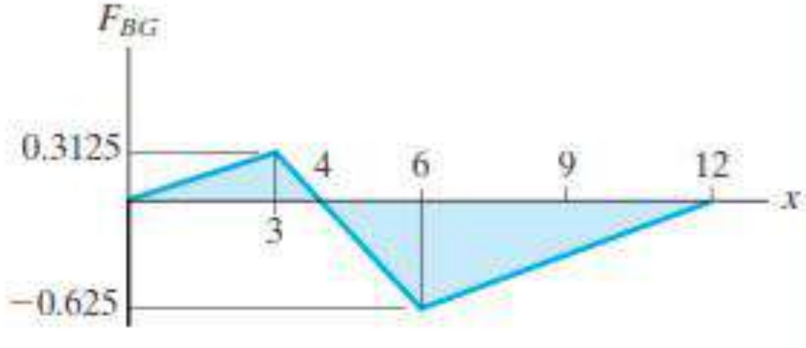
$$(M_B)_{\max} = 8(1.20) + 3(0.4) = 10.8\ \text{kN}\cdot\text{m} \qquad \textit{Ans.}$$

EXAMPLE 6.20

Determine the maximum compressive force developed in member *BG* of the side truss in Fig. 6–32*a* due to the right side wheel loads of the car and trailer. Assume the loads are applied directly to the truss and move only to the right.



(a)



influence line for F_{BG}

(b)

Fig. 6–32

SOLUTION

The influence line for the force in member *BG* is shown in Fig. 6–32*b*. Here a trial-and-error approach for the solution will be used. Since we want the greatest negative (compressive) force in *BG*, we begin as follows:

1.5-kN Load at Point C. In this case

$$F_{BG} = 1.5\text{ kN}(-0.625) + 4(0) + 2\text{ kN}\left(\frac{0.3125}{3\text{ m}}\right)(1\text{ m})$$

$$= -0.729\text{ kN}$$

4-kN Load at Point C. By inspection this would seem a more reasonable case than the previous one.

$$F_{BG} = 4\text{ kN}(-0.625) + 1.5\text{ kN}\left(\frac{-0.625}{6\text{ m}}\right)(4\text{ m}) + 2\text{ kN}(0.3125)$$

$$= -2.50\text{ kN}$$

2-kN Load at Point C. In this case all loads will create a compressive force in BC.

$$F_{BG} = 2\text{ kN}(-0.625) + 4\text{ kN}\left(\frac{-0.625}{6\text{ m}}\right)(3\text{ m}) + 1.5\text{ kN}\left(\frac{-0.625}{6\text{ m}}\right)(1\text{ m})$$

$$= -2.66\text{ kN} \qquad \textit{Ans.}$$

Since this final case results in the largest answer, the critical loading occurs when the 2-kN load is at C.

6.7 Absolute Maximum Shear and Moment

In Sec. 6–6 we developed the methods for computing the maximum shear and moment at a *specified point* in a beam due to a series of concentrated moving loads. A more general problem involves the determination of both the *location of the point* in the beam *and the position of the loading* on the beam so that one can obtain the *absolute maximum* shear and moment caused by the loads. If the beam is cantilevered or simply supported, this problem can be readily solved.

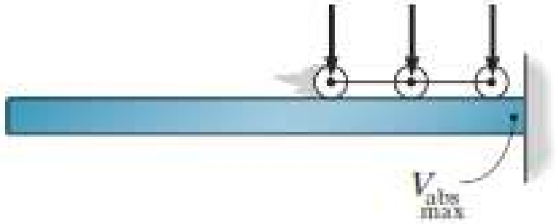


Fig. 6–33

Shear. For a *cantilevered beam* the absolute maximum shear will occur at a point located just next to the fixed support. The maximum shear is found by the method of sections, with the loads positioned anywhere on the span, Fig. 6–33.

For *simply supported beams* the absolute maximum shear will occur just next to one of the supports. For example, if the loads are equivalent, they are positioned so that the first one in sequence is placed close to the support, as in Fig. 6–34.

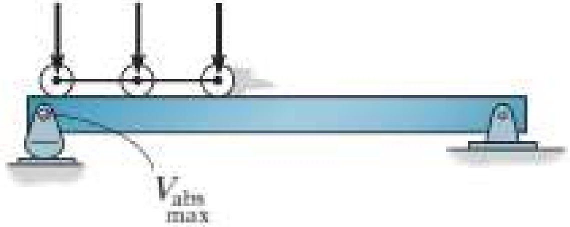


Fig. 6–34

Moment. The absolute maximum moment for a *cantilevered beam* occurs at the same point where absolute maximum shear occurs, although in this case the concentrated loads should be positioned at the *far end* of the beam, as in Fig. 6–35.

For a *simply supported beam* the critical position of the loads and the associated absolute maximum moment cannot, in general, be determined by inspection.

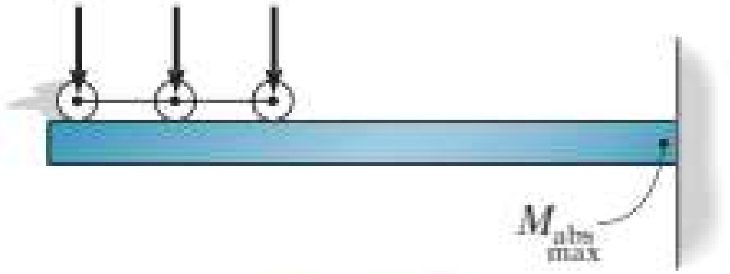


Fig. 6–35

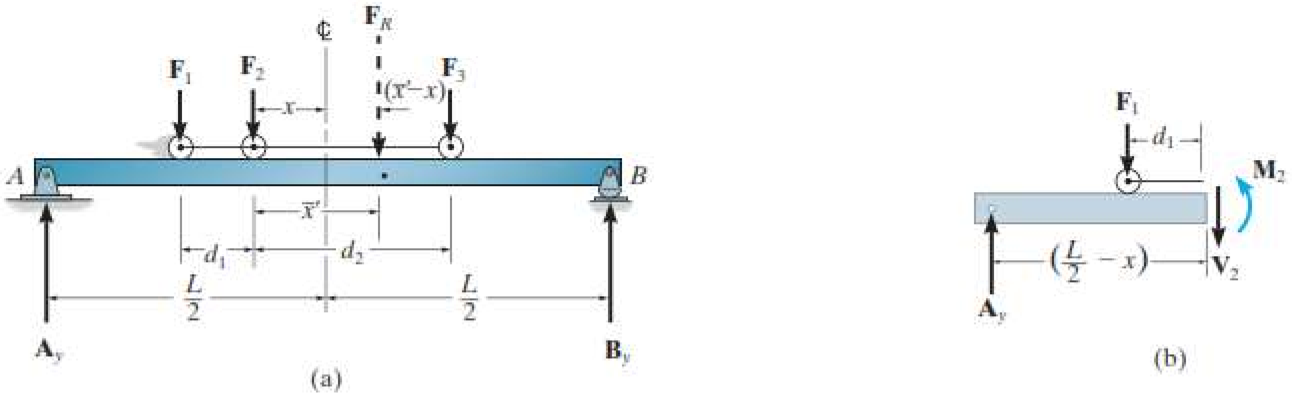
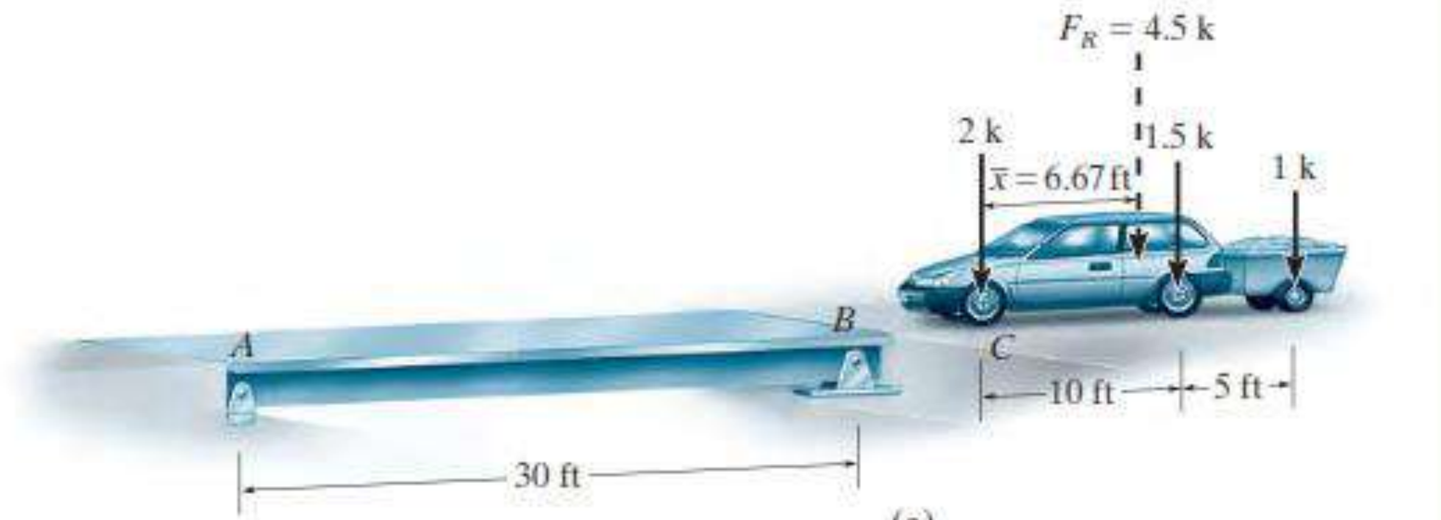


Fig. 6–36

Envelope of Maximum Influence-Line Values. Rules or formulations for determining the absolute maximum shear or moment are difficult to establish for beams supported in ways other than the cantilever or simple support discussed here. An elementary way to proceed to solve this problem, however, requires constructing influence lines for the shear or moment at selected points along the entire length of the beam and then computing the maximum shear or moment in the beam for each point using the methods of Sec. 6–6. These values when plotted yield an "envelope of maximums," from which both the absolute maximum value of shear or moment and its location can be found. Obviously, a computer solution for this problem is desirable for complicated situations, since the work can be rather tedious if carried out by hand calculations.

EXAMPLE 6.21

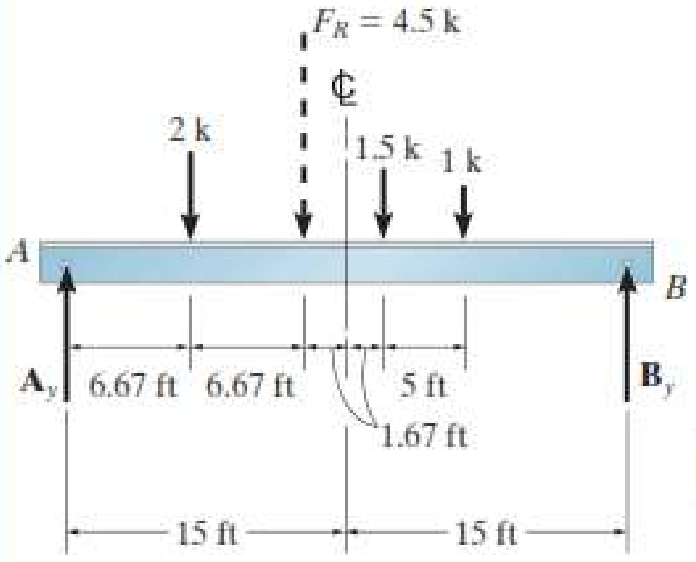
Determine the absolute maximum moment in the simply supported bridge deck shown in Fig. 6–37*a*.



(a)

SOLUTION

The magnitude and position of the resultant force of the system are determined first, Fig. 6–37*a*. We have



(b)

$+\downarrow F_R = \Sigma F;$ $\qquad F_R = 2 + 1.5 + 1 = 4.5$ k

$\curvearrowleft + M_{R_C} = \Sigma M_C;$ $\qquad 4.5\bar{x} = 1.5(10) + 1(15)$

$$\bar{x} = 6.67 \text{ ft}$$

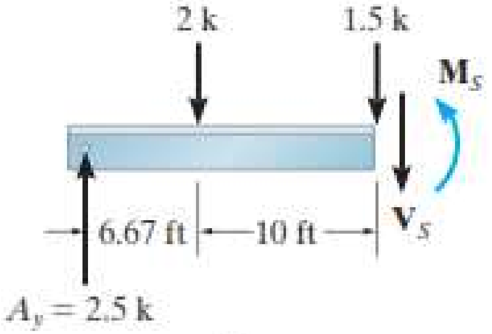
Let us first assume the absolute maximum moment occurs under the 1.5-k load. The load and the resultant force are positioned equidistant from the beam's centerline, Fig. 6–37*b*. Calculating A_y first, Fig. 6–37*b*, we have

$\curvearrowright + \Sigma M_B = 0;$ $\qquad -A_y(30) + 4.5(16.67) = 0 \qquad A_y = 2.50$ k

Now using the left section of the beam, Fig. 6–37*c*, yields

$\curvearrowright + \Sigma M_S = 0;$ $\qquad -2.50(16.67) + 2(10) + M_S = 0$

$$M_S = 21.7 \text{ k} \cdot \text{ft}$$



(c)

Fig. 6–37

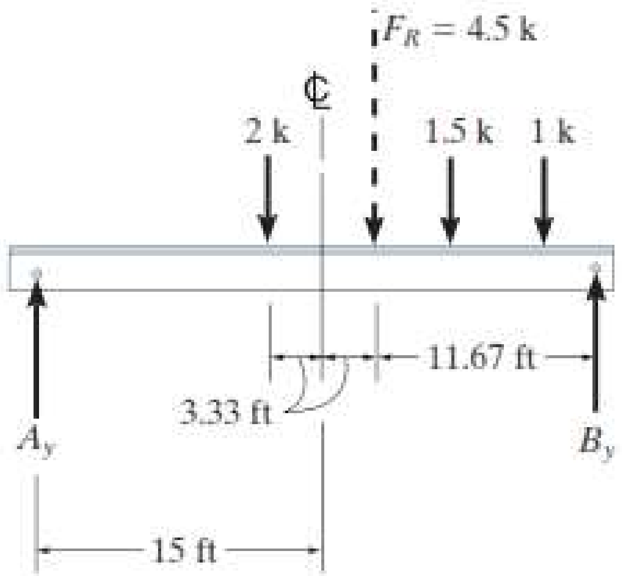
There is a possibility that the absolute maximum moment may occur under the 2-k load, since 2 k $>$ 1.5 k and $\mathbf{F}_R$ is between both 2 k and 1.5 k. To investigate this case, the 2-k load and $\mathbf{F}_R$ are positioned equidistant from the beam's centerline, Fig. 6–37*d*. Show that $A_y = 1.75$ k as indicated in Fig. 6–37*e* and that

$$M_S = 20.4 \text{ k} \cdot \text{ft}$$

By comparison, the absolute maximum moment is

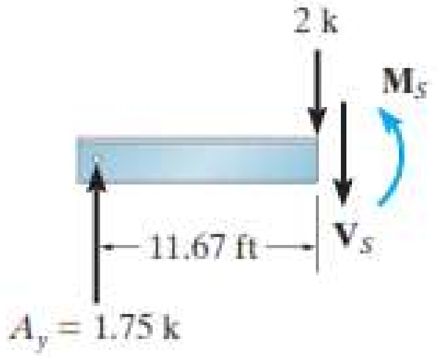
$$M_S = 21.7 \text{ k} \cdot \text{ft} \qquad \textit{Ans.}$$

which occurs under the 1.5-k load, when the loads are positioned on the beam as shown in Fig. 6–37*b*.



$F_R = 4.5$ k
℄
2 k
1.5 k
1 k
11.67 ft
3.33 ft
A_y
B_y
15 ft

(d)



2 k
$\mathbf{M}_S$
11.67 ft
$\mathbf{V}_S$
$A_y = 1.75$ k

EXAMPLE 6.22

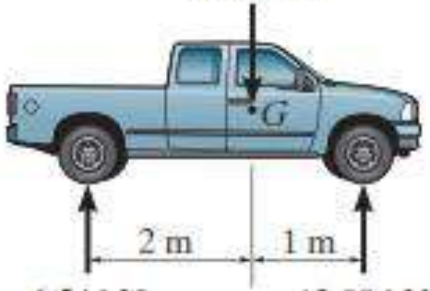
The truck has a mass of 2 Mg and a center of gravity at G as shown in Fig. 6–38a. Determine the absolute maximum moment developed in the simply supported bridge deck due to the truck's weight. The bridge has a length of 10 m.

SOLUTION

As noted in Fig. 6–38a, the weight of the truck, $2(10^3)\text{ kg}(9.81\text{ m/s}^2) = 19.62\text{ kN}$, and the wheel reactions have been calculated by statics. Since the largest reaction occurs at the front wheel, we will select this wheel along with the resultant force and position them *equidistant* from the centerline of the bridge, Fig. 6–38b. Using the resultant force rather than the wheel loads, the vertical reaction at B is then

$$\circlearrowright + \Sigma M_A = 0; \qquad B_y(10) - 19.62(4.5) = 0$$

$$B_y = 8.829\text{ kN}$$

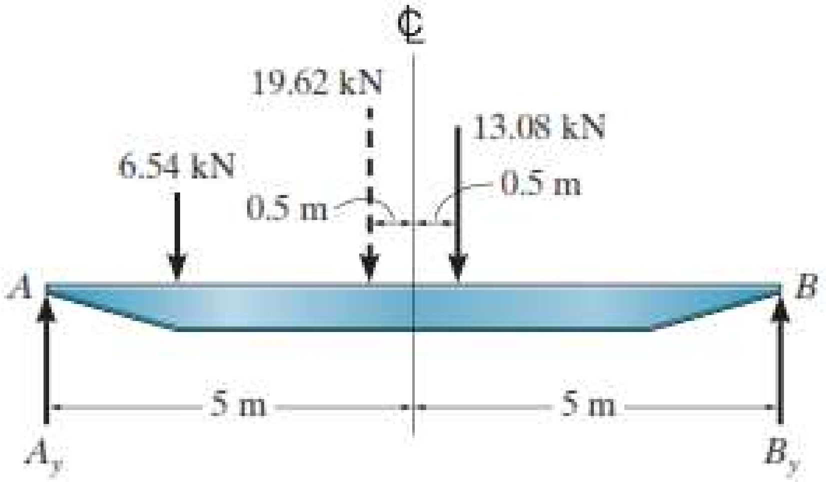


(a)

The maximum moment occurs under the front wheel loading. Using the right section of the bridge deck, Fig. 6–38c, we have

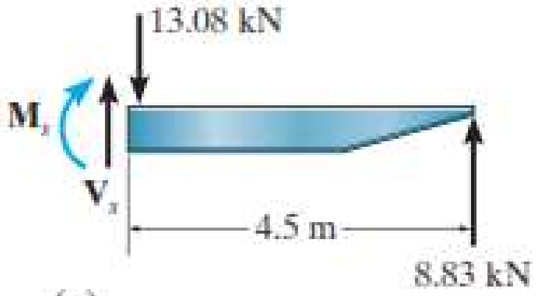
$$\circlearrowright + \Sigma M_s = 0; \qquad 8.829(4.5) - M_s = 0$$

$$M_s = 39.7\text{ kN}\cdot\text{m} \qquad \textit{Ans.}$$



19.62 kN
13.08 kN
6.54 kN
0.5 m
0.5 m
A
B
5 m
5 m
A_y
B_y

(b)



13.08 kN
$\mathbf{M}_s$
$\mathbf{V}_s$
4.5 m
8.83 kN

(c)

Fig. 6–38